MESHFREE FORMULATIONS OF KINEMATIC WAVE FOR CHANNEL FLOW ROUTING

HALINAWATI BINTI HIROL

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Civil Engineering)

Faculty of Civil Engineering
Universiti Teknologi Malaysia

SEPTEMBER 2016

To my thoughtful and lovely husband, parents, children and family.

ACKNOWLEDGEMENT

Praise to Allah, the All Mighty who sparks my intuition to pursue my PhD study and provides me with invaluable guidance throughout my study and life.

Thanks to Universiti Teknologi Malaysia and the Government of Malaysia, for providing the financial support throughout my study.

Thanks to my supervisor, Prof. Dr Zulkifli Yusop and Dr Ahmad Kueh Beng Hong for their supervision and support in seeing this work through to completion.

For my lovely husband, Dr Airil Yasreen Mohd Yassin and cheerful kidsAiril Haziq, Hasya ‘Ainaa, Nuha Liyana and Hafsa Binish, thanks for cheering up my life.

To my past parent, Hirol Bol Hasan and Salbia Jini, my parent in law, Mohd Yassin Shariff and Halimah Saidin and my family, my deepest appreciation goes out to all of you.

Thanks also to all my research colleagues especially Mohd Zafri Jamil Abd Nazir and Mohd Al-Akhbar Mohd Noor for their work support and assistances.

ABSTRACT

This study concerns the development of various Meshfree formulations, namely Point Interpolation Method (PIM), Radial Point Interpolation Method (RPIM) and Element Free Galerkin (EFG) in solving numerically, St Venant's kinematic wave equations for the hydrologic modeling of surface runoff and channel flow. It involves problem formulations derivation of governing equations, provision of the corresponding solutions by generating Matlab source codes, verification of results against established data, parametric study and assessment of performance of the newly derived Meshfree formulations against established numerical methods, namely Finite Element Method (FEM) and Finite Difference Method (FDM). The originality and the main contribution of the study are solving the Meshfree formulations of the kinematic wave equations numerically. The formulations are verified when it is found that the results produced by the source codes are in general in close agreement with the benchmark data. Although slight discrepancies have been observed in some cases, these are later validated as due to several factors, namely shape parameters values which are yet to be optimized, different number of nodes used for comparison and manual discretization of input data. In obtaining the best performance of the methods, optimum values of the shape parameters have been determined through a parametric study which once obtained are used in the performance assessment. RPIM and PIM are found to be less sensitive to the optimum values as compared to EFG. Two types of performance are assessed; the convergence rate and the computer resource consumption in terms of CPU time. Based on this study, it can be concluded that, in general, Meshfree methods perform comparably with the established methods in terms of convergence rate despite the fact it does not need the construction of mesh which can save modelling time. This shows the potential of Meshfree as numerical methods for its future development.

ABSTRAK

Kajian ini adalah berkenaan penerbitan formulasi beberapa kaedah *Meshfree* iaitu *Point Interpolation Method (PIM), Radial Point Interpolation Method (RPIM)* dan *Element Free Galerkin (EFG)* dalam menyelesaikan secara numerikal persamaan ombak kinematik St Venant untuk model hidrologi air larian permukaan dan aliran alur. Kajian ini melibatkan penerbitan formulasi, penyediaan penyelesaian dan penulisan kod komputer menggunakan Matlab, pengesahan keputusan melalui perbandingan dengan data sediada, kajian parameter dan penilaian kemampuan kaedah-kaedah yang baharu dihasilkan melalui perbandingan dengan kaedah-kaedah numerikal sediada seperti kaedah unsur terhingga dan kaedah pembeza. Keaslian dan sumbangan utama kajian ini adalah formulasi beberapa kaedah *Meshfree* yang dihasilkan dengan menukar persamaan ombak kinematik ke dalam bentuk matrik. Formulasi-formulasi yang diterbitkan telah disahkan apabila keputusan-keputusan yang terhasil didapati menyamai data sediada. Walaupun terdapat perbezaan kecil untuk beberapa kes, ia telah dijelaskan sebagai kesan dari beberapa faktor seperti nilai parameter bentuk yang belum optimum, perbezaan bilangan nod sewaktu perbandingan dibuat dan penentuan input data sediada yang dibuat secara manual. Kemampuan terbaik kaedah-keadah yang baharu dihasilkan ini diperoleh dengan penentuan nilai optimum parameter bentuk melalui kajian parameter yang telah dijalankan. *PIM* dan *RPIM* didapati kurang dipengaruhi oleh nilai optimum berbanding *EFG.* Melalui penggunaan nilai-nilai optimum ini, kajian kemampuan telah dijalankan dimana ia melibatkan dua bentuk kajian iaitu kadar penumpuan dan kadar penggunaan sumber komputer. Berdasarkan kajian ini boleh disimpulkan bahawa secara umumnya kaedah-kaedah *Meshfree* mempunyai kemampuan yang sama dengan kaedah-kaedah numerikal sediada walaupun ia tidak memerlukan penyediaan mesh lantas mengurangkan masa untuk kerja permodelan dan ini menunjukkan potensi untuk penggunaan akan datang.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

A	-	cross-sectional area of the flow
BC	-	background cell
GP	-	number of gauss point
g	-	gravitational pull
L	-	Length of domain
m	-	the number of polynomial terms used in RPIM interpolation
n	-	number of the field node in the support domain
P	-	Wetted perimeter
Q	-	flow rate
q	-	MQ-RBF dimensionless shape parameter
$q\ (x)$	-	forcing term (i.e. precipitation, lateral flow).
r	-	iteration
t	-	Time
x	-	spatial coordinate
y	-	depth of water
Δx	-	size of spatial increment
Δt	-	time step
α	-	$\left(\eta P^{2/3}/\left(1.49\sqrt{S_o}\right)\right)^{0.6}$
β	-	0.6 (factor in Manning equation)
η	-	Manning roughness coefficient

ϵ	-	error criterion
π	-	MLS interpolation potential
ξ	-	natural coordinate
$\backslash$	-	Matlab command for Gauss elimination procedure
d_s	-	size of support domain
d_c	-	averaged distance between adjacent nodes
d_i	-	distance from node to the point of interest (i.e. $\lvert x_n - x_{pi} \rvert$)
N_j	-	shape functions
Q_j	-	degree of freedoms (nodal values) of flow rate
S_f	-	frictional slope
S_0	-	bed slope
x_{pi}	-	coordinates of point of interest
pi		point of interest
α_s	-	size of support domain
α_c	-	MQ-RBF dimensionless shape parameter
δ_{lj}	-	Kronecker delta
$[A]$	-	weighted moment matrix
$a_m(x)$	-	MLS non-constant coefficient
$\{a\} = a_n$	-	vector of interpolation coefficient
$\{b\} = b_m$	-	Vector of polynomial coefficients in RPIM interpolation
$F_i = \{F\}$	-	load vector
$K_{ij} = [K]$	-	stiffness matrix
$M_{ij} = [M]$	-	mass matrix
$Q_j = \{Q\}$	-	vector of degree of freedoms

$R = \{R\}$	-	residual of partial differential equations
$\{\Delta Q\}$	-	incremental degree of freedoms
$T_{ij} = [T]$	-	tangent stiffness
$R(Q_{i+1}^{t+1})$	-	residual error in finite difference scheme
$\{P\}$	-	vector of monomials built from Pascal triangle
$[P\|_n]$	-	evaluated values of the monomials at nodes (also termed as PIM moment matrix)
$\{N\|_{pi}\}$	-	shape functions evaluated at point of interests
$\{P\|_{pi}\}$	-	monomials evaluated at point of interests
$\{R\} = R_n$	-	vector of radial basis function (RBF)
$\{r\}$	-	vector of radial distance of point of interest
$[R\|_n]$	-	evaluated radial basis function at nodes
$[G\|_n]$	-	RPIM moment matrix
$\{R\|_{pi} \quad P\|_{pi}\}$	-	Evaluated values of $\{R\}$ and $\{P\}$ at point of interests
$[W(x)]$	-	MLS weight functions
$\|J_f\|$	-	Jacobian for f^{th} background cell
$\widehat{W_g}$	-	Gauss weighting factor for the g^{th} Gauss point

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Kinematic Wave Equations

Hydrologic modeling concerns the study of hydrologic processes such as evapotranspiration, subsurface flow, surface runoff and channel flow. Methods of study can be either stochastic or deterministic or combination of the two. Whilst stochastic method employs probabilistic (statistical) approach, deterministic method basically involves attempt to solve a set of partial differential equation which describes the behavior of the flow. This study concerns the latter.

The deterministic approach, on the other hand can be further divided into two groups, lumped and distributed. The main advantage of distributed modeling over lumped is that, it is easier to allow for variation in the properties of parameters such as variation in cross-sectional area, intensity of precipitation, soils coefficients, slopes and many others.

However, such an advantage requires the solution of a set of one-dimensional nonlinear partial differential equations known as St. Venant equations. These equations are actually the simplification of the two-dimensional shallow water theory derived from the general Navier-Stokes equations.

St. Venant equations themselves can be further classified into full dynamics, diffusive and kinematic wave equations. Full dynamics equations allow for complete consideration of the flow, whilst diffusive equations able to capture backwater effect. If the slope of the plane is assumed as equaled to the frictional slope, such an assumption would uncouple the continuity equation from the momentum equation hence the prevalence of the kinematic wave equations.

Despite being the simplest case of St Venant equations, there is no closed form solution available for the kinematic wave except for the simplest case of no lateral flow and constant wave celerity. The difficulty is due to the nonlinearity as well as the unsteady state of the equation. As a result, kinematic wave equations are commonly solved numerically with the help of computer programming.

1.2 Numerical Methods

Physical phenomenon is usually described by a set of partial differential equations (PDEs). By solving the equations, information of interest can be obtained. For simple set of equations, closed form solution may be available. But, for complex problems, solutions are commonly obtained by solving the equations numerically rather than analytically. Methods used in obtaining such solutions are classified as numerical methods.

At present, there are various numerical methods have been developed such as Finite Difference Method (FDM), Finite Element Method (FEM), Boundary Element

Method (BEM), Finite Volume Method (FVM) and many more. Amongst these methods, the most established and famous are FDM and FEM.

FDM can be considered as the earliest form of numerical method which history of development can be traced back as early as 1930's (Thomee, 2001). The basic idea of FDM is to convert the continuous nature of PDE into algebraic equations in matrix forms by replacing the derivative terms with forward, backward or central difference equations. The advantage of FDM relies on its straightforward implementation as well as on the fact that it operates directly of the PDEs, hence the term strong form. However, the disadvantage of FDM is in the modeling of irregular domain. Although there are several mapping techniques have been developed, these techniques are not as convenient as FEM when it comes to modeling irregular domain.

FEM, as mentioned, is a numerical method which advantage is in its efficiency in modeling irregular body shapes and problem domains. Such efficiency is due to the use of interpolation functions to approximate the problems variables. Historically, FEM was developed during the 1950's (Bathe, 1996). Whilst the first reported work on FEM can be attributed to the work of the famous mathematician, R. Courant in 1943, the major development of the method, especially in the engineering fields began with the work of Turner et.al (1956) and the separate work by Argyris and Kelsey (1954). The basic idea in FEM is to discretize the continuous nature of the PDEs by weaken into a weak statement. This can be obtained by employing either weighted residual approach or variational approach. Either approach will yield similar algebraic equations in matrix forms. With the advent of computer technology, FEM has become an established numerical methods applied in various fields to include engineering, physics, chemistry and biology (Mackerle, 2002).

Despite the establishment of FDM and especially FEM, researches have been conducted in finding new numerical methods and looking at other possibilities for better algorithms. The most notable would be BEM which development was at its height in the 1970's (Brebbia and Dominguez, 1977). The basic idea of BEM is to convert the continuous nature of the PDEs by conducting integration by parts until the differential operators on the unknown variables completely transferred onto the weighting functions. Such act allows the problem to be defined by the boundary terms only. However, this method suffered from the need to employ fundamental solutions or Green functions as the weighting functions which are difficult to be dealt with.

Further research works in the field of numerical method development then led to the introduction of a new family of numerical methods termed as Meshfree or Meshless methods in the 90's (Liu and Gu, 2005). This is the interest of this study thus is discuss next.

1.3 Meshfree Methods

Meshfree methods can be considered as the latest output in the research development of numerical techniques. The inventions were motivated by the attempt to remove the need for predefined meshes which are required in FEM. It is argued that, with the removal of the mesh, computer cost in the mesh development as well as in mesh refinement can be omitted. Therefore, since there could be various ways in doing this, Meshfree methods do not refer to specific method but to a family of methods. Methods that fall under this family, amongst others are; Point Interpolation Method (PIM), Radial Point Interpolation Method (RPIM), Element Free Galerkin (EFG), Smooth Particle Hydrodynamic (SPH), Meshless Local Petrov Galerkin (MLPG), Diffuse Element Method (DEM), and Boundary Node Method (BNM). However, due to constraints, this study only considers PIM, RPIM and EFG.

Since Meshfree methods do not require predefined mesh, the construction of shape functions must be carried out afresh for every analysis. This then becomes the major work in any Meshfree formulations. In PIM, the shape functions are constructed by using polynomial interpolation whilst in RPIM, a special interpolation is employed termed as radial basis function. For EFG, the construction of the shape functions involve the use of quartic function and the imposition of stationary condition of weighted discrete norms. All these are going to be detailed in the upcoming chapter.

Another major topic in the development of PIM, RPIM and EFG is the effect of several parameters during the construction of the shape functions. Optimum values of the parameter are required which are best determined by conducting a series of numerical test on a typical problem as these values can be different from one case to another (Liu and Gu, 2005).

Since Meshfree methods, especially PIM, RPIM and EFG, are relatively new, more studies are needed to investigate the robustness and generality of the methods especially in practical application (Liu and Gu, 2005).

1.4 Problem Statement

The hydrologic phenomenon of surface runoff and channel flow can be studied by solving kinematic wave equation. However, due to the nonlinearity and the unsteady state of the equation, no closed form solution is available except for the simplest case of no lateral flow and constant wave celerity. Therefore, in obtaining a more general solution, at present, kinematic wave equation is commonly solved either by using FDM (Chow et.al, 1988) or FEM (Vieux et.al (1990), Litrico et.al (2010)).

However, despite the various works and formulations of FDM and FEM on kinematic wave equation, there are yet PIM, RPIM and EFG formulations for the equation. Such undertake is thus important as not only can it provide alternative methods in the field of hydrology but also assists in the establishment of the Meshfree methods by widening its study and development into the field of civil engineering, in particular hydrology and river engineering. Also, by carrying out such undertaking, the study can also be among the first to provide data on optimum values of parameters which govern the performance of the Meshfree methods especially in the field of hydrology and river engineering.

Based on these, it is therefore the main interest and purpose of this study to the develop PIM, RPIM and EFG formulations for kinematic wave equation.

1.5 Objectives of Study

i. To derive and formulate PIM, RPIM and EFG formulations for kinematic wave equation and write the corresponding Matlab source-codes. For performance assessment purposes, source codes for FEM and FDM are also written.

ii. To validate the formulation against previous works (benchmark problems)

iii. To conduct parametric study (numerical test) to determine the optimum range and value of parameters in ensuring the best performance of the Meshfree methods

iv. To conduct performance study in terms of convergence rate and computer resource consumption in assessing the potential of the Meshfree methods against the established numerical methods; FDM and FEM

1.6 Scope and Limitation of Study

i. To limit the scope of the study, only three type of Meshfree methods are considered which are PIM, RPIM and EFG

ii. The study strictly involves with mathematical derivations and computer programming thus no direct experimental works are conducted due to time constraint. However, the absence of direct experimental work is compensated by the validation and verification which are carried out against the actual gauged data provided by one of the benchmark problem

iii. All assumptions in St Venant equations and kinematic wave equation holds

iv. Although one of the main advantage of Meshfree methods is in the ease of treating irregular arrangement of nodes hence refinement process, due to the pioneering nature of this study, only uniform distribution of nodes is considered and no consideration is given in the refinement process

v. Despite the availability of various nonlinear schemes and time-integration methods available, this study only employs Picard and Newton-Raphson as iterative schemes and backward difference as the time-integration methods because of their simplicity but good convergence.

vi. Despite the availability of various radial basis functions and spline functions for the construction of shape functions of RPIM and EFG respectively, this study only employ multi-quadric function for the former and quartic spline function for the latter because they are the most basic function and generally used.

1.7 Significance of Study

This study is one of pioneering work of PIM, RPIM and EFG methods in hydrologic modeling especially in the solution of kinematic wave. It provides insight into the performance of the methods mentioned in terms of convergence rate and computer resource consumption as well as one of the first to report on the optimum ranges and values of parameters of the methods. Such information would be useful, not only for future studies of Meshfree as numerical methods but also in practical realm of civil engineering and hydrology.

1.8 Outline of Thesis

This thesis comprises of six chapters outlined as follows.

CHAPTER 1: This chapter introduces the general idea of hydrologic modeling and corresponding methods of study. It describes relevant theories and equations. An introduction is also given on various numerical methods to include their brief history, basic idea and current state of development and application. Problem statement in then outlined in detailing the need for the study to be conducted followed by the objectives of the study. To clarify the framework of the study, scope and limitation are detailed out. The significance of study is then highlighted.

CHAPTER 2: In this chapter, relevant previous works are reviewed and discussed. The discussion begins with works related to St Venant equations especially on kinematic wave equation. Then, previous works on FDM and FEM related to the solution of the equation are reviewed and discuss. The final part of the chapter focuses on the current state of knowledge on Meshfree methods especially PIM, RPIM and EFG.

CHAPTER 3: This chapter concerns the mathematical derivations of both, the kinematic wave equation and the relevant numerical formulations. Established numerical method are derived first; FEM followed by FDM. Then detailed derivation of the shape functions leading to the discretized algebraic equations in matrix forms are given for PIM, RPIM and EFG.

CHAPTER 4: In this chapter, all formulations and their corresponding source codes are validated by comparing their results against three benchmark problems; Chow et. al. (1988), Vieux et.al (1990) and Litrico et.al (2010). Reasons for the selection of these problems as benchmark are detailed. Besides validation of the formulation, validation on the use of different iterative schemes is also provided.

CHAPTER 5: This chapter is divided into two parts. The first parts concerns the parametric studies in which series of numerical tests are conducted in determining the optimum ranges and values of parameters affecting the performance of the Meshfree methods. In the second parts, the optimum values are then used in the formulation to assess the performance of the Meshfree methods relative to the established ones (FDM and FEM). Their performance in terms of convergence rate and computer resource consumption (CPU time) are assessed.

CHAPTER 6: This is the final chapter of the thesis. In this chapter, findings obtained from the study are summarized and concluded. Suggestions for future works are given at the end of the chapter.

6.4 Suggestions for Future Works

As mentioned, this study is a pioneering work in the discretization of kinematic wave equations by Meshfree methods specifically PIM, RPIM and EFG. Due to its pioneering nature, this work has limitations which can be extended in future works. It is suggested that the formulations be extended:

i. To allow for irregular distribution of nodes and automated for adaptive analysis where decision on the distribution and number of nodes is automatically made based on the needs of the analysis i.e. at the region of high gradient, moving boundaries, discontinuities etc.

ii. For network system where a number of branches (representing watershed draining or rivers) can be modeled. This will make the formulation more general and able to capture greater spatial variability of parameters.

iii. To other forms of St. Venant equations namely diffusion and full dynamics as this will make the formulation more general and practical (i.e. allow backwater)

iv. To other Meshfree methods such as smoothed particle hydrodynamics (SPH) and Reproducing Kernel Particle Method (RKPM).

REFERENCES

Abbott, M. B., & Ionescu, F. (1967). On the numerical computation of nearly horizontal flows. *Journal of Hydraulic Research*, *5*(2), 97-117.

Amein, M., & Fang, C. S. (1970). Implicit flood routing in natural channels. *Journal of the Hydraulics Division*. *96(12), 2481-2500*

Argyris, J. H., & Kelsey, S. (1954). Energy theorems and structural Analysis. Part II. Application to thermal stress problem and Saint Venant's torsion.*Aircraft Engng*, *26*, 413-416.

Balloffet, A. (1969). One-dimensional analysis of floods and tides in open channels. *Journal of the Hydraulics Division*, *95*(4), 1429-1454.

Baltzer, R. A., & Lai, C. (1968). Computer simulation of unsteady flows in waterways. *Journal of the Hydraulics Division*, *94*(4), 1083-1120.

Bathe, K. J. (1996). *Finite element procedures*. Klaus-Jurgen Bathe.

Belytschko, T., Krysl, P., & Krongauz, Y. (1997). A three-dimensional explicit element-free galerkin method. *International Journal for Numerical Methods in Fluids*, *24*(12), 1253-1270.

Belytschko, T., Lu, Y. Y., & Gu, L. (1994). Element free Galerkin methods.*International journal for numerical methods in engineering*, *37*(2), 229-256.

Beven, K. (1979). On the generalized kinematic routing method. *Water Resources Research*, *15*(5), 1238-1242.

Brebbia, C. A., & Dominguez, J. (1977). Boundary element methods for potential problems. *Applied Mathematical Modelling*, *1*(7), 372-378.

Chang, T. J., Kao, H. M., Chang, K. H., & Hsu, M. H. (2011). Numerical simulation of shallow-water dam break flows in open channels using smoothed particle hydrodynamics. *Journal of Hydrology*, *408*(1), 78-90.

Chaudhry, Y. M., & Contractor, D. N. (1973). Application of the implicit method to surges in open channels. *Water Resources Research*, *9*(6), 1605-1612.

Chen, X. L., Liu, G. R., & Lim, S. P. (2003). An element free Galerkin method for the free vibration analysis of composite laminates of complicated shape.*Composite Structures*, *59*(2), 279-289.

Chow, V. T., Maidment, D. R., & Mays, L. W. (1988). Applied hydrology, 572 pp. *Editions McGraw-Hill, New York*.

Contractor, D. N., & Wiggert, J. M. (1972). Numerical studies of unsteady flow of the James River, 56 pp. *Water Resources Research Center, Virginia Polytechnic Institute and State University*.

Dakssa, L. M., & Harahap, I. S. H. (2013). The Performance of The SPH Method In Simulating Surface Runoff Along a Saturated Soil Slope. Int. J. of Safety and Security Eng., 3(2), 77–87

Dronkers, J. J. (1969). Tidal Computations for Rivers, Coastal Areas, and Sea.*Journal of the Hydraulics Division*, *95*(1), 29-78.

Firoozjaee, A. R., & Afshar, M. H. (2011). Discrete Least Squares Meshless (DLSM) method for simulation of steady state shallow water flows. *Scientia Iranica*, *18*(4), 835-845.

Franke, C., & Schaback, R. (1998). Solving partial differential equations by collocation using radial basis functions. *Applied Mathematics and Computation*,*93*(1), 73-82.

Garrison, J. M., Granju, J. P. P., & Price, J. T. (1969). Unsteady Flow Simulation in

River and Reservoirs. *Journal of the Hydraulics Division*, *95*(5), 1559-1576.

Govindaraju, R. S., Jones, S. E., & Kavvas, M. L. (1988). On the diffusion wave model for overland flow: 1. Solution for steep slopes. *Water Resources Research*, *24*(5), 734-744.

Govindaraju, R. S., Kavvas, M. L., & Jones, S. E. (1990). Approximate analytical solutions for overland flows. *Water Resources Research*, *26*(12), 2903-2912.

Greco, F., & Panattoni, L. (1975). An implicit method to solve Saint Venant equations. *Journal of Hydrology*, *24*(1), 171-185.

He, Y., & Zhang, J. M. (2014). The Development and Application of Meshless Method. In *Applied Mechanics and Materials* (Vol. 444, pp. 214-218).

Henderson, F. M. (1989). Open channel flow. In *McMillan series in civil engineering*. MacMillan.

Hjelmfelt, A. T. (1981). Overland flow from time-distributed rainfall. *Journal of the Hydraulics Division*, *107*(2), 227-238.

Homayoon, L., Abedini, M. J., & Hashemi, S. M. R. (2013). RBF-DQ solution for shallow water equations. *Journal of Waterway, Port, Coastal, and Ocean Engineering, 139(1), 45-60*

Hon, Y. C., Cheung, K. F., Mao, X. Z., & Kansa, E. J. (1999). Multiquadric solution for shallow water equations. *Journal of Hydraulic Engineering, 125(5), 524-533*

Isaacson, E., Stoker, J. J., & Troesch, B. A. (1954). Numerical solution of flood prediction and river regulation problems (Ohio-Mississippi floods). *Report II, Inst. Math. Sci. Rept. IMM-NYU-205, New York University*.

Jaber, F. H., & Mohtar, R. H. (2002a). Stability and accuracy of finite element schemes for the one-dimensional kinematic wave solution. *Advances in water resources*, *25*(4), 427-438.

Jaber, F. H., & Mohtar, R. H. (2002b). Dynamic time step for one-dimensional overland flow kinematic wave solution. *Journal of Hydrologic Engineering*, *7*(1), 3-11.

Jayawardena, A. W., & White, J. K. (1977). A finite element distributed catchment model, I. Analytical basis. *Journal of Hydrology*, *34*(3), 269-286.

Jayawardena, A. W., & White, J. K. (1979). A finite-element distributed catchment model, II. Application to real catchments. *Journal of Hydrology*,*42*(3), 231-249.

Kansa, E. J. (1990). Multiquadrics—A scattered data approximation scheme with applications to computational fluid-dynamics—II solutions to parabolic, hyperbolic and elliptic partial differential equations. *Computers & mathematics with applications*, *19*(8), 147-161.

Kibler, D. F., & Woolhiser, D. A. (1970). *The kinematic cascade as a hydrologic model*. Fort Collins, Colorado: Colorado State University.

Krysl, P., & Belytschko, T. (1999). The element free Galerkin method for dynamic propagation of arbitrary 3-D cracks. *International Journal for Numerical Methods in Engineering*, *44*(6), 767-800.

Kurt, H. (1985). Application limits for the kinematic wave approximation. *Nordic Hydrology*, *16*(4), 203-212.

Lal, W. and Toth, G. (2013). Implicit TVDLF Methods for Diffusion and Kinematic Flows, *Journal of Hydraulic Engineering, 139, 974.*

Liggett, J. A., & Woolhiser, D. A. (1967). Difference solutions of the shallow-water equation. *Journal of the Engineering Mechanics Division*, *93*(2), 39-72.

Lighthill, M. J., & Whitham, G. B. (1955, May). On kinematic waves. I. Flood movement in long rivers. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* (Vol. 229, No. 1178, pp. 281-316). The Royal Society.

Litrico, X., Pomet, J. B., & Guinot, V. (2010). Simplified nonlinear modeling of river flow routing. *Advances in Water Resources*, *33*(9), 1015-1023.

Liu, G. R. (2002). *Meshfree methods: moving beyond the finite element method*. Taylor & Francis.

Liu, G. R., & Gu, Y. T. (1999). A point interpolation method. In*Proc. 4th Asia-Pacific Conference on Computational Mechanics, December, Singapore* (pp. 1009-1014).

Liu, G. R., & Gu, Y. T. (2001). A local radial point interpolation method (LRPIM) for free vibration analyses of 2-D solids. *Journal of Sound and vibration*, *246*(1), 29-46.

Liu, G. R., & Gu, Y. T. (2003). A matrix triangularization algorithm for the polynomial point interpolation method. *Computer Methods in Applied Mechanics and Engineering*, *192*(19), 2269-2295.

Liu, G. R., & Gu, Y. T. (2004). Boundary meshfree methods based on the boundary point interpolation methods. *Engineering Analysis with Boundary Elements, 28 (5), 475–487.*

Liu, G. R., & Gu, Y. T. (2005). *An introduction to meshfree methods and their programming*. Springer Science & Business Media.

Liu, G. R., Gu, Y. T., & Dai, K. Y. (2004). Assessment and applications of point interpolation methods for computational mechanics. *International Journal for Numerical Methods in Engineering*, *59*(10), 1373-1397.

Mackerle, J. (2002). Finite elements in the analysis of pressure vessels and piping, an addendum: a bibliography (1998–2001). *International Journal of Pressure Vessels and Piping*, *79*(1), 1-26.

Meenal, M., & Eldho, T. I. (2011). Simulation of groundwater flow in unconfined aquifer using meshfree point collocation method. *Engineering Analysis with Boundary Elements*, *35*(4), 700-707.

Parlange, J. Y., Rose, C. W., & Sander, G. (1981). Kinematic flow approximation of runoff on a plane: an exact analytical solution. *Journal of Hydrology*, *52*(1), 171-176.

Ponce, V. M. (1991). Kinematic wave controversy. *Journal of Hydraulic Engineering*, *117*(4), 511-525.

Quinn, F. H., & Wylie, E. B. (1972). Transient analysis of the Detroit River by the implicit method. *Water Resources Research*, *8*(6), 1461-1469.

Ross, B. B., Contractor, D. N., & Shanholtz, V. O. (1979). A finite-element model of overland and channel flow for assessing the hydrologic impact of land-use change. *Journal of Hydrology*, *41*(1), 11-30.

Saint-Venant, B. D. (1871). Theory of unsteady water flow, with application to river floods and to propagation of tides in river channels. *French Academy of Science*, *73*(1871), 237-240.

Shakibaeinia, A., & Jin, Y. C. (2012). MPS mesh-free particle method for multiphase flows. *Computer Methods in Applied Mechanics and Engineering*,*229*, 13-26.

Singh, V. P. (2001). Kinematic wave modelling in water resources: a historical perspective. *Hydrological processes*, *15*(4), 671-706.

Singh, V. P. (2003). Kinematic wave modeling in hydrology. *Bridges*, *10*(40685), 165.

Singh, V. P., & Prasana, M. (1999). Generalized flux law, with an application.*Hydrological processes*, *13*(1), 73-87.

Stoker, J. J. (1957). Water Waves: The Mathematical Theory with Applications.

Tayfur, G., Kavvas, M. L., Govindaraju, R. S., & Storm, D. E. (1993). Applicability of St. Venant equations for two-dimensional overland flows over rough infiltrating surfaces. *Journal of Hydraulic Engineering*, *119*(1), 51-63.

Taylor, C., & Huyakorn, P. S. (1978). A comparison of finite element based solution schemes for depicting overland flow. *Applied Mathematical Modelling*,*2*(3), 185-190.

Thomas, A., Eldho, T. I., & Rastogi, A. K. (2014). A comparative study of point collocation based MeshFree and finite element methods for groundwater flow simulation. *ISH Journal of Hydraulic Engineering*, *20*(1), 65-74.

Thomée, V. (2001). From finite differences to finite elements: A short history of numerical analysis of partial differential equations. *Journal of Computational and Applied Mathematics*, *128*(1), 1-54.

Turner, M. J., Clough, R.W., Martin, H.C. and Topp, L.J. (1956). Stiffness and deflection analysis of complex structures.*Journal of the Aeronautical Sciences (Institute of the Aeronautical Sciences)*, *23*(9).

Vieira, J. D. (1983). Conditions governing the use of approximations for the Saint-Venant equations for shallow surface water flow. *Journal of Hydrology*,*60*(1), 43-58.

Vieux B.E. (1988), Finite element analysis of hydrologic response areas using geographic information systems. Dissertation presented to Michigan State University, at East Lansing, Michigan, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Vieux, B. E., & Gauer, N. (1994). Finite‐Element Modeling of Storm Water Runoff Using GRASS GIS. *Computer‐Aided Civil and Infrastructure Engineering*, *9*(4), 263-270.

Vieux, B. E., Bralts, V. F., Segerlind, L. J., & Wallace, R. B. (1990). Finite element watershed modeling: one-dimensional elements. *Journal of water resources planning and management*, *116*(6), 803-819.

Wang, J. G., & Liu, G. R. (2000). Radial point interpolation method for elastoplastic problems. In *ICSSD 2000: 1 st Structural Conference on Structural Stability and Dynamics* (pp. 703-708).

Wang, J. G., & Liu, G. R. (2002a). A point interpolation meshless method based on radial basis functions. *International Journal for Numerical Methods in Engineering*, *54*(11), 1623-1648.

Wang, J. G., & Liu, G. R. (2002b). On the optimal shape parameters of radial basis functions used for 2-D meshless methods. *Computer methods in applied mechanics and engineering*, *191*(23), 2611-2630.

Wasantha Lal, A. M., & Toth, G. (2013). Implicit TVDLF methods for diffusion and kinematic flows. *Journal of Hydraulic Engineering*, *139*(9), 974-983.

Wendland, H. (1995). Piecewise polynomial, positive definite and compactly supported radial functions of minimal degree. *Advances in computational Mathematics*, *4*(1), 389-396.

Wendland, H. (1998). Error estimates for interpolation by compactly supported radial basis functions of minimal degree. *Journal of approximation theory*, *93*(2), 258-272.

Woolhiser, D. A., & Liggett, J. A. (1967). Unsteady, one-dimensional flow over a plane—The rising hydrograph. *Water Resources Research*, *3*(3), 753-771.

Yu, C., & Duan, J. G. (2014). High resolution numerical schemes for solving kinematic wave equation. *Journal of Hydrology*, *519*, 823-832.