FINITE DIFFERENCE METHOD FOR NUMERICAL SOLUTION OF A GENERALIZED BURGERS-HUXLEY EQUATION

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To my beloved parents, family and friends

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ABSTRACT

There are many applications of the generalized Burgers-Huxley equation which is a form of nonlinear Partial Differential Equation such as in the work of physicist which can effectively models the interaction between reaction mechanisms, convection effects and diffusion transports. This study investigates on the implementation of numerical method for solving the generalized Burgers-Huxley equation. The method is known as the Finite Difference Method which can be employed using several approaches and this work focuses on the Explicit Method, the Modified Local Crank-Nicolson (MLCN) Method and Nonstandard Finite Difference Schemes (NFDS). In order to use the NFDS, due to a lack of boundary condition provided in the problem, this research used the Forward Time Central Space (FTCS) Method to approximate the first step in time. Thomas Algorithm was applied for the methods that lead to a system of linear equation. Computer codes are provided for these methods using the MATLAB software. The results obtained are compared among the three methods with the exact solution for determining their accuracy. Results shows that NFDS has the lowest relative error and one of the best way among these three methods in order to solve the generalized Burgers-Huxley equations.

ABSTRAK

Terdapat banyak aplikasi persamaan umum Burgers-Huxley yang merupakan Persamaan Perbezaan Separa berbentuk linear seperti dalam kerja-kerja ahli fizik bagi memodelkan interaksi antara mekanisme tindak balas, kesan perolakan dan penyebaran mengangkut secara berkesan. Kajian ini mengenai pelaksanaan kaedah berangka untuk menyelesaikan persamaan umum Burgers-Huxley. Kaedah ini dikenali sebagai Kaedah Beza Terhingga yang menggunakan beberapa pendekatan dan kerja-kerja ini memberi tumpuan kepada Kaedah Explicit, Kaedah Tempatan Crank-Nicolson (MLCN) yang diubahsuai dan Skim Perbezaan Terhingga Tidak Standard (NFDS). Dalam usaha untuk menggunakan NFDS, disebabkan kekurangan keadaan sempadan yang diperuntukkan dalam masalah ini, kajian ini menggunakan Kaedah Masa Kehadapan Ruang Tengah (FTCS) untuk langkah pertama dalam masa. Thomas Algoritma telah digunakan untuk kaedah yang membawa kepada sistem persamaan linear. Kod komputer disediakan untuk kaedah ini menggunakan perisian MATLAB. Keputusan yang diperolehi dibandingkan antara tiga kaedah dengan penyelesaian tepat untuk menentukan ketepatan mereka. Hasil kajian menunjukkan bahawa NFDS mempunyai ralat relatif yang paling rendah dan salah satu cara yang terbaik di antara ketiga-tiga kaedah untuk menyelesaikan persamaan umum Burgers-Huxley.

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solution and Nonstandard Finite Difference Schemes, Explicit Method and Modified Local Crank Nicolson at $\Delta t = 0.001$.

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LIST OF SYMBOLS

Nomenclature

R	-	region

Greek Letters

∂	-	del
α	-	alpha, Advection Coefficient
β	-	beta, Coefficient of Reaction
δ	-	delta
γ	-	gamma
λ	-	lambda
ψ	-	psi
ξ	-	xi, Amplification Factor

LIST OF ABBREVIATIONS

PDE	Partial Differential Equation
ADM	Adomian Decomposition Method
DQM	Differential Quadrature Method
FDM	Finite Difference Method
ODE	Ordinary Differential Equation
NFDS	Nonstandard Finite Difference Scheme
MLCN	Modified Local Crank Nicolson
FTCS	Forward Time Central Space
SFDS	Standard Finite Difference Scheme
LCN	Local Crank Nicolson

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CHAPTER 1

INTRODUCTION

1.0 Research Background

Most of the science and engineering problems might encounter a single or a system of partial differential equations (PDE). This is a study of the generalized Burgers-Huxley equation for fluid dynamics. The generalized Burgers-Huxley equation is a nonlinear parabolic PDE that models various mechanisms in engineering field describing the interaction between convection, diffusion and reaction.

The generalized Burgers-Huxley equation is in the form:

$$\frac{\partial u}{\partial t} + \alpha u^{\delta} \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} - \beta u f(u) = 0, \tag{1.1}$$

for every $x \in I$ and every $t \geq 0$, where

$$f(u) = (1 - u^{\delta})(u^{\delta} - \gamma) \tag{1.2}$$

with the initial condition

$$u(x,0)=(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh[a_1 x])^{\frac{1}{\delta}},\ \text{for every } x\in l,$$

(1.3)

and the boundary conditions

$$u(a,t)=(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh[a_1(a-a_2t)])^{\frac{1}{\delta}},\ \text{for every } t\geq 0,$$

(1.4.a)

and

$$u(b,t)=(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh[a_1(b-a_2t)])^{\frac{1}{\delta}},\ \text{for every } t\geq 0,$$

(1.4.b)

where

$$a_1=\frac{-\alpha\delta+\delta\sqrt{\alpha^2+4\beta(1+\delta)}}{4(1+\delta)}\gamma,$$
$$a_2=\frac{\gamma\alpha}{1+\delta}-\frac{(1+\delta-\gamma)(-\alpha+\sqrt{\alpha^2+4\beta(1+\delta)})}{2(1+\delta)}$$

(1.5)

while α is the advection coefficient and β is the coefficient of reaction. The function (1.2) is the factor of nonlinear reaction. In this study, the parameters are α as a non-negative real number, $\beta\geq 0,\ \delta\geq 1,$ and $\gamma\in(0,1)$.

The role of the parameters on exact solutions was analyzed by Efimova and Kudryashov in [1]. The particular solution of (1.1) is

$$u(x,t)=(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh[a_1(x-a_2t)])^{\frac{1}{\delta}},\ \text{for every } x\in l \text{ and every } t\geq 0,$$

(1.6)

Nonlinear PDE are encountered in various fields of science. As stated by Sari and Gurarslan in [2], the generalized Burgers-Huxley equation as in equation (1.1) can be utilized to model the interaction between reaction mechanisms, convection effects and diffusion transports. Since there exists no general technique for finding analytical solutions of nonlinear diffusion equations so far, numerical solutions of nonlinear differential equations are of great importance in physical problems [3].

According to Sari and Gurarslan in [2] and Inan and Bahadir in [4], when $\alpha = 0$ and $\delta = 1$, (1.1) is reduced to the Huxley equation which describes nerve pulse propagation in nerve fibers and wall motion in liquid crystals

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = \beta u(1-u)(u-\gamma). \tag{1.7}$$

When $\beta = 0$ and $\delta = 1$, (1.1) is reduced to the Burgers equation which describes the far field of wave propagation in nonlinear dissipative systems

$$\frac{\partial u}{\partial t} + \alpha u \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = 0. \tag{1.8}$$

It is known that nonlinear diffusion equations (1.7) and (1.8) plays important role in nonlinear physics. They are of special significance for studying nonlinear phenomena.

If we take $\alpha \neq 0,$ and $\beta \neq 0,$ (1.1) becomes generalized Burgers-Huxley equation:

$$\frac{\partial u}{\partial t} + \alpha u^{\delta} \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta u(1-u^{\delta})(u^{\delta} - \gamma). \tag{1.9}$$

This research will focus on solving the generalized Burgers-Huxley equation of the form (1.9) which can also be used to show a prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transport.

Many numerical methods have been proposed for approximating the solution of the generalized Burgers-Huxley equation as listed below. Ismail *et al.* in [5] and Hashim *et al.* in [6] solved the generalized Huxley, Burgers-Huxley and Burgers-Fisher equations by using the Adomian decomposition method (ADM). Likewise, Hashim *et al.* in [7] used the decomposition scheme obtained from the ADM yields an analytical solution in the form of a rapidly convergent series for the numerical solutions of the generalized Burgers-Huxley equation. The discrete ADM was applied to a fully implicit scheme of the generalized Burgers-Huxley equation by Al-Rozbayani in [8].

As stated in Javidi in [9, 10] presented methods for solving of the equation by using the collocation formula for calculating spectral differentiation matrix for Chebyshev-Gauss-Lobatto point. Spectral collocation method and Darvishi's preconditioning to solve the generalized Burgers-Huxley equation was used by Darvishi *et al*. in [11].

Whereas Batiha *et al.* in [12, 13] used the variational iteration method, which are based on the incorporation of a general Lagrange multiplier in the construction of correction functional for the equation. Numerical solutions of the equation was obtained using a polynomial differential quadrature method (DQM) by Sari and Gurarslan in [2]. DQM is an extension of finite difference method (FDM) for the highest order of finite difference scheme [14]. As stated by Mittal and Jiwari in [3], this method linearly sum up all the derivatives of a function at any location of the function values at a finite number of grid points, then the equation can be transformed into a set of ordinary differential equations (ODE) or a set of algebraic equations. The set of ODE or algebraic equations is then treated by standard numerical methods such as the implicit Runge-Kutta method in order to obtain the solutions.

For numerical solution of the equation, based on collocation method using radial basis functions, called Kansa's approach was used by Khattak in [15]. As Javidi and Golbabai in [16], presented the spectral collocation method using Chebyshev polynomials for spatial derivatives and fourth order Runge-Kutta method for time integration to solve the generalized Burgers-Huxley equation. El-Kady *et al.* in [17] proposed based on cardinal Chebyshev and Legendre basis functions with Galerkin method for solution of the equation. Chebyshev Wavelet collocation method for solving generalized Burgers-Huxley equation proposed by Celik in [18].

Another method as used by Biazar and Mohammadi in [19] is the differential transform method for solving of the equation. A fourth order finite-difference scheme in a two-time level recurrence relation was proposed for the equation by Bratsos in [20]. Dehghan *et al.* in [21] found a numerical solution of the generalized Burgers Huxley equation using three methods based on the interpolation scaling functions and the mixed collocation finite difference schemes. Whereas in [22] used Haar wavelet method for solving the equation. Mittal and Tripathi in [23] used the collocation of cubic B-splines method for numerical solution of the generalized Burgers-Fisher and generalized Burgers-Huxley equations.

The explicit exponential FDM was originally developed by Bhattacharya in [24] for solving the heat equation. Whereas Handschuh and Keith in [25] used exponential FDM for the solution of Burgers equation. The Korteweg-de Vries equation is solved by Bahadır in [26] using the exponential finite difference technique. Whereas implicit exponential FDM and fully implicit exponential FDM was applied to the Burgers equation in [27].

According to Sari and Gurarslan in [2] and Wang *et al.* in [28] investigated the solitary wave numerical solution of the generalized Burgers-Huxley equation and Estevez in [29] introduced non-classical symmetries and the singular manifold method on the modified Burgers and Burgers-Huxley equation.

In the past few years, various powerful mathematical methods have been used such as homotopy analysis method by Molabahramia and Khani in [30], the tanh-coth method by Wazwaz in [31], and Hopf-Cole transformation by Efimova and Kudryashov in [1] have been used in attempting to solve the generalized Burgers Huxley equation.

Generally, the closed form solution for most problems involving the nonlinear PDE are not easily obtained. This fact makes the scientists realize the importance of developing another alternative to approximate the solutions of these PDE. After years of researches, scientists therefore approximate the solution of the system of PDE by using numerical discretization techniques on some function values at certain discrete points, so-called grid points or mesh points. FDM is one of the most widely used in engineering problems.

FDM known as the simplest method where the functions are represented by their values at certain grid points and approximate the derivatives through differences in these values namely Newton’s forward difference, backward difference and central difference method for the first order and also the second order. The aim of this research is to solve the nonlinear generalized Burgers-Huxley equation using the nonstandard finite difference scheme (NFDS) that combine Newton's forward and central difference methods for time derivatives.

In this work, the NFDS is applied to solve numerically the nonlinear generalized Burgers-Huxley equation. The results will be compared with the explicit method and the modified local Crank-Nicolson (MLCN) method. The Newton's forward difference method is been used in the explicit method for the first order derivatives, whereas the Newton's central difference method is been employed for the second order derivative. In the MLCN method, the approximation use Newton's forward difference method for the time derivatives. In the discretization of the MLCN method, the average of the current time $k+1$ and the previous time k is taken as a collocation point where the spatial derivatives at those points are obtained by taking the average of Newton's central difference method at time k and $k+1$.

Some examples are presented to demonstrate the effectiveness of these methods to solve the equation. This research will show that the numerical solution of these methods are reasonably in good agreement with the exact solution.

1.1 Problem Statement

This study will described a numerical method known as Finite Difference Method (FDM) for solving the generalized Burgers-Huxley equation. The FDM can be employed in several ways such as the explicit method, the MLCN method and the NFDS. This research will solve the generalized Burgers-Huxley problem with these three methods, highlighting the NFDS. In order to use the NFDS, when there is a lack of boundary condition provided in the problem, this research will use the

Forward Time Central Space (FTCS) method to approximate the first step in time. The results obtained are compared among the three methods with the exact solution to imply on their accuracy. Thomas Algorithm is employed in these methods that lead to a system of linear equations. Computer codes are developed for these schemes using the MATLAB software.

1.2 Objectives of Study

This study is to achieve the following objectives:

1) To construct the NFDS using the FTCS method for the calculations at the first time step in solving the generalized Burgers-Huxley equation.
2) To employ the NFDS, the explicit method and the MLCN method for solving the generalized Burgers-Huxley problem.
3) To analyse the numerical results obtained by the three methods and compare its accuracy relative to the exact.
4) To present the stability analysis of the three methods.
5) To provide computer codes for the three methods by using MATLAB software.

1.3 Scope of Study

In this study, the main numerical approach that will be discussed is the NFDS. Also, the scope of the study will focus on solving the generalised Burgers-Huxley equation with Dirichlet's boundary conditions. However, a slight modification of this method is done by using the FTCS method to approximate first time step due to lack of boundary condition provided in the problem. The explicit method and MLCN method will be used for comparison on their accuracy. Next, MATLAB codes will be developed for the Thomas Algorithm in order to solve those three methods.

1.4 Significant of Study

The importance for this study is to provide numerical solution for the Burgers-Huxley equations using FDM in several different ways. The main method that is discussed in this thesis is NFDS which has been used in science and engineering fields in solving nonlinear PDEs and here, will be applied to the generalized Burgers-Huxley equations with the Dirichlet's boundary conditions. This research will also discuss and apply the explicit method and the MLCN method to solve the problem. Then, the accuracy of these three methods are determined by comparing each with the exact solution. For the methods that lead to a system of linear equations, Thomas Algorithm will be used. This research will provide numerical codes for all methods of solving the generalized Burgers-Huxley equations using the MATLAB software.

1.5 Thesis Organization

The organizations of this study are divided into five chapters. Those chapters will discussed about the generalized Burgers-Huxley equation that apply three types of FDM such as the NFDS, the explicit method and the MLCN method. In addition, Thomas Algorithm is applied in order to solve those three methods that leads to a system of linear equations.

Chapter 1 gives a brief description of the main problem that will be solved throughout the thesis. This chapter representing the backgrounds of the problem, the statement of the problem, the objectives and scope of the study and the significance of the study of the thesis.

The literature survey of the generalized Burgers-Huxley equations and available methods used to solve the problem is provided in Chapter 2, mostly from books, research papers and journals. Here a detail on the NFDS, the explicit method and the MLCN method and their characteristics are implemented.

Next in Chapter 3 presents the discretization of the steps involved in order to implement of the three methods numerically solve the generalized Burgers-Huxley equation. For the NFDS, this scheme is combined with the Newton's forward and central difference methods for time derivatives. However, due to the lack of boundary condition provided in the problem, the FTCS method is used to approximate first time step.

Another method employed is the explicit method which uses the Newton's forward difference method for finding the first order derivatives, whereas the Newton's central difference method is been employed for finding the second order derivatives. In the discretization of the MLCN method, the average of the current time $k+1$ and the previous time k is taken as a collocation point where the spatial derivatives at those points are obtained by taking the average of Newton's central difference method at time k and $k+1$.

The stability analysis of those three methods has also been illustrated in Chapter 3. The stability of each method is analysed by the Fourier series method, also known as Von Neumann's method.

Chapter 4 provides a problem that will be solved numerically using those three methods; the NFDS, the explicit method and the MLCN method. This study also develop computer codes for Thomas Algorithm which is implemented for solving the generalized Burgers-Huxley equations by using MATLAB software. To measure the accuracy of the results, comparison with exact solution is made.

Last but not least, this research will also provide a conclusion for the overall of the thesis and also provide some recommendations for future research in Chapter 5.

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