PARAMETER ESTIMATION FOR GENERALIZED EXTREME VALUE AND GENERALIZED PARETO IN EXTREME RAINFALL ANALYSIS

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To my beloved father, mother, fiancé, brother and sister

Thank you for all your love and support

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ABSTRACT

Analyzing extreme rainfall events is important as the results found can be advantageous for civil engineers and town planners to estimate the strength of building under the most extreme conditions. The annual maximum series (AMS) data of daily rainfall in Malacca were fitted to two distributions, namely Generalized Extreme Value (GEV) and Generalized Pareto (GP) distribution, by using Method of Moments (MOM), Maximum Likelihood Estimator (MLE) and Bayesian Markov Chain Monte Carlo (MCMC) simulations. Previous studies have shown that the performance of the parameter estimations by using MOM was better than MLE. However, some researchers acknowledged that the parameter estimations of MLE can be additionally improved and developed by using Bayesian MCMC. To get posterior densities, non-informative and independent priors were used. The performance of the results were tested by several goodness of fit (GOF) tests. The results showed that GEV distribution is a better distribution than GP distribution in estimating the parameters of extreme daily rainfall amount in Malacca. MOM had maximum value return level for 10, 25, 50 and 100 years for most of stations followed by Bayesian MCMC. Conclusively, the results showed that Bayesian MCMC and MLE were equally better than MOM in estimating GEV and GP parameters.

ABSTRAK

Analisis peristiwa hujan melampau amat penting, memandangkan hasil kajian mampu membantu jurutera awam dan pakar runding untuk menjangka kebolehan struktur sesebuah bangunan untuk bertahan dalam situasi yang paling melampau. Siri maksimum tahunan (AMS) data hujan harian di Melaka telah disuaikan dengan menggunakan dua taburan, iaitu taburan nilai melampau teritlak (GEV) dan taburan Pareto teritlak (GP). Kajian ini dijalankan untuk menentukan kaedah terbaik untuk menganggarkan parameter bagi taburan GEV dan GP dengan menggunakan kaedah momen (MOM), penganggaran kebolehjadian maksimum (MLE) dan kaedah simulasi Markov Chain Monte Carlo (MCMC) Bayes. Sesetengah penyelidik mengenalpasti bahawa anggaran parameter MLE boleh ditambahbaik dengan menggunakan (MCMC) Bayes. Untuk mendapatkan taburan posterior, taburan prior tak-bermaklumat dan tak-bersandar digunakan. Prestasi anggaran parameter dengan MOM, MLE dan simulasi Bayes dibandingkan ujian kebagusan penyuaian (GOF). Taburan GEV merupakan taburan terbaik dalam menganggar parameter taburan hujan harian melampau di Melaka. MOM mempunyai tahap pulangan nilai maksimum selama 10, 25, 50 dan 100 tahun bagi kebanyakan stesen diikuti dengan MCMC Bayes. Hasil kajian daripada ujian tersebut menunjukkan bahawa kaedah simulasi MCMC Bayes dan MLE sama-sama lebih baik daripada MOM dalam menganggarkan GEV dan GP parameter.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Introduction

Malaysia, situated close to the north of the equator, is largely influenced by the northern hemisphere summer and winter monsoon. Hence, Malaysia's climate can be classified as equatorial, as it is hot and humid throughout the year. Hence, heavy rainfall is bound to happen in regular intervals every year, resulting in local tropical wet season. There are two monsoon winds seasons in Malaysia, which are Southwest Monsoon, which happens from late May to September and the Northeast Monsoon, which happens from November to March. The Northeast Monsoon, which originates from China and North Pacific, brings in more rainfall as compared to the Southwest Monsoon.

Rainfall is the quantity of rain fallen within a given area in a given time. Estimation of the rainfall distribution and identification of wet and dry events in a particular day is part of the rainfall process. There are 189 water basins in Malaysia and with an average rainfall of over 2000 – 4000 mm per year, Malaysia is liable to have flooding. Since 1962, there have been 15 cases of major flooding happening in Malaysia. Flooding is a major concern today due to rapid development in river catchment areas, which increase the river runoff and decrease the river capacity.

Extreme rainfall is estimated to take place regularly annually, as Malaysia is situated in a tropical climate zone. This results in a local tropical wet season. Extreme rainfall events is connected to climate change that is significant effect such as rising sea levels and rainfall. These rapid climate change leads to increasing flood events and large droughts in Malaysia. The unpredictable extreme rainfall phenomena has costs millions of Malaysian ringgit of damages to the government.

Occurrence of flood causes displacement of people, damaged infrastructures and losses in agricultural production, especially when there are inadequate measures taken. According to Baharuddin M K (2007), about 9% of the land area in Malaysia (approx. 2.97 million ha) is flood prone. It is difficult to estimate the cost of flood damage, but a conservative figure of RM 100 million has been used to estimate the average flood damage per year. This high losses in economic and agriculture sectors have significant impacts to the country development and these inspire researchers to investigate and model the rainfall process to comprehend the fluctuations and characteristics of rainfall previously.

Suitable statistical distributions, which gives the best inferences about the patterns of extreme rainfall, is needed to model extreme rainfall data. Many distributions are able to be used in analyzing the extreme rainfall analysis, such as Generalized Extreme Value (GEV) distribution (Kim *et. al*, 2013), Generalized Pareto (GP) distribution (Coles *et. al*, 2003) and Lognormal distribution (Adetan and Afullo, 2012). Studies, which represents the daily rainfall amount, that have been conducted in Malaysia includes the commonly used GEV, GP and Mixed Exponential. As Malaysia is situated in a tropical climate zone, the rainfall distributions is classified as extreme events. In this study, GEV and GP distributions will be used to fit the extreme daily rainfall in Malacca.

The rainfall data for Southern Peninsular Malaysia, Malacca had been modelled for the daily rainfall data in this study. Malacca experiences a tropical rainforest climate with Northeast Monsoon rain from November until March and Southwest Monsoon rain from late May to September. The mean annual rainfall in Malacca for 2013 is 1434.5 mm. Moreover, continuous heavy downpour had occurred in Malacca and Johor in December 2006, which has led to one of the worst floods in Malaysia, which is the 2006/2007 Malaysia floods. Floods with water levels as high as 10 feet (3.0 m) above the ground level had occurred in Muar and Segamat, which is very close to Malacca.

1.2 Background of the Study

Floods are one the most common hazard in many areas of the world, which includes Malaysia. Some floods develop slowly, while others can develop in a few minutes without visible signs of rain, like a flash flood. Moreover, floods can be local, which impact a neighborhood, community or on the other hand, it can be very large, which can affect multiple states.

In recent times, the increase of global temperatures has led to increase in the amount and intensity of rainfall in different regions. Activities such as deforestation due to illegal logging, clearance of land for various usage like agriculture, housing and industrial purposes had ruined the ecosystem. Therefore, analysis of rainfall behavior is most valuable for experts to be able to give reliable predicting about extreme weather events.

The recent events of a heavy flood that happened in Johor, Malaysia during mid-December 2006 was one of the most tragic natural disaster that happened. The flooding began when a continuous, torrential downpour since the week before caused rivers and dams to overflow. As a result of this, there is a loss of lives and properties, a decrease in purchasing and production power, mass migration and hindered economic growth in the country.

In this research, the study gives an understanding to the potential change in the events of extreme rainfall and dry spell for the past 34 years. By understanding the probability distribution, predicting of flood events and their characteristics can be determined. With this, prevention measures can be taken and flash flood warning models can be built easily. The study aims to benefits engineer in designing drainage, retaining walls, bridge, dam systems based on the expected rainfall amount over a certain period of time.

Extreme events are a major concern in statistical modeling and to derive estimations of the extreme properties of various phenomena from complex observations, appropriate statistical methods are needed (Raillard *et. al*, 2013). According to Ailliot *et. al* (2008), the data that is available is often limited to extreme values. Hence, fitting of an extreme value distribution can be difficult, especially for the estimation of extreme quantiles.

To estimate the parameters for GEV and GP distributions, this research conducted in Malaysia has focused on three methods, namely Method of Moments (MOM), Maximum Likelihood Estimator (MLE) and Bayesian inferences. The advantages of using MOM is that the method is fairly simple and produce consistent estimators under weak assumption. However, MOM can often be quite biased. Thus, MOM is often superseded by MLE as MLE has higher probability of being near to the estimated quantities and are often unbiased. Bayesian approach estimates parameters by using observations to update estimates unknown parameters.

According to Adetan and Afullo (2012), parameter estimation by means of moment-based techniques may give better estimation over MLE since the latter contributes to large deviation error as it is incapable to obtain parameter estimation for small sample. However, as compared to MLE, Bayesian approach has no particular advantage over it in the results of parameter estimation, as MLE is an all-around utility and adaptable to model change (Coles, 2001).

Smith (1985) showed that asymptotic properties related with MLE are violated when estimating GEV parameters from the assumption of a restricted parameter space. Moreover, MLE was incapable of obtaining estimates for small samples (Rao and Hamed, 2000). Alternatively, Bayesian approach is chosen to estimates the parameters of GEV and GP distributions. Bayesian approach has the ability to specify prior distributions, which means more information can be incorporated in an inference.

1.3 Problem Statement

As Malaysia is located in the tropical climate zone, extreme rainfall is expected to occur frequently. Hence, the trend of rainfall in Malaysia is different depending on the location, area and surrounding factors. At certain regions in the country, flash floods, droughts and landslides happened regularly. In Smith (2005), the inference on the extreme of environmental processes is essential for design-specification in civil engineering is explained.

Modelling any extreme environmental event is erratic undoubtedly. In modelling extreme rainfall data, suitable statistical distribution have been used for modelling data to give the best inferences of the behavior of extreme rainfall. For example, Gumbel, Generalized Extreme Value (GEV), generalized Pareto distribution (GPD), generalized logistic distribution (GLO), Pearson Type III and lognormal distribution are usually used to analyse extreme rainfall data. Several studies have been conducted have suggested that GEV and GP distributions are two suitable distributions to fit the annual extreme series of rainfall data.

The estimation of parameters by using Method of Moments (MOM) is worthy to be used when the sample size is modest. However, MOM is the oldest method of deriving point estimators and not necessarily the best method since MOM always produce some asymptotically unbiased estimators. MOM provides estimators, which are fairly consistent, but not as efficient as Maximum Likelihood Estimators (MLE). MLE arise as a flexible and powerful modelling tool, but its performance with smaller samples has been shown to be a poor alternative in relative to a fitting procedure based on probability weighted moments. Despite the fact that MLE had many advantages, its poor performance when dealing with small samples is improved by conducting Bayesian MCMC, especially after the discovery of Markov Chain Monte Carlo (MCMC) simulation, which makes complex computation easier (Coles, 2001).

Thus, the predicting of future rainfall distribution for different climate changes situations is needed to offer information for high quality climate related studies. In this study, the probability distribution for modelling regional data is of concerns to engineers and hydrologists. Moreover, information related to distributions of rainfall data is valuable towards designing water-related structures.

1.4 Study Objectives

1. To estimate parameters of Generalized Extreme Value (GEV) and Generalized Pareto (GP) distributions using Method of Moments (MOM), Maximum Likelihood Estimation (MLE) and Bayesian Markov Chain Monte Carlo (MCMC).
2. To compare the methods of Method of Moments (MOM), Maximum Likelihood Estimation (MLE) and Bayesian Markov Chain Monte Carlo (MCMC) used in estimating the parameters of Generalized Extreme Value (GEV) and Generalized Pareto (GP)
3. To evaluate the estimated parameters of GEV and GP distribution using Relative Mean Square Error (RRMSE) and Relative Absolute Square Error (RASE).
4. To predict the return levels values for the next 10, 25, 50 and 100 years based on 30-year history.

1.5 Scope of Study

Suitable statistical distribution will be needed to model extreme rainfall data. In this study, GEV and GP distribution are the most suitable distributions as it gives the best inferences of the behavior of extreme rainfall events (Zalina *et. al*, 2002 and Zin *et. al*, 2009). The daily rainfall of 4 rain gauge stations in Malacca which have records for 34 years from 1975 to 2008 is analyzed. The data of annual maximum rainfall data is then selected for each year.

Three methods, namely Method of Moments, Maximum Likelihood Estimation and Bayesian approach are applied to estimate the parameters of GEV and GP distribution. For the Bayesian approach, Markov Chain Monte Carlo simulation is used to estimate its parameter as suggested by Eli *et. al* (2012) and the simulation of MCMC is supported by R statistical software as suggested by Gamerman and Lopes (2006). To find the best model in this study, Relative Root Mean Square Error (RRMSE) and Relative Absolute Square Error (RASE) have been done. Finally, by using the parameters that have been estimated, the predicted return levels values for the next 10, 25, 50 and 100 years would be calculated based on the 30-years history annual maximum rainfall.

1.6 Significance of Study

Analysis of rainfall occurrence is significant to many as it is beneficial for managing the consumption of water, estimating the ability of building structures and also help predicting the extreme weather events. The analysis and information about rainfall can be used to evaluate and measure the danger of heavy downpour flood and landslides.

Moreover, civil engineers will be able to predict the practical material that should be used to design bridges drainage, retaining wall and dam systems from the information received. Besides, it is hoped that the study may help our country from unnecessary costs and economic damages as well as avoiding danger and hazard as a result of overflow of water in the country.

The predicted return levels values had been calculated by using the estimation of parameters by using MOM, MLE and Bayesian approach for the next 10, 25, 50 and 100 years which were evaluated based on 30-year history annual maximum rainfall. Therefore, the results enable civil engineers and town planners to estimate the ability of building structures which can survive under extreme events (Eli *et. al*, 2012).

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