

MATHEMATICAL MODELING OF OIL POLLUTION DISSEMINATION IN
RIVER

NORSHELA BINTI MOHD NOH

A dissertation submitted in partial fulfilment
of the requirements for the award of the degree of
Master of Science (Engineering Mathematics)

Faculty of Science
University Teknologi Malaysia

JUNE 2014

For dearest,

my husband, Abu Samah Bin Ibrahim

my parents, Mohd Noh Bin Abdul Karim & Jamaliah Binti Salleh,

my lovely daughter, Nur Imtiaz

my siblings,

Norita, Mohd Farid, Mohd Hafiz, Nuraini

ACKNOWLEDGEMENT

First of all, thanks to Almighty Allah s.w.t. gives me opportunity to undertake this study. I take this opportunity to express my profound gratitude and deep regards to my guide Professor Madya Dr. Shamsuddin Bin Ahmad for his exemplary guidance, monitoring and constant encouragement throughout the course of this thesis. The blessing, help and guidance given by him time to time shall carry me a long way in the journey of life on which I am about to embark.

Lastly, I thank my husband, parents, brother, sisters and friends for their constant encouragement without which this assignment would not be possible.

ABSTRACT

In this study, analytical solutions are shown for the spreading of hydrocarbon oil in 2-dimensional river which in x and z coordinates. Both the conservative and non-conservative forms of the advection diffusion equation are solved analytically using the method of Laplace transform. The results of concentration are presented graphically. Results show that the conservative model concentration of hydrocarbon oil that spreads into the river is higher compared to the non-conservative model. The oil concentration is decreasing as distance increasing.

ABSTRAK

Dalam kajian ini , penyelesaian analisis ditunjukkan untuk proses penyebaran minyak haidrokarbon di dalam sungai secara dua dimensi iaitu x dan z koordinat. Bentuk konservatif dan bentuk bukan konservatif persamaan resapan adveksi diselesaikan secara analisis menggunakan kaedah perubahan Laplace. Keputusan kepekatan ditunjukkan secara grafik. Keputusan menunjukan bahawa model konservatif mempunyai kepekatan minyak haidrokarbon yang lebih tinggi berbanding model bukan konservatif. Kepekatan minyak itu akan terus berkurang apabila jarak semakin bertambah.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	TITLE	i
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENTS	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF FIGURES	x
	LIST OF ABBREVIATIONS	xi
	LIST OF SYMBOLS	xii
	LIST OF TABLES	xiv
	LIST OF APPENDICES	xv

1	INTRODUCTION	
1.0	Introduction	1
1.1	Background of the study	2
1.2	Statement of the study	3
1.3	Objectives of the study	3
1.4	Scope of the study	3
1.5	Significance of the study	4
1.6	Outline of the study	4
2	LITERATURE REVIEW	
2.0	Introduction	5
2.1	The Oil Pollution Dissemination in River Model	6
2.2	History of Mathematical Model Oil Pollution In River	7
2.3	The Fick's Law	8
2.4	Advection Diffusion Equation	11
2.5	Two dimensional conservative and non-conservative Advection Diffusion Equation	12
2.6	Solution of the Advection Diffusion Equation	13
2.7	Laplace Transform	14
3	FORMULATION OF THE PROBLEM	
3.0	Introduction	16
3.1	Research Instruments and Data Analysis	17
3.2	Mathematical Model	17
3.2.1	Advection	18
3.2.2	Diffusion	18

4	ADVECTION DIFFUSION MODEL	
4.0	Introduction	21
4.1	Advection Diffusion Model	22
4.2	Analytical approach	22
4.2.1	Analytical Solution for Conservative Model	23
4.2.2	Analytical solution for Non-Conservative Model	30
5	RESULT AND DISCUSSION	
5.1	Introduction	37
5.2	Results for the Models	38
5.2.1	Analytical Solution for Two-Dimensional Conservative Advection Diffusion Model	38
5.2.2	Analytical Solution for Two-dimensional Non-Conservative Advection Diffusion Model	40
6	CONCLUSION AND RECOMMENDATION	
6.0	Introduction	42
6.1	Summary	43
6.2	Conclusion	44
6.3	Suggestion for Future Research	44
	REFERENCES	45
	Appendices	49

LIST OF FIGURES

FIGURE	TITLE	PAGE
2.1	The control volume	9
5.1	Graph between oil concentration and distance in x and z direction for conservative model	38
5.2	Graph between oil concentration and distance in x and z direction for non-conservative model	40

LIST OF ABBREVIATIONS

Abbreviations		Significant
1D	-	One Dimensional
2D	-	Two Dimensional
ADE	-	Advection diffusion equation
ADM	-	Advection diffusion model
PDE	-	Partial differential equation

LIST OF SYMBOLS

Notations	Signification
C	- Hydrocarbon oil concentration
D, γ	- Diffusion coefficient
erfc	- Complement error function
H	- Mixing height of oil in z coordinates
M	- Mass of oil spilled
s	- Laplace parameter
t	- Time
u	- Speed of water current along x coordinates
w	- Speed of water current along z coordinates
w_0	- Speed of oils sludge sedimentation
x	- x coordinates axes
z	- z coordinates axes
Δx	- Change in x
δ	- Dirac delta function
σ	- Speed of admixture loss due to evaporation
γ_1	- Diffusion coefficient along x coordinate
γ_3	- Diffusion coefficient along z coordinate

Subscripts c

-

 p

-

 t

-

 x

-

 xx

-

Significant

Complimentary solution

Particular integral solution

First differentiate respect to t First differentiate respect to x Second differentiate respect to x

LIST OF TABLE

TABLE	TITLE	PAGE
5.1	Value of parameters for conservative model	39
5.2	Value of parameters for non-conservative model	41

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Laplace Transform	49
B	Error Function	51

CHAPTER 1

INTRODUCTION

1.0 Introduction

This chapter contains six sections which will be introduction of oil pollution dissemination in river, background of this study, the statement of this study, the objectives of this study, the scope of this study and the significance of this study. The focal point of this study is to learn mathematical model of oil pollution dissemination in river.

1.1 Background of the Study

Oil spilling in river and sea has become common and this is because of the rapid development of oil industry, especially seagoing oil transportations. For example, in the region of Caspian Sea, the existing reservoir for oil and gas has been renewed; new Baku-Supsa and Baku-Tbilisi-Ceyhan oil pipelines and Baku-Tbilisi-Erzurum gas pipeline have been constructed and have been put into operation. The volume of oil products from Tengiz (Kazakhstan) and Khanchagal (Azerbaijan) fields, transported by railway from Baku to the Black Sea coast, has increased (Begalishvili *et al*, 2012).

The activities mentioned above may cause the risk of emergency and catastrophic oil spilling increase drastically. The existence of oil spills in the river creates an unpleasant sight and odour as well as actual harm to marine life. The need for an early warning system to detect the violator and to initiate clean up procedures is brought out by the cost of cleanup in some recent oil spill cases where millions of dollars has been expended by major oil companies to cleanup various coastlines (Marshall, 1978). Thus an accurate prediction of oil spill is very important to minimize river damage due to unexpected oil spilling accident (Cho *et al*, 2012).

Developing a model to enable us to understand how to predict and control oil pollution dissemination is a significant interest. A lot of researches are carried out in order to provide information of oil spilling, the quality of river water and possible ways to improve. The model of oil pollution dissemination is useful for the simulation of accidental hydrocarbon oil release in river water as well as for the simulation or ordinary discharges.

1.2 Statement of the study

The quality of river become worst caused by emergency or catastrophic oil spill. In order to study oil pollution dissemination in river, we need to study a suitable mathematical model that represents the physical phenomenon. In this study, we formulate two-dimensional mathematical model of oil pollution dissemination in river.

1.3 Objectives of the study

The research objectives are:

- 1) To study the model of oil pollution dissemination in river
- 2) To formulate two-dimensional mathematical model of oil pollution dissemination in river.
- 3) To find the analytical solution of two-dimensional conservative advection diffusion equation for oil pollution dissemination in river.
- 4) To find the analytical solution of two-dimensional non-conservative advection diffusion equation for oil pollution dissemination in river.

1.4 Scope of the study

The scope of the study is to present Advection Diffusion Model (ADM) for two-dimensional conservative and non-conservative of oil pollution dissemination in river. Results will be shown in graphs by using Matlab software.

1.5 Significance of the study

The study of mathematical model of oil pollution dissemination in river is important because we can accurately predict the oil pollution movement. By knowing the spreading of oil we can give an idea of the oil spilling impact and a proper design for cleanup recovery operations and protect the ecological sensitive zone. In addition to the significance of this study, the management team can make quick decision making on policy of oil pollution on river.

1.6 Outline of the study

This study contains six chapters including introduction and conclusion. First chapter describes the background of this study, problem statements, objectives, scope and significance of this study.

Literature review of this study will be discussed in Chapter 2. This chapter contains the conceptual studies of oil pollution dissemination in river model, advection diffusion equation, history of oil pollution model. Derivation of the governing equation of oil pollution dissemination in river model will be discussed in Chapter 3. Results and data analyses will be done in Chapter 4. Finally, the conclusion and recommendation of this study will be discussed in chapter 5 and 6.

REFERENCES

- Banks, R. B., and Ali, J. (1964) Dispersion and Adsorption in Porous Media Flow. *J. Hydr. Div*, 90, 13-31
- Begalishvili, N. A., Tsintsadze, T., Begalishvili, N. N., Tsintsadze, N. (2012) Mathematical Modelling of Oil Pollution Dissemination in River and Sea Waters of the western Georgia. *Journal of Fisheries and Aquatic Sciences*. vol (12), 493-496
- Boxall, J. B., and Guymer, I. (2007) Longitudinal mixing in meandering channels,: new experimental dataset and verification of a predictive technique. *Water research* 41, 341-354.
- Brajesh, K. J., Neeru, A., and Mehta, M. N. (2012) Analytical solution of Two dimensional Advection Diffusion Equation Arising in Cytosolic Calcium Concentration Distribution. *International Mathematical forum*, vol.7 (3), 135-144
- Brannan, J. R., and Boyce, W. E. (2006) *The differential equations :An Introduction to modern & Application*. New Jersey, USA: John Wiley & Son.
- Bruce, H., Oyvind, B., and Cecille, W. (2006) Forecasting the drift of objects and substances in the ocean. Chassignet, E. P., Jacues, V.(Eds). *Ocean weather forecasting* (pp 507-523).Netherland: Springer.

- Cho, Y. S., Kim, T. K., Jeong, W., and Ha, T. (2012). Numerical simulation of oil spill in ocean. *Journal of Applied Mathematics*, 1-7.
- Govindaraju, R. S., and Bhabani S. D. (2007) *Moment Analysis for Subsurface Hydrologic Applications*. First Edition, Springer, Netherlands, 120-125
- Guerrero, J. S. P., Pimental, L. C. G., Skaggs, T. H. and Van Genuchten, M. T. (2009) Analytical solution of the advection-diffusion transport equation using a change of variable and integral transform technique. *International Journal of Heat and Mass Transfer* 52, 3297-3304
- Jaiswal, D. K., Kumar, A., and Yadav, R. R. (2011) Analytical solution to the one-dimensional Advection Diffusion Equation with temporally dependent coefficient. *Journal of water Resource and Protection*, vol (3), 76-84
- Jirka, G. H., and Weitbrecht, V. (2005) Mixing models for water quality management in rivers: continuous and instantaneous pollutant releases . In: Czernuszenko W, ROwinski PM (Eds). *Water Quality Hazards and dispersion of pollutants*. Springer, New York, pp 1-34.
- Joachim W. D. *Mathematical modelling of the transport of pollution in water*. Baltic Sea Research Institute Warnemunde, Germany. Encyclopedia of life support system.
- Kumar, A., Jaiswal, D. K. and Kumar, N. (2010) Analytical Solution of one dimensional Advection-diffusion equation with variable coefficients in a Finite Domain. *J. Earth Syst. Sci.*, volume 118(5) 539-549.
- Lee, T. C. (1999) *Applied Mathematics in Hydrogeology*. Boca Raton, Fla, USA:lewis.
- Mark, R., Oisten, J., Per J. B., Per, D., Alun, L., Robert, F., Don, M., and Richard, P. (1999) Oil spill Modeling towards the close of 20th century: overview of the state of the art. *Spill Science & Technology Bulletin*, Vol.50 (1), 3-16.
- Marshall, S. (1978) *Petroleum Transportation and Production, Oil Spill and Pollution Control*. New Jersey, USA: Noyes data corporation.

- Marsili-Libelli, S., and Giusti, E. (2008) Water quality modelling for small river basins. *Environmantel Modelling Software*. 23, 451-463.
- Massabo', M. Cianci, R. and Paladino, O. (2010) An Analytical Solution of The Advection Dispersion Equation in a Bounded Domain and its Application to Laboratory Experiments. *Hindawi Publishing Corporation Journal of Applied Mathematics*. 14
- Mazahari, M., Samani, J. M. V., and Samani, H. M. V. (2012). Analytical Solution to One Dimensional Advection-diffusion Equation with Several Point Sources through Arbitrary Time-dependent Emission Rate Patterns. *J. Agr. Sci. Tech.* (2013) vol 15, 1231-1245
- Nornadiah, K. (2012). Mathematical Modeling of Water Pollutant Transport.
- Nurul Syazwani, M. A. (2012). Analytical Solution Of Contaminant Transport in Streams.
- Pujol, L. I., and Sanchez-Cabeza, J. A. (2000). Use of tritium to predict soluble pollutants transport in Ebro River waters (Spain). *Environmental Pollution* 108, 257-269.
- Quang, A. D., Matthias, E., Gia, L. T. and Duc, L. (2011) *Mathematical Modelling and Numerical Algorithms for Simulation of Oil Pollution*. Springer science.
- Rowinski, P. M., Guymer, I., Bielonko, A., Napiorkowski, J. J., and Piotrowski, A. (2007). Large scale tracer study of mixing in a natural lowland river. Proceeding of the 32nd IAHR Congress, Venice, paper 297.
- Socolofsky, S. A., Jirka, G. H. (2002) *Environmental Fluid Mechanics, Mass transfer and diffusion*. Germany :
- Tyrell H. J. V. (1964). The origin and present Status of Fick's law diffusion. *Journal of Chemical Education*. 41, 397-400
- Wang S. D., Shen, Y. M., Zeng, Y. H. (2005) Two dimensional numerical solution for transport and fate of oil spills in seas, *Ocean Engineering*, 32 ,1556-1571

Wardley- Smith, J. (1976) *The Control of Oil Pollution on the sea and inland waters*.
Graham & Trotman Ltd, United Kingdom.

Yadav, R. R., Jaiswal, D. K., Gulrana and Yadav, H. K. (2010). Analytical solutions of one dimensional temporally dependent advection dispersion equation in Homogeneous Porous Media. *International Journal of Engineering, Science and Technology*. vol 2(5), 141-148.

Yadav, R. R., Jaiswal, D. K., (2011). Two-dimensional Analytical solutions for point source contaminants transport in semi-infinite Homogeneous Porous Medium. *Journal of engineering science and technology*. Vol 6 (4), 459-468.