

MATHEMATICAL MODELING OF TRANSIENT FLOW AND HEAT TRANSFER IN  
A PRODUCING GAS WELL

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To my late father Garba Wutama, Mother Khadija, wife, children and entire family.

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## ABSTRACT

This thesis focuses on the mathematical modeling of transient flow in a producing gas well. A producing gas well is a well that has enough pressure to push the gas from the reservoir to the wellhead. Most previous models considered steady state conditions with simplified governing equations that neglect some important parameters such as gas compressibility and gas flow propagations. This situation led to partial prediction or poor perception of the behavior of the flow characteristics, making it difficult to predict the productive capability of the producing well and could also lead to its early or premature closure. The present work incorporates a source term in the governing equations which take into account the inflow, well geometry, temperature difference and sound wave propagation. Other parameters considered include the thermal conductivity of the flow environment, the formation permeability, the inclination angles of well, and the diameter of the tubing, casing and wellbore. The governing equations comprising non-linear partial differential equations are solved numerically using the finite difference implicit Steger-Warming flux vector splitting method (FSM). Validation with an existing work has been carried out with a well-depth of 3000 m, wellhead and bottomhole pressure 9.5 of Mpa, well temperature 273 K, and mass flow rate of 9 kg/s. In determining the productive capability of the gas well, the inflow characteristics from reservoir to the bottom of the wellbore at different mass flow rates and inclinations of the well are considered. At reservoir pressure of 6000 Mpa, wellbore pressure 5800 Mpa, inlet velocity 40 m/s and productivity index of 20 Mpa, with the mass flow rate varying from 2 kg/s to 9 kg/s, and inclination angles taken to vary at 15° intervals, the inflow characteristics from the reservoir to the bottom of the wellbore are observed to be increasing with depth, which indicate that the gas well under consideration is capable of production. However, when the mass flow rate is less than 2 kg/s, the inflow pressure, temperature and velocity decrease indicating the inability of the well to produce. Temperature changes between the flowing fluid and formation interface are observed to influence the inflow characteristics which also depend on the outer diameter of tubing, casing and their thermal conductivities.

## ABSTRAK

Tesis ini memberi tumpuan kepada model matematik aliran sementara dalam telaga pengeluar gas. Telaga pengeluar gas merupakan telaga yang mempunyai tekanan yang cukup untuk menolak gas dari reservoir ke kepala telaga. Kebanyakan model sebelum ini mengambil kira keadaan mantap dengan persamaan menakluk yang lebih ringkas dengan mengabaikan beberapa parameter penting seperti kebolehmampatan gas dan perambatan aliran gas. Keadaan ini membawa kepada ramalan separa atau persepsi salah terhadap sifat-sifat aliran, yang boleh menyukarkan untuk meramal produktiviti telaga pengeluar dan boleh juga menyebabkan telaga pengeluar ditutup lebih awal sebelum masanya. Kajian ini mempertimbangkan sebutan sumber dalam persamaan menakluk yang mengambil kira aliran masuk, geometri telaga, perbezaan suhu dan perambatan gelombang bunyi. Parameter lain yang dipertimbangkan termasuk kekonduksian haba di persekitaran aliran, kebolehtelapan pembentukan, sudut kecondongan telaga dan garis pusat tiub, *casing* dan lubang telaga. Persamaan menakluk terdiri daripada persamaan pembezaan separa tak linear telah diselesaikan secara berangka menggunakan kaedah perbezaan terhingga tersirat *Steger-Warming flux vector* (FSM). Hasil kajian disahkan dengan kajian sedia pada kedalaman telaga 3000 m, tekanan kepala telaga dan dasar telaga 9.5 Mpa, suhu 273 K, kadar aliran jisim 9 kg/s. Dalam menentukan kemampuan produktif telaga gas, sifat-sifat aliran masuk dari reservoir ke lubang bawah telaga pada kadar aliran jisim dan kecondongan telaga yang berbeza diambil kira. Pada tekanan reservoir 6000 Mpa, tekanan lubang telaga 5800 Mpa, halaju masuk 40 m/s dan indeks produktiviti 20 Mpa, dengan kadar aliran jisim berubah dari 2 kg/s hingga 9 kg/s, dan sudut kecondongan berubah pada selang 15°, ciri-ciri aliran masuk dari reservoir ke bahagian bawah lubang telaga meningkat terhadap kedalaman telaga, yang menunjukkan bahawa telaga gas mempunyai keupayaan pengeluaran. Namun, apabila kadar aliran jisim kurang dari 2kg/s, tekanan aliran masuk, suhu dan halaju menurun menunjukkan telaga tidak berupaya mengeluarkan gas. Perubahan suhu antara aliran bendalir dan pembentukan antaramuka didapati mempengaruhi ciri-ciri aliran masuk yang bergantung juga kepada diameter luar tiub, *casing* dan kekonduksian haba.

## TABLE OF CONTENTS

<b>CHAPTER</b>	<b>TITLE</b>	<b>PAGE</b>
	<b>DECLARATION</b>	ii
	<b>DEDICATION</b>	iii
	<b>ACKNOWLEDGEMENT</b>	iv
	<b>ABSTRACT</b>	v
	<b>ABSTRAK</b>	vi
	<b>TABLE OF CONTENTS</b>	vii
	<b>LIST OF TABLES</b>	x
	<b>LIST OF FIGURES</b>	xi
	<b>LIST OF SYMBOLS</b>	xiv
	<b>LIST OF APPENDICES</b>	xvii
<b>1</b>	<b>INTRODUCTION</b>	<b>1</b>
	1.1 Research Background	1
	1.2 Statement of Problem	5
	1.3 Research Objectives	6
	1.4 Scope of the Research	7
	1.5 Significance of the Study	7
	1.6 Organization of Thesis	7
<b>2</b>	<b>LITERATURE REVIEW</b>	<b>9</b>
	2.1 Introduction	9
	2.2 Analytical Model	9
	2.3 Pressure Model in Producing Well	10
	2.4 Temperature Model in Producing Well	14
	2.5 Gas Flow Models	18
	2.6 Gas State Equation	21

2.7	Gas compressibility	21
2.8	Incline Well	22
2.9	Methods of Solution	24
<b>3</b>	<b>MATHEMATICAL FORMULATIONS</b>	<b>27</b>
3.1	Introduction	27
3.2	Derivation of Governing Equations	27
3.3	Continuity Equation	30
	3.3.1 Differential form of Continuity Equation	32
	3.3.2 Continuity Equation with Source Term	33
3.4	Momentum Equation	34
	3.4.1 Momentum Equation with Inclination	38
3.5	Energy Equation	41
	3.5.1 Energy Equation with Heat Transfer	44
3.6	Initial Conditions	47
3.7	Boundary Conditions	48
<b>4</b>	<b>SOLUTION PROCEDURE</b>	<b>50</b>
4.1	Introduction	50
4.2	Flux Vector Form	50
4.3	Flux Splitting Technique	52
4.4	Implicit Steger-Warming Procedure	54
	4.4.1 Linearization of FSM	57
	4.4.2 Tridiagonal Decomposition	59
4.5	Treatment of Boundary Conditions	60
4.6	Formula for Calculating Wellhead and Bottomhole	63
4.7	Analytical Solution	65
4.8	Stability Analysis	65
<b>5</b>	<b>PREDICTION OF TRANSIENT FLOW IN PRODUCING AND INJECTION WELLS</b>	<b>68</b>
5.1	Introduction	68
5.2	Results for Producing and Injection Wells	68
	5.2.1 Results Comparison Small and Large Flow Rate Conditions	69
	5.2.2 Wellhead Pressure in Production Well	74

5.2.3	Bottomhole Pressure in Injection Well	75
5.2.4	Temperature in Producing and Injection Well	76
5.2.5	Density and Velocity Profile in Producing Well.	78
5.3	Results for Producing Well with inflow and in Inclined Position	81
5.3.1	Pressure Comparison	82
5.3.2	Temperature Comparison	83
5.4	New Results	84
5.5	Results Comparison with and without Inflow for Producing Well	89
5.6	Summary	93
<b>6</b>	<b>TRANSIENT FLOW CHARACTERISTICS IN FULLY DEVELOPED FLOW</b>	<b>94</b>
6.1	Introduction	94
6.2	Results and Discussion	94
6.2.1	Temperature Comparison	95
6.2.2	Pressure Comparison	96
6.3	New Results for Thermal Conductivities, Inclination and Area Changes	97
6.4	Comparison of Present Model, FSM and Steady State	106
6.5	Stability of the Analysis	106
6.5	Summary	108
<b>7</b>	<b>SUMMARY AND CONCLUSION</b>	<b>109</b>
7.1	Summary of Work	109
7.2	Conclusion	113
7.3	Suggestion for Feature Work	114
	<b>REFERENCES</b>	<b>116</b>
	Appendices A - I	130- 179



**LIST OF TABLES**

<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE</b>
4.1	Control direction	64
5.1	Well parameters and fluid properties	82
5.2	Comparison of Steger-Warming flux vector splitting method with finite difference.	92
5.3	Computational time with respect to temperature	93
6.1	Producing well structure	107
6.2	Comparative results in respect of wellhead temperature and pressure	108

## LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Producing Gas well	2
1.2	Gas flaring (Climate Guest Contributor, 2011)	3
1.3	Sound Sensor to Detect Gas Reservoir (IAGC, 2014)	4
2.1	Rock permeability	23
2.2	Horizontal and vertical drilling	23
3.1	Moving control volume (Anderson, et al, 2009)	28
3.2	Model used for the derivation of momentum equation in x-component direction	35
3.3	Control Volume for gravitation (Jiuping et al., 2013)	38
3.4	Schematic diagram of producing gas well with heat transfer	44
3.5	Domain of Boundary Condition	49
4.1	A computational domain for equation (4.20), (Hoffman and Chain, 2000)	60
4.2	Computational Algorithm Chart	67
5.1	Pressure profile of injection well with small mass flow rate of 2.2135 kg/s condition	69
5.2	Pressure profile of injection well with different injection temperature under large flow rate of 9.057.2 kg/s condition.	70
5.3	Pressure profile of production well with different injection temperature under small mass flow rate of 2.2135 kg/s condition.	71
5.4	Pressure profile of production well with different injection temperature under large flow rate of 9.057.2 kg/s condition.	72
5.5	Variation of wellhead pressure with the increase of mass flow rate	73

5.6	Variation of bottom-hole pressure with the increase of mass flow rate	74
5.7	Wellhead pressure of in injection well under large flow rate of 9.0572kg/s	75
5.8	Bottomhole pressure in injection well with mass flow rate 9.0572kg/s	76
5.9	Temperature of injection well under large flow rate 9.0572kg/s	77
5.10	Temperature of production well under large flow rate 9.0572 kg/s	77
5.11	Density profile at different time	78
5.12	Velocity profile at different time	79
5.13	Temperature profile with geothermal gradient	80
5.14	Density profile with geothermal gradient at $t=0.5s$	80
5.15	Compressibility factor on density profiles.	81
5.16	Pressure comparison	83
5.17	Temperature comparism Steger-Warming and (Pinanm et al., 2003)	84
5.18	Inflow velocities at well pressure $P=5800 Mpa$	85
5.19	Temperature with or without inflow kinetic energy	86
5.20	Temperature with or without axial kinetic energy	86
5.21	Inflow temperature at reservoir pressure $P_R=6000 Mpa$	87
5.22	Well temperature at $P=5800 Mpa$	88
5.23	Inflow temperature at different time	89
5.24	Pressure profile with and without inflow	90
5.25	Temperature profile with and without inflow	90
5.26	Pressure comparisons of Steger Warming and Analytical Solutions	91
5.27	Velocity comparisons of Steger Warming and Analytical Solutions	92
6.1	Temperature distributions at different depth and time	95
6.2	Pressure distribution at different depth and time	96
6.3	Earth thermal conductivities at different time.	97
6.4	Thermal conductivity of cement on temperature distribution	98
6.5	Thermal conductivity of tube	99
6.6	Overall heat transfer on velocity	99
6.7	Pressure at different inclination angles	100
6.8	Velocity at different inclination angles	101
6.9	Density at different inclination angles	101
6.10	Temperature at $15^\circ$ inclination	102

6.11	Well temperature at different tubing diameter with different production time.	103
6.12	Well temperature at different casing outer diameter with production time.	103
6.13	Well temperature at different wellbore diameter with production time.	104
6.14	Sound wave profiles at the upper part of the well at Different times (min)	105
6.15	Sound wave profiles at the lower part of the well at different times (min)	105
6.16	Comparison of the present model, FSM and steady state	106
6.17	Eigenvalue at Z-plane used in (FSM) Computation	107

## LIST OF SYMBOLS

$A$	-	Area of Wellbore
$A_{LR}$	-	Relaxation Distance Coefficient
$a$	-	Sound Wave (Speed of Sound)
$C_p$	-	Specific Heat at Constant Pressure
$C_v$	-	Specific Heat at Constant Volume
$d$	-	Diameter
$f$	-	Fanning Friction Factor
$g$	-	Gravitational Acceleration
$h_f$	-	Fluid Film Coefficient
$h_r$	-	Radiation Heat Transfer Coefficient
$h_c$	-	Convection Heat Transfer Coefficient
$J$	-	Productivity Index
$K_{JT}$	-	Joule Thomson Coefficient
$k_e$	-	Thermal Conductivity of Earth
$k_{cem}$	-	Thermal Conductivity of Cement
$k_{cas}$	-	Thermal Conductivity of Casing
$k_{an}$	-	Thermal Conductivity of Annulus
$k_{tub}$	-	Thermal Conductivity of Tubing
$L$	-	Well Length
$r_{ti}$	-	Radius of Inner Tubing
$r_{to}$	-	Radius of Outer Tubing
$r_{ci}$	-	Radius of Inner Casing
$r_{co}$	-	Radius of Outer Casing
$r_w$	-	Radius of Wellbore
$R$	-	Natural Gas Constant
$R$	-	Reservoir Radius
$T$	-	Temperature
$T_f$	-	Temperature of Flowing Fluid

$T_{ei}$	-	Temperature of Undisturbed Earth Formation (initial)
$T_r$	-	Temperature of Radiation
$T_w$	-	Temperature of Wellbore
$t$	-	Time
$F(t_D)$	-	Dimensionless Function Time
$U_{to}$	-	Overall Heat Transfer Coefficient of Outer Tubing
$u_I$	-	Inflow Velocity
$u$	-	Fluid Velocity
$M$	-	Molar Mass
$\dot{M}_x$	-	Mass Flow Rate
$P$	-	Pressure
$x$	-	Space
$Z$	-	Compressibility Factor
$\theta$	-	Inclination Angles
$e$	-	Internal Energy
$\varepsilon$	-	Pipe Roughness
$\gamma_g$	-	Specific Heat of Gas
$\rho$	-	Density of Gas
$\rho_I$	-	Inflow Density
$\mu$	-	Viscosity
$\rho g$	-	Mass $\times$ Gravitational Acceleration
$\mathbf{V}$	-	Velocity Vector $\rightarrow \mathbf{V} = \mathbf{V}(x, y, z, t)$
$v$	-	Control Volume
$S$	-	Control Surface
$dS$	-	Change in Surface
$\nabla \cdot \mathbf{V}$	-	Time Rate of Change of the Volume of a Moving Fluid Element.
$\mathbf{n}$	-	Unit Vector Perpendicular to the Surface
$\nabla p$	-	Pressure Gradient
$dv$	-	Incremental Area inside the Finite Volume
$\nabla$	-	Gradient
$\nabla \bullet$	-	Divergence
$\nabla \times$	-	Curl
$\iint_s$	-	Integration over Surface

$\iiint_v$	-	Integration over Volume
$\rho \mathbf{V} \cdot d\mathbf{S}$	-	Positive $\rightarrow$ Outflow
$\rho \mathbf{V} \cdot d\mathbf{S}$	-	Negative $\rightarrow$ Inflow
$\rho dv$	-	Mass within Element

## LIST OF APPENDICES

<b>APPENDIX</b>	<b>TITLE</b>	<b>PAGE</b>
A1	Natural Gas Reserves	130
A2	Natural Gas Production	131
A3	Natural Gas Utilization	132
A4	Flow of Gas in Wells	134
A5	Flowing Bottomhole Pressure	135
A6	Prediction of Flowing Temperature	137
A7	Heat Transfer in Wellbore	137
A8	Gas Pressure	139
A9	Pressure Drop in Gas Wells	139
A10	Solution of Energy Generation Problem	140
B1	Physical Laws for Motion of a System	141
B2	Reynolds Transport Theorem	144
C	Perturbation Procedure	148
D	Wellhead and Bottomhole Steady State Equations	154
E	Boundary Flow Equation	157
F	Inflow Equation	159
G	Flow Chart for Numerical Procedure	168
H	Matlab M-File Code for Computing One Dimensional Transient Compressible Gas Flow in a Well	169
I	List of Published and Accepted Articles	179



## **CHAPTER 1**

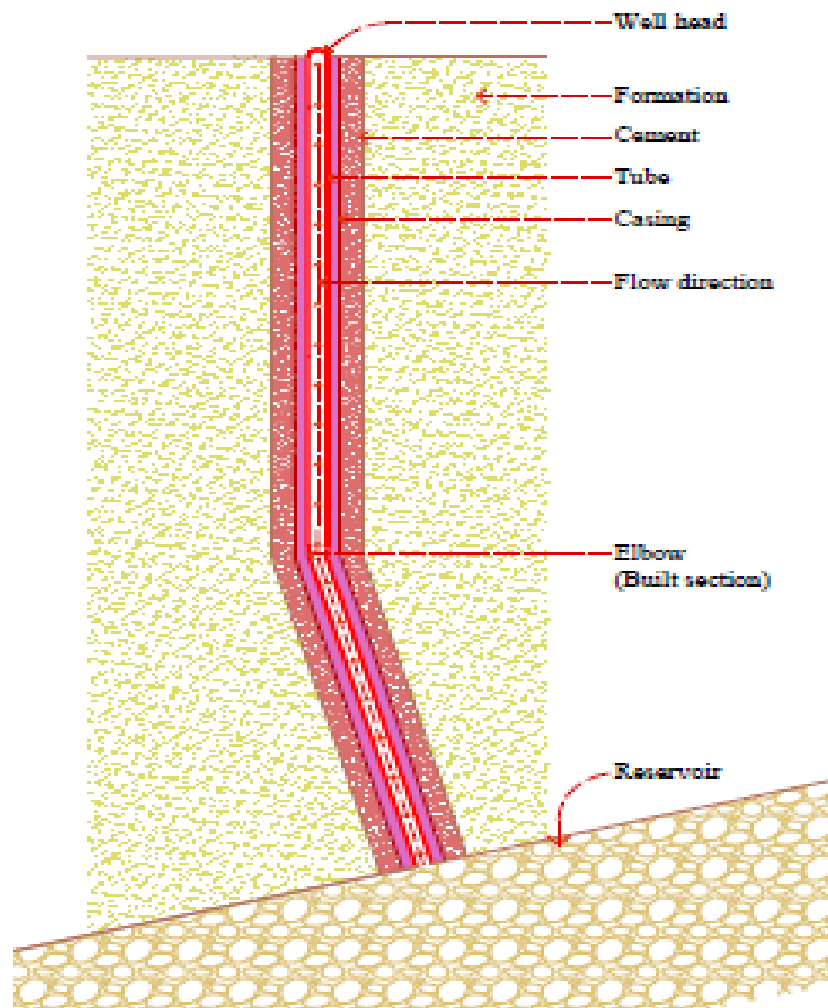
### **INTRODUCTION**

#### **1.1 Research Background**

A producing well refers to the type of well that has enough pressure to push the gas to the surface (wellhead), but when the pressure is low and cannot push the fluid to the wellhead a gas is injected into the reservoir to boost the pressure and the well is now refers to as injection well. In a continuous gas production, the formation gas is supplemented with additional high pressure gas from outside surface by injecting gas into the producing conduit at a maximum pressure that depend on the injection pressure and depth. The injected gas mixed with the produced well fluid and decreases the density and subsequently the flowing pressure gradient which causes the flowing bottomhole pressure to create a pressure differential that allows the fluid to flow into the wellbore. On the other hand when it is completely flowing, such that no fluid is injected it is refers to as fully developed well flow. Producing wells are drilled thousands of feet into the earth directly into gas rich deposits contained in underground formations but where rock permeability is strong or in offshore (water well) they are drilled in inclined positions (Figure 1.1). In producing gas well there is no oil production.

Presently producing gas wells are of increasing importance because it is the most energy efficient fossil fuel offering important energy solving benefit in the field of economic, industrial, power, agriculture and environmental considerations when used instead of oil or coal. It is available, accessible, versatile and clean. This has

been highlighted by the International Energy Agency (IEA) in May 2013, that the global demand for gas could likely rise more than 50% by 2035 and over 56% by 2040 from 2010 levels. These reasons motivated engineers to search for gas reservoirs and to drill gas wells in order to meet the world demand as gas produced along with oil are flared as unwanted products.



**Figure 1.1:** Producing Gas well

In homogenous flow of oil and gas, gas density and viscosity are much lighter compared to oil, making flaring easier (see Figure 1.2) and simultaneous production difficult.



**Figure 1.2:** Gas flaring (Climate Guest Contributor, 2011)

The flaring of gas in oil production has led to search for reservoirs containing only gas. Several methods are used in searching for gas reservoirs and subsequent drilling (see Figure 1.3). Most of these methods are carried out experimentally using Seismic sound sensor or laboratory equipment. The sensor can be inserted into the ground surface where pressure movement can be detected or mud can be tested in a laboratory which will shows presence of oil but since oil well flare gas the sound sensor is most applied for gas reservoir detection. The tool in (Figure 1.3) is use to study underground movement of fluid such as earth quake, Oil or gas that are located deep below the earth surface (Bob, 2013). Seismic wave are sent deep into the earth and allowed to bounce back. Geophysicists record the wave to learn about oil and gas reservoirs located beneath Earth's surface. Due its sensitivity it is usually stop when wind get up to 20 miles an hour or higher because wind shake grass and affects its signal. These reservoirs are located deep or shallow depending on the environment and gas is obtained as a result of sedimentary processes that occur over an extensive geological history both on land (onshore) and on sea (offshore).



**Figure 1.3:** Seismic Sound Sensor to detect Gas Reservoir (Bob, 2013)

Transient flow in a gas well is a situation where the flow characteristics are changing with both space and time. Well geometry, friction, starting or stopping of a pump, well inclination, thermal conductivities of the flow environment are some of the factors that causes changes to the flow characteristics with space and time. Transient condition in producing gas well is only applicable for relatively short period after disturbances has been created in the reservoir.

The complex well geometry consists of different layers where each layer has different thermal conductivities that cause temperature of a flowing fluid to change with both space and time. In the wellbore heat transfer also occurs when there is an exchange of temperature between the flow environment and the undisturbed formation. In heat transfer application it has been observed the steady state transient over estimates the temperature increase in the wellbore at early time but at long time it disappear (Wu, 1990). For this reason the steady state solution is always considered as initial when solving the transient condition (Zhou and Adewumi, 1995).

## 1.2 Statement of the Problem

During production or injection, flow characteristics of gas moving from reservoir to the wellhead change as time goes on because the distance it covers has different thermal conductivities due to their permeable formations. In the past focus was mainly on studying the producing well and its formation under steady state conditions but this may lead to partial prediction of the distribution of the flow characteristics since they occurs in transient conditions. Traditionally, it relies on pressure gauge, thermometer and empirically derived graph based on steady state flow conditions but the disadvantage of these methods is that in high pressure and high temperature gas wells, it is not easy to install these materials and sometimes may lead to well accident or inaccurate prediction. There are many factors in the well which may cause the changes in the flow with time during production; these factors are well roughness, casing and tubing, cementing, thermal conductivities, thermal resistance and convection. To predict the transient flow in a producing well these factors which are always neglected in the past analysis must to be considered because knowing them will help to improve, analyze and forecast reservoir/well performance.

Wellhead and bottomhole has been in used for calculating the behavior of static flow characteristics but the transient aspect has always been neglected it is therefore important to study the behavior of the flow characteristics at the wellhead and the bottomhole under the transient condition.

Compressibility of a fluid is basically a measure of change in density that will be produced from the well by a specified change in pressure. Gases are, in general, highly compressible and assuming it to be constant will not give its actual flow behavior during production. This is because pressure change will induce density changes since it is a function of density and sound wave (celerity), therefore this will have an influence on the flow.

Due to permeability of formation gas wells are not always vertical but sometimes are drilled in an inclined position, therefore including inclination in gas

well analysis will give the general behavior of the flowing fluid. What effect does well inclination have on the transient flow characteristics? Most analysis shows that the inflow properties from the reservoir to the producing well are always constant; this assumption also cannot give the real situation of the flow due the change in the flow characteristics.

In a fully developed flow the assumption that it is in uniform flow will not give a reliable results because of the flow geometry it is therefore important to investigate the behavior of the flow characteristics when the flow is fully developed. Well geometry consists of transient temperature distribution due to thermal conductivity effect between the formation and the flow medium (tubing) where the flow occurs and such heat exchange between the flowing fluid and formation experiences changes both in time and space. For good understanding of flow within the producing well, the effect these changes on the flow characteristics need to be investigated. Most producing well analysis uses different method of solutions depending on their problem without considering sound wave propagation conditions. A method that can numerically solve a model with propagation effect is most needed therefore this research will apply the most suitable method that will solve our model which has propagation effect.

### **1.3 Objectives of the Study**

Based on the problems above, the main objective of the research is to develop a mathematical model that can determine the behavior of the flow characteristics in a producing gas well. It can be achieved by incorporating sound wave, inclination, inflow, friction and thermal conductivities of the flow environment.

The specific objectives are;

- To investigate the behavior of the flow characteristics at the wellhead and bottomhole of a producing well.
- To investigate the behavior of the flow characteristics in a fully develop flow

- To develop a code based on mathematical model that can compute and simulate the behavior of the flow characteristics in a producing gas well.

#### **1.4 Scope of the Research**

One dimensional model is widely accepted to be sufficient in the analysis of transient compressible flow of gaseous fluid because it gives satisfactory solutions to many problems in fluid flow analysis (Daneshyar, 1976; Osiadacz, 1984). In this study a one-dimensional transient compressible model has been used for investigating the behavior of flow characteristics in a producing gas well. We consider vertical formation temperatures as linearly distributed and the well is assumed to be surrounded by homogeneous rock formation that extends to infinity. In the absence of field data the boundary conditions are numerically treated as provided by (Zhou and Adewumi, 1995).

#### **1.5 Significance of the Study**

The significance of this work is that **one** want to determine gas well performance and the behavior of the flow characteristics during the production period. Of significance also is the determination of thermal conductivities of the flow environment and the influence of the tubing, casing and well size on the flow characteristics.

#### **1.6 Organization of the Thesis**

In Chapter 1 we present the background of this work and based on the background, a statement of problem, objectives, scope and significance of the study were presented.

In Chapter 2 literature review on producing gas well for different works were presented. Analytical and numerical solutions for problems in producing wells as presented by different authors have been reviewed. Methods of solution for pipeline flow analysis has also been reviewed where the best method for the present work has been selected.

Chapter 3 presents mathematical formulation for transient flow of gas in a producing well with heat transfer because once the effect of this parameter is integrated into the tubing performance prediction; the overall well performance can be calculated. We begin by presenting the general form of the governing equations where a source term was incorporated to account for flow parameters such as sound wave, heat transfer, inflow and inclination effects. Initial and boundary conditions for this work were also presented.

Chapter 4 describes the numerical procedure of implicit Steger-Warming flux vector splitting method to solve the governing equations. Tridiagonal decomposition followed by treatment of boundary conditions was also presented.

Chapter 5 presents the transient results of the flow characteristics obtained using the implicit Steger-Warming flux vector splitting method and were validated with existing ones. The dynamic behavior of gas flow from the bottomhole to wellhead was presented on a rigid pipe of diameter 0.2158 m, depth 3000 m installed in the well. The behavior of the flow characteristics are calculated based on known values of injection temperature i.e. ( $T = 273$  (K), 283 (K), 293(K), 303(K) and 313 (K)) and two values of mass flow rate;  $w = 2.1133$  kg/s and  $w = 9.0572$  kg/s with  $P_0 = 9.5$  (Mpa). Other well parameters are as follows;  $r_{rto} = 0.073$  m,  $r_h = 0.2159$  m,  $r_{co} = 0.1397$  m,  $r_{ci} = 0.12426$  m,  $dx = 0.01$ ,  $Gt = 209$  (Kw/mk), Geothermal temperature is  $T = 0.023x + b$  where  $b = 13.13$ .

Chapter 6 presents transient behavior of the flow characteristics when the flow is fully developed and the well is in inclined position with influence of heat transfer between the flowing fluid and formation, casing and tubing. The behavior of the flow characteristics were also observed on different values of inclination In Chapter 7 we present the summary and conclusion of this work.





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