FRACTAL THEORY OF A PROPAGATING CRACK IN AUSTENITIC STAINLESS STEEL

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To

My beloved family and all my lecturers and friends

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ABSTRACT

Classical fracture mechanics have limitation when it comes to solving real world applications. Factors such as material properties, probabilistic aspects make it difficult for classical fracture mechanics to be used in fatigue life prediction on working components. The introduction of fractal theory provides a better alternative tool for fatigue life prediction. Therefore, this research aims to investigate the relationship between stress intensity factor range, ΔK and local fractal dimension on the crack surface, D_f. Compact tension (C(T)) specimen was used for fatigue crack growth rate test in accordance to ASTM E647 to obtain crack length against number of cycles i.e. "a vs N" curve. In the constant growth rate stage ranging from 2.0 (10^{-8}) to 2.5 (10^{-7}) m/cycle, the crack growth rate behavior can be represented by Paris Law equation with coefficient of 3.0 (10^{-12}) and 3.2851. This linear region ranging from ΔK 16 MPa√m to 28 MPa√m will be considered for fractal dimension evaluation. Fractal dimension evaluation was conducted using the Box-counting method. The process was made for different sampling sizes, Δx to find the optimum range for this method. Results have shown that for every sampling size, increment of D_f is fairly consistent with the value of 0.065. We can also deduce that a correlation can be made between D_f and ΔK where a linearly increasing relationship was obtained. This shows that the crack tip driving force leaves behind a local uniqueness on the crack surface which varies along the crack length. If was also found that sampling sizes ranging from 0.025mm to 0.055mm have achieved the best consistency for evaluating D_f. Fractal dimension for this range only varies from 1.7274 to 1.7293. It contributes to only 5% of the entire D_f range that was tested. Improvement to achieve a single value (lower percentage) for D_f could be possible if the mentioned recommendations will be considered for future works

ABSTRAK

Mekanik patah klasikal mempunyai kelemahannya apabila digunakan untuk menyelesai masalah dunia realiti. Antara factor-faktor bahan, dan kebarangkalian menjadikan mekanik patah klasikal ini sukar untuk membuat ramalan akan jangka hayat lesu sesuatu komponen. Pengenalan kepada teori fraktal boleh menjadikan ia sebagai alat alternative yg berkesan untuk membuat ramalan jangka hayat lesu ini. Oleh itu, penyelidikan ini bertujuan untuk menyelidik hubungan antara faktor tekanan keamatan, ΔK dan dimensi fraktal pada permukaan retak, D_f. Spesimen tegangan padat (C(T)) telah digunakan untuk eksperimen kadar perkembangan retak lesu berpandukan ASTM E647 untuk memperoleh graf panjang retak terhadap bilangan kitaran, graf "a vs N". Pada rantau kadar pertumbuhan yang berterusan bermula dari 2.0 (10^{-8}) hingga 2.5 (10^{-7}) m/cycle di mana ia boleh diwakili oleh persamaan Paris Law bersama dengan pekali 3.0 (10^{-12}) dan 3.2851. Rantau kadar pertumbuhan yang berterusan ini, ΔK dari 16 MPa√m hingga 28 MPa√m sahaja akan diambil kira untuk evaluasi dimensi fraktal. Evaluasi dimensi fraktal telah dilaksanakan dengan mengunakan kaedah *Box-counting.* Kaedah ini telah dilaksanakan untuk pelbagai saiz persampelan, Δx supaya dapat memperoleh Δx yang optimum. Keputusan telah tunjuk bahawa untuk setiap saiz persampelan, perbezaan D_f agak konsisten dengan jumlah 0.065. Kita boleh jugak membuat konklusi bahawa terdapat hubungan linear antara D_f dan ΔK. Kajian juga telah didapati bahawa Δx antara 0.025mm hingga 0.055mm telah mencapai konsistensi terbaik untuk memperoleh D_f. Fraktal dimension bagi jarak ini adalah antara 1.7274 hingga 1.7293. Ia hanya menyumbang kepada 5% daripada keseluruhan jangka D_f yang telah dikaji. Penambahbaikan boleh dibuat jika cadangan-cadangan yang telah disenarikan akan dipertimbangkan.

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LIST OF ABBREVIATIONS

C(T)	-	Compact Tension
FCGR	-	Fatigue Crack Growth Rate
ASTM	-	American Society of Testing of Material
EDM	-	Electrical Discharge Machining
SEM	-	Scanning Electron Microscopy
LEFM	-	Linear Elastic Fracture Mechanics

LIST OF SYMBOLS

D_f	-	Fractal dimension
ΔK	-	Stress intensity factor range
K_{IC}	-	Stress intensity factor
ΔK_{th}	-	Stress intensity factor threshold
E	-	Euclidean dimension
a	-	Crack length
α	-	Geometry factor
β	-	Thickness of specimen
W	-	Width of specimen

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CHAPTER 1

INTRODUCTION

1.1 Research Background

A crack is assumed in fracture mechanics to be a smooth traction free surface of stress with discontinuity in displacement field. This assumption is contradictory to the real nature of cracks, where crack surfaces are shows a high degree of irregularity. In some cases a crack can be idealized as a concentrated distribution of damage on the material. In such cases, a blunt crack would be a more realistic model. One reason of assuming a sharp crack is the singularity in the stress field at the crack tip. Even with this unrealistic characteristic the sharp crack stress solutions are useful if there is a very small plastic region around the crack tip embedded in the elastic stress filed (small-scale yielding). The assumption made that cracks are sharp creates fracture mechanics problems mathematically controllable and this is the main reason for this assumption in most of the literature of fracture mechanics.

Fractal theory appears to be a new approach in fracture mechanics as a physical aspect through mathematical expressions integrated into classical analysis. Fractal Geometry is a non-Euclidean geometry that uses non-integer dimension as a quantifier. Mathematicians have developed equations to quantify 1-dimensional, 2-dimensional and 3-dimensional objects. In the case of fracture analysis i.e. fracture surfaces, when

This proceeding study will integrate fractal theory into traditional fracture mechanics to incorporate the roughness of fracture surface rather than the treatment of smooth surface. The first part of this project is the review on basis governing equation of fractal fracture mechanics. In the second part, experimental evidence will be collected in laboratory test to proof that crack surface features contain some orderliness of self-similar characteristics. Then the correlation between material toughness and fractal dimension will be established to predict the fracture crack growth base on the crack pattern.

1.2 Statement of Research Problem

In its current form, fracture mechanics have too many limitations to accommodate factors such as material parameters, probability, that makes it difficult for fracture mechanics to be used as a basis to propel microscopic/mesoscopic approach to understand failure theory. It needs fundamental improvements to catch up with other engineering methods to be able to predict the fatigue life better. The idea here is to combine classical Fracture Mechanics with Fractal Mathematics. How could the crack on a mechanical structure/component be quantified using fractal fracture mechanics approach?

The aim to incorporate fractal dimension, D_f into classical fracture mechanics is to eliminate the limitations of classical fracture mechanics where there are many uncertainties because of real-world conditions are not purely ideal. The idea is when plot of D_f and ΔK is already established, an engineer goes to the crack, and then he takes a high-resolution image of it and then evaluate the fractal dimension of that crack for that particular material. Once D_f has been computed, the value will be used in the D_f vs ΔK to obtain the current stress intensity factor range. From there, the value of ΔK will be used to predict the fatigue crack growth rate, da/dN of the component. Figure 1.1 shows how the method of utilizing D_f will be used in real world applications where there is a crack subjected to some internal pressure causing a cyclic loading.

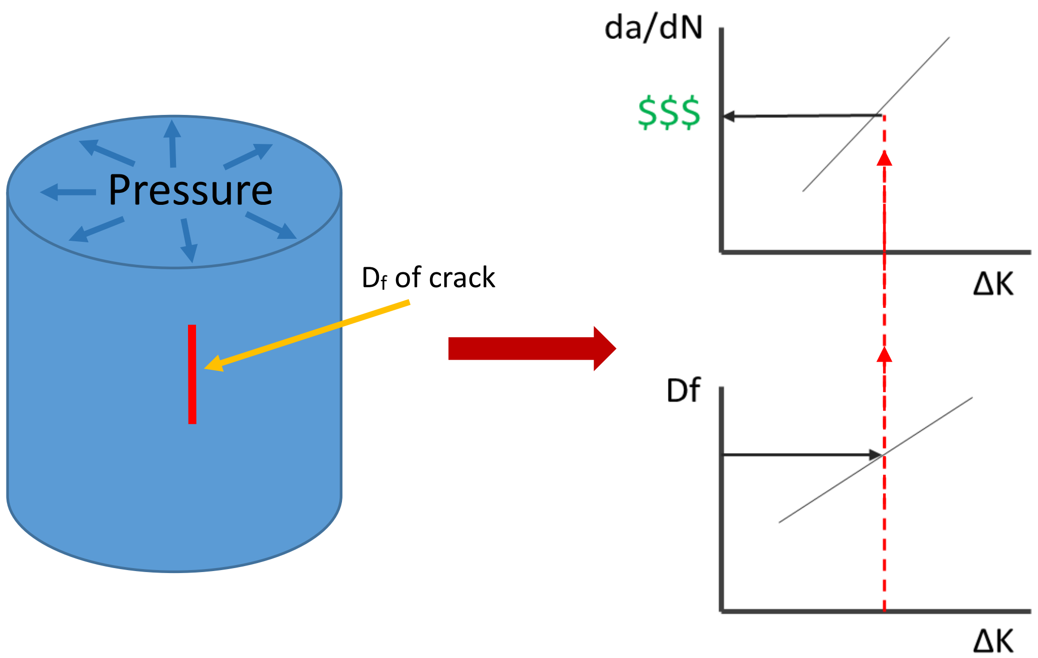

Figure 1.1: Brief illustration on how fractal dimension as a tool works.

1.2.1 Research Questions

1. Why classical fracture mechanics is not able to correctly predict fatigue crack propagation for Austenitic Stainless Steel during normal operating conditions?

2. Does cracks in austenitic steels exhibit fractal (self-similarity), and is it possible to model crack surface with non-Euclidean dimension as geometry factor?

3. How to quantify fractal dimension? Will it have a possibility to correlate fractal dimension with crack growth rate?

4. To what extent can this new theory improve the fatigue knowledge in engineering/science for Austenitic Stainless Steel during normal working conditions?

1.2.2 Research Hypothesis

To investigate the theory of fractal in fracture mechanics, some hypothesis are establish as follow:

- It is expected that crack in the Paris Law region will exhibit fractal characteristics.
- Fractal dimension can be correlated with stress intensity factor range to use in crack growth rate prediction.

1.3 Research Objectives

The main objectives to be achieved in this study is incorporating fractal theory into fracture mechanics as a parameter for crack growth rate prediction. Along with that, specific aims of this study are:

- To investigate whether crack exhibits fractal characteristics.
- To determine the fractal dimension of a Mode-1 crack in austenitic stainless steel.
- To correlate the fractal dimension with the applied stress intensity factor range through experimental work.
- To determine the optimum sampling size for fractal evaluation.

1.4 Scope of Research

The scope of study covers the followings:

1. Type 316 stainless steel will be the material of choice.

2. Mechanical and fracture properties will be established in accordance to ASTM standards or equivalent. Experimental works will be conducted at room temperature and laboratory air/ humidity environment.

3. Analytical review of fracture mechanics with the integration of fractal fracture approach.

4. Perform fatigue crack growth test to a specific crack length. Capture high-resolution images of the crack. Determine fractal dimension and the stress intensity factor range for that crack length. Repeat for a minimum of three crack lengths.

5. Deduce whether a correlation can be made between stress intensity range and fractal dimension.

6. The scope of this study is within the region of Linear Elastic Fracture Mechanics (LEFM).

1.5 Significance of Research

This research addresses various industrial sectors' strategic objectives. It includes achieving maximum plant useful life and cost/risk-focused decision making in regulation, operation, and design. This research also focuses on developing a methodology to address materials degradation/aging.The fractal approach can be a cost effective, less tedious method to predicting the life of a component/structure. Conventional method such as utilizing a strain gauge rosette may be appropriate for some cases but not all of them. Thus making fractal approach a more suitable alternative.

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