# UNSTEADY MHD FLOW OF VISCOUS AND SECOND GRADE FLUIDS IN A POROUS MEDIUM

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This work is dedicated to the Almighty God, ALLAH SWT. To my beloved wife - Suhana who has been a great source of motivation and inspiration. To my children - Ammar Zikri, Ammar Zakwan, Ammar Zaimi, Amni Sufya and Ammar Zafran, they are my strength to accomplish my study, as well as this thesis. And to my parents and siblings, the vehicle that conveyed my existence.

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## ABSTRACT

In this thesis, the unsteady magnetohydrodynamic (MHD) free convection flows of viscous and second grade fluids past an infinite inclined plate in a porous medium are studied. These viscous and second grade fluids are under the conditions of ramped wall temperature and isothermal plate. Analytic solutions are developed by using Laplace transform technique. The main finding of this thesis is to determine the expressions of exact solutions for velocity, temperature and concentration profiles. All these profiles are graphically plotted for various physical parameters such as radiation, heat absorption, porosity, rotation and second grade parameters. The results show that when temperature decreases, high radiation and heat absorption occurs which consequently decreases the velocity. For larger values of magnetic parameter, the fluid velocity decreases. The velocity is found to increase with increasing values of the porosity parameter. It is also observed that when the second grade parameter increases, the velocity shows an oscillating behavior where the velocity first decreases and then increases. An interesting result for the velocity is observed from the comparison of ramped wall temperature and isothermal. It is found that fluid velocity retarded in the case of ramped wall temperature compared to isothermal case. In limiting cases, the present solutions are reduced in order to compare with existing results. As expected, the results are found identical, verifying the validity of the obtainable solutions. The numerical results of skin-friction, Nusselt number and Sherwood number are also computed and displayed in tables, and also analyzed in details.

# ABSTRAK

Dalam tesis ini, aliran tak mantap olakan bebas magnetohidrodinamik (MHD) bendalir likat dan gred kedua yang melintasi plat condong tak terhingga dalam bahantara berliang dikaji. Bendalir likat dan gred kedua ini di bawah syarat suhu tanjakan dinding dan plat isoterma. Penyelesaian analisis dibangunkan dengan menggunakan teknik jelmaan Laplace. Dapatan utama tesis ini adalah untuk penentuan ungkapan penyelesaian tepat bagi profil halaju, profil suhu dan profil kepekatan. Tingkah laku semua profil ini diplot secara grafik dengan parameter fizikal seperti parameter radiasi, parameter penyerapan haba, parameter keliangan, parameter putaran dan parameter gred kedua. Keputusan menunjukkan apabila suhu menurun, radiasi dan penyerapan haba yang tinggi didapati berlaku yang mengakibatkan pengurangan halaju. Untuk nilai parameter magnet yang besar, halaju bendalir berkurangan. Halaju bendalir diperhatikan meningkat apabila parameter keliangan meningkat. Didapati juga peningkatan parameter gred kedua menunjukkan tingkah laku halaju yang berayun di mana pada mulanya halaju berkurangan dan kemudian meningkat. Keputusan yang menarik bagi halaju dapat diperhatikan daripada perbandingan antara suhu tanjakan dinding dan plat isoterma. Suhu tanjakan dinding telah melambatkan halaju bendalir apabila dibandingkan dengan plat isoterma. Dalam kes mengehad, penyelesaian yang diperoleh diturunkan untuk dibandingkan dengan keputusan yang telah diterbitkan. Seperti dijangka, keputusan serupa diperoleh, yang membuktikan kesahihan penyelesaian yang diperoleh. Keputusan berangka untuk geseran kulit, nombor Nusselt dan nombor Sherwood telah juga dihitung dan dipersembahkan dalam bentuk jadual serta dianalisis secara terperinci.

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# LIST OF SYMBOLS

**Roman Letters**

| | | |
|---|---|---|
| $\mathbf{A_1}$ | - | first Rivilin-Ericksen tensor |
| $\mathbf{A_2}$ | - | second Rivilin-Ericksen tensor |
| $\mathbf{B}$ | - | Total magnetic field |
| $\mathbf{B_0}$ | - | applied magnetic field |
| $B_0$ | - | magnitude of applied magnetic field |
| $\mathbf{b}$ | - | body force |
| $C$ | - | concentration of the fluid |
| $c_p$ | - | specific heat at constant pressure |
| $\frac{d}{dt}$ | - | material time derivative |
| $\mathbf{div}$ | - | divergence |
| $\mathbf{E}$ | - | electric field |
| erf | - | error function |
| erfc | - | complementary error function |
| $\exp$ | - | exponential function |
| $Gr$ | - | thermal Grashof number |
| $Gm$ | - | mass Grashof number |
| $\mathbf{g}$ | - | gravitational acceleration |
| $H(.)$ | - | Heaviside function |
| $\mathbf{I}$ | - | identity tensor |
| $\mathbf{J}$ | - | current density |
| $K$ | - | dimensionless porosity parameter |
| $k$ | - | thermal conductivity |
| $L$ | - | characteristic length |
| $\mathcal{L}$ | - | Laplace transform |
| $\mathcal{L}^{-1}$ | - | Inverse Laplace transform |

| | | |
|---|---|---|
| $M$ | - | dimensionless magnetic parameter |
| $Nu$ | - | Nusselt number |
| $\mathrm{Pr}$ | - | Prandtl number |
| $p$ | - | scalar pressure |
| $p^*$ | - | modified pressure gradient |
| $\mathbf{R}$ | - | Darcy's resistance |
| $R$ | - | radiation parameter |
| $s$ | - | Laplace transform parameter |
| $Sc$ | - | Schmidt number |
| $\mathbf{T}$ | - | Cauchy stress tensor |
| $T$ | - | temperature of the fluid near the plate |
| $t$ | - | dimensionless time |
| $t_0$ | - | characteristic time |
| $F$ | - | complex velocity |
| $\mathbf{F}$ | - | body force vector |
| $u$ | - | velociy in $x$-direction |
| $v$ | - | velociy in $y$-direction |
| $\mathbf{V}$ | - | velocity vector field |
| $\boldsymbol{\Omega}$ | - | angular velocity vector |
| $\Omega$ | - | constant angular velocity |
| $x$ | - | dimensionless coordinate axis of the plate |
| $y$ | - | dimensionless coordinate axis of the plate |
| $z$ | - | dimensionless coordinate axis normal to the plate |

**Greek Letters**

| | | |
|---|---|---|
| $\alpha_1, \alpha_2$ | - | material moduli or normal stress moduli |
| $\alpha$ | - | dimensionless second grade parameter |
| $\beta$ | - | volumetric coefficient of thermal expansion |
| $\rho$ | - | density |
| $\sigma$ | - | electrical conductivity |
| $\nu$ | - | kinematic viscosity |
| $\mu$ | - | dynamic viscosity |
| $\omega$ | - | dimensionless rotating parameter |
| $\phi$ | - | porosity of the medium |
| $\tau$ | - | dimensionless skin-friction |
| $\theta$ | - | dimensionless heat absorption coefficient |
| $\varphi$ | - | porosity of porous medium |
| $I_0(.)$ | - | modified bessel function of order zero |
| $I_1(.)$ | - | modified bessel function of order one |

**Subscripts**

| | | |
|---|---|---|
| $w$ | - | conditions on the wall |
| $\infty$ | - | free stream condition |

**Superscript**

| | | |
|---|---|---|
| $T$ | - | transpose operation |
| $*$ | - | dimensional sign |

## LIST OF APPENDICES


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# CHAPTER 1

# INTRODUCTION

## 1.1 Introduction

This chapter contains a review of unsteady MHD free convection flow of viscous and non-Newtonian fluids namely second grade fluids with heat and mass transfer together. A background of the research is presented in Section 1.2. The statement of problem is given in Section 1.3. The objectives and scope of the study are given in Sections 1.4 and 1.5 respectively. Sections 1.6 and 1.7 described the research methodology and significance of the study. At the end of this chapter, thesis organization are addressed in Section 1.8.

## 1.2 Research Background

In fluid behaviour study, generally there are two types of fluid, Newtonian non-Newtonian fluids. Newtonian fluid or viscous fluid is a fluid which obeys the linear relations where Newton first postulated between sheer stress and rate of deformation. Fluids such as air, water and most of gases are Newtonian. This means that, a plot of shear stress against shear rate at a certain temperature is a straight line with a constant slope that is independent of the shear rate. This slope is called as the viscosity of the fluid. Furthermore, low molecular weight liquids, and solutions of low molecular weight substances in liquids are usually Newtonian. Two examples of viscous fluid are aqueous solutions of salt and sugar.

A fluid that does not obey the Newtonian relationship between the shear stress and shear rate is called non-Newtonian fluids. The subject of ”Rheology” is dedicated to the study of the behaviour of such fluids. High molecular weight liquids which include polymer melts and polymer solutions, as well as liquids in which fine particles are suspended, such as slurries and pastes, are usually non-Newtonian (El-Shahed, 2004). In this case, the slope of the shear stress versus shear rate curve will not be constant as the changes of shear rate. When the viscosity increases with decreasing shear rate, the fluid is shear-thinning. In the opposite case where the viscosity decreases as the fluid is subjected to a lower shear rate, the fluid is called shear-thickening. Shear-thinning behaviour is more common than shear-thickening. Shear-thinning fluids also are called pseudo plastic fluids. This type of fluids is more complex and interesting to be studied. Amongst the many fluid models which have been classes as non-Newtonian, the fluids of differential type that have received special attention (Erdogan, 1995; Erdogan, 2003). The second grade fluid, which are a subclass of the differential type fluids, has been successfully gained attention in various kinds of flows by different researchers (Fetecau *et al.*, 2002; Fetecau and Fetecau, 2005).

To study fluid flow with various effects, researchers must consider a transport phenomena, heat and mass transfer, and fluid dynamics. Heat transfer concerned with the transport of energy, mass transfer involves with the transport of mass of various chemical species, and fluid dynamics dealt with transport of momentum. These three transport phenomena should be studied together because they frequently occur simultaneously. The mathematical concepts needed for defining these phenomena are very similar, where the basic equations that describe the three transports phenomena are closely related. When transports phenomena happen, especially heat transfer, mechanism of fluid motion is generated density differences in the fluid occurring due to temperature gradients. This mechanism is called convection.

The convection of heat transfer generally divided into two basic processes, which are free or natural convection, and forced convection. Natural convection or free convection is the motion of fluids initiated exclusively due to the density difference during the heating or cooling of the fluid. When heated, the density change in the boundary layer causes the fluid to rise and be replaced by a cooler fluid. This continues as a phenomena called free convection. Most common example of free convection is air rising above sunlight-warmed land or water, a major feature of all weather systems.

In forced convection or heat advection, fluids movement results are due on the external forces. The typical use of the forced convection is to increase the rate of heat exchange. In any forced convection situation, free convection effect is also present under the presence of gravitational body forces. In addition, when the effect of force flow in free convection becomes significant, the process is then called mixed convection flow which is a combination of free and force convection flows.

Nowadays, many researchers in fluid mechanics branches have been considering various physical parameters to study the behaviour of the free convection flow, such as the effects of magnetic field, porosity of the media, thermal radiation, physical geometry, rotation, type of boundary conditions, and fluid type, either in viscous or in second grade fluids. This is because natural convection presence both in nature and engineering applications.

Magnetohydrodynamics (MHD) is the interaction between magnetic fields and fluid dynamics. The concept of MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn create forces on the fluid and also changes the magnetic field itself. Examples of such fluids include liquid metals, plasmas and salt water. The influence of magnetic field is observed in many natural and man-made flows. Magnetic fields are commonly studied in solar physics, aeronautics, chemical engineering and electronics (Davidson, 2001; Parvin and Nasrin, 2011). Apart from MHD, the flow of fluids through porous media has become an important topic because of the wide applications in geothermal and the recovery of crude oil from the pores of the reservoir rocks (Makinde and Mhone, 2005; Mebine and Adigio, 2011). A porous medium is a substance that contains pores, or spaces between solid materials through which fluids can pass. Examples of naturally occurring porous media include sand, soil, and rock. Sponges, ceramics, and reticulated foam are also manufactured for use as a porous media. Physically, a porous medium can be distinguished from other materials including other porous media, by the porosity, or the size of the pores.

Materials with low porosity are less permeable and typically have smaller pores, making it more difficult for fluids to pass through them, while materials with high porosity have huge pores and are easily permeated. Porosity is important, especially in filtering, since if particles must be removed by a porous medium, the pores must be small enough to effectively trap them. Geologists also consider the porosity of the surrounding stone and soil when conducting observations of oil and natural gas reservoirs.

On the other hand, thermal radiation is a radiation of electromagnetic produced by the thermal motion of charged particles in matter. All matters with a temperature greater than absolute zero discharges thermal radiation. The mechanism is that bodies with a temperature above absolute zero have molecules with kinetic energies which are changing, and these changes result in charge-acceleration and dipole oscillation of the charges that compose the molecules. The objects give off radiation or thermal radiation, like modern smart-phone, also absorb such radiation from their surroundings. If a body is hotter than the surroundings it produces more radiation than it absorbs, and tends to cool. But if a body is cooler than the environments it absorbs more radiation than it emits, and tends to warm. This situation is called as generating and absorption of heat by thermal radiation. Furthermore, extensive study has been done in radiation interaction with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing emitting fluids (Kesaviah *et al.*, 2013).

Further, the rotating flow fluids have stimulated the interest of researchers in fluid studies and is an area of research undergoing rapid growth in the modern fluid mechanics. This is due to their wide range of scientific applications in various fields. The specific applications of rotating fluids are encountered in geophysics, especially in the study of wind generating ocean currents on rotating earth. A rotation is a circular movement of an object around a point of rotation. A three-dimensional object rotates always around an imaginary line called a rotation axis. If the axis is within the body, and passes through centre of the mass, it said to rotate upon itself. A rotation about an external point such as the earth about the sun, it is termed a revolution or orbital revolution, typically when it is created by gravity. A moving object in a rotating reference frame is called Coriolis effect, and widely use in fluid flow study with rotating effects.

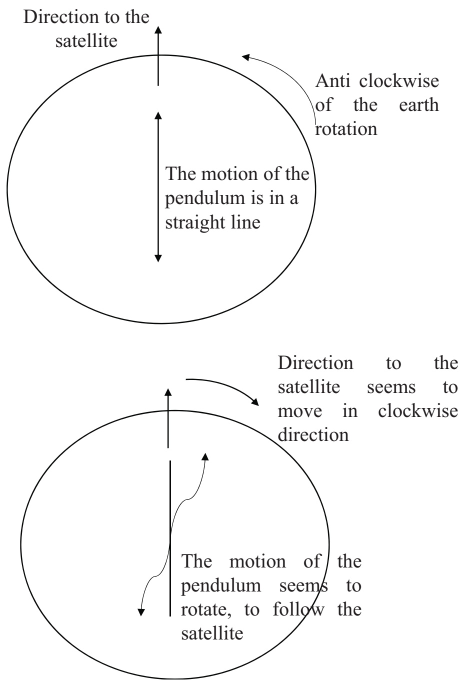


**Figure 1.1:** Coriolis effect

Figure 1.1 illustrate an imaginary force that happen in Coriolis effect. If we viewed from above, the motion of the pendulum is along a straight line. At the pole, the pendulum motion would always move along the straight line to the satellite. Meanwhile, the Earth is rotating anti clockwise under the pendulum. When we use the rotating earth as our reference frame, the satellite appears to move in clockwise direction. The pendulum, maintaining its alignment with the satellite, appears to move in clockwise direction as well. This illusion or imaginary, created by using the reference of the earth rotating frame, is known as Coriolis effect. Such studies of Coriolis effect on unsteady free convection flow was reported by Islam and Alam (2008) and Vijayalakshmi and Kamalam (2013). Beside the flow study past a vertical and horizontal geometry, the study of fluid flow past an inclined surface starts to gain more attention from the researchers (Ganesan and Palani, 2004; Aboeldahab and Azzam, 2005). An inclined surface is a flat supporting surface tilted at an angle, with one end higher than the other. Fluid flow up an inclined plane requires less force than lifting it vertically, especially from the gravitational force. Due to conservation of energy, the same amount of mechanical energy is required to lift a given object by a given vertical distance, disregarding losses from friction, but the inclined plane allows the same work to be done with a smaller force exerted over a greater distance.

It is interesting to study the behaviour of fluid motion in rotating systems along an inclined surfaces imposed by varieties of boundary conditions. Boundary conditions are set of conditions specified for the behaviour of the solution to a set of differential equations at the boundary of domains. Boundary conditions are important in determining the mathematical solutions to many engineering applications. An example of boundary condition is isothermal plate and wall ramped temperature. Isothermal, also known as uniform temperature, is the system when the temperature remains constant as other quantities changed. On the other hand, ramped wall temperature is a system where temperature is changing over the time. Ramped wall temperature is more attracted to researchers compared to isothermal system because this type of temperature condition profiles are likely to be of relevance in several industrial applications, especially where the initial temperature profiles assume to be important in the stage of design processes.

## 1.3 Statement of Problems

Previous study shows that the flows of Newtonian and non-Newtonian fluid play an important role in fluid flow problems. Non-Newtonian fluid such as second grade fluid has attracted many researchers to study their fluid behaviour when various embedded fluid flow characteristic parameters are considered. Mostly, the theoretically study of unsteady free convection flow of viscous and second grade fluids have been conducted in vertical plates. However, only a few researchers considered the problem of convective flow involve with inclined and rotating plate. Even, most of the results obtained in the previous study for the problem of the flow in infinite inclined and rotating plates are conducted in numerical analysis. Therefore, the study to explore the mathematical model for the problem of unsteady free convection flow of viscous and second grade fluids in an infinite inclined plate and rotating frame is significant. Hence, this thesis emphasized this matter and investigated the behaviour of viscous and second grade fluids in relation to the issue of unsteady MHD free convection flow passing an infinite inclined plate embedded in a porous medium with ramped and isothermal temperature, specifically, on the problems of:

(i) Problem 1: Unsteady MHD free convection flow of viscous and second grade fluids in a porous medium with ramped wall temperature.

(ii) Problem 2: Unsteady MHD free convection flow of viscous and second grade fluids in a porous medium with isothermal plate.

(iii) Problem 3: Unsteady rotating MHD free convection flow of viscous and second grade fluids in a porous medium with ramped wall temperature.

(iv) Problem 4: Unsteady rotating MHD free convection flow of viscous and second grade fluids in a porous medium with isothermal plate.

## 1.4 Objectives of the Study

This study investigated unsteady MHD free convection flow of viscous and second grade fluids in a porous medium with ramped and isothermal temperature. This thesis was extending and analyzing the mathematical modelling by formulating the appropriate governing equations with some physical conditions, as well as solving the resulting governing equation analytically:

(i) To obtain exact solutions of the velocity, temperature and concentration profiles by using the Laplace transform method.

(ii) To compute the skin-friction, Nusselt and Sherwood numbers from the obtained velocity, temperature and concentration profiles, respectively.

(iii) To analyze graphically the obtained exact solutions of velocity, temperature and concentration profiles as well as computed skin-friction, Nusselt and Sherwood numbers presented in tables.

## 1.5 Scope of the Study

This study is focused on unsteady free convection flows of Newtonian and non-Newtonian fluids. Second grade fluids are taken as non-Newtonian fluids. Free convection flows of Newtonian and second grade fluids past an infinite inclined plate are investigated in the presence of MHD and porosity with combined effects of heat and mass transfer. Different motions of both Newtonian and second grade fluids are investigated under the conditions of ramped wall temperature and isothermal plate. Analytical solutions of all the problems are obtained by using the method of Laplace transform. Mathematica and Mathcad softwares are used for plotting an exact results of velocity, temperature and concentration fields including computation of numerical results of skin-friction, Nusselt number and Sherwood number.

## 1.6 Research Methodology

The governing equations of momentum, energy and concentration are modeled for both viscous and second grade fluids. Under the assumption of incompressible fluid, the continuity equation is identically satisfied. The fluid is assumed electrically conducting under the assumption of a uniform magnetic field, and external magnetic field are neglected. Darcy’s law for viscous fluids and modified Darcy’s law for second grade fluids are incorporated. Initial and boundary conditions are introduced in each cases. The non-dimensional equations for each problem are found by defining some suitable dimensionless variables. The partial differential equations and the appropriate initial and boundary conditions governing the flows are transformed into a set of three and four ordinary differential equations with the transformed initial and boundary conditions. This transformation is achieved by using the Laplace transform technique. Laplace transform is an integral transform of a positive real variable $t$ (often time) to a function of a complex variable $s$ (frequency). The Laplace transform of $f(t)$ is defined as

$$\mathcal{L}\left\{f\left(t\right)\right\} = \int_{0}^{\infty} e^{-st} f\left(t\right) dt. \tag{1.1}$$

Usually, Laplace transform often denotes as

$$\mathcal{L}\left\{f\left(t\right)\right\} = F(s). \tag{1.2}$$

The transformed ordinary differential equations with initial and boundary conditions will form a well-posed mathematical model in each case. Finally, the solutions are obtained by finding the inverse Laplace transform, notated by

$$\mathcal{L}^{-1}\left\{F\left(s\right)\right\} = f(t). \tag{1.3}$$

These solutions are verified by the graphs itself, and also reducing them to the known published results as their limiting cases. Each results for skin-friction, Nusselt number and Sherwood number are also calculated. Physical aspects of the present work for velocity, temperature and concentration profiles are plotted graphically using Mathematica and Mathcad softwares. Other than that, graphical results can aid for accuracy purpose of the obtained solutions by satisfying all the imposed initial and boundary conditions. In addition, the numerical results of skin-friction, Nusselt number and Sherwood number are calculated and presented in tables.

## 1.7 Significant of Research

This study significantly provides the profound understanding of heat and mass transfer analysis on unsteady free convection fluid flow. The unsteady free convection problems have attracted a considerable amount of interest because of the importance in atmospheric and oceanic circulations, control of crystal growth system, electromagnetic material processing, lubrication control of high speed spinning machine components with magnetic fields, magneto-astronautical, magnetohydrodynamic energy generators, planetary fluid dynamics, nuclear reactors, power transformers and vortex chambers (Takhar *et al.*, 1987; Thakur and Mishra, 1988; Han *et al.*, 1988; Friedrich *et al.*, 1997; Vogin and Alemany, 2007; Yasuda, 2007; Toki, 2009b; Zueco and Beg, 2010).

This study also provided accurate exact solutions for the mathematical models containing ramped wall temperature and isothermal plate. These exact solutions can be used to check the correctness of the results obtained through numerical schemes.

## 1.8 Thesis Organization

Chapter 1 begins with a brief introduction, then research background. After that it followed by problem statements, research objectives, scope of study, research methodology and significance of the present research.

Chapter 2 concentrates on literature review of the research problems acknowledged in the objectives.

Chapter 3 focuses on mathematical modeling of governing equations for the convection flow of MHD viscous and second grade fluids past an infinite inclined plate in a porous medium with ramped wall. Radiation effects are considered in the energy equation.

Chapter 4 is an extension of work studied in Chapter 3 by considering isothermal plate.

Chapter 5 presents mathematical modeling of governing equations for the rotating free convection flow of MHD viscous and second grade fluids past an infinite inclined plate in a porous medium with heat generation/absorption. Rotating in the momentum equation and heat generation/absorption in the energy equation and constant concentration equation are also considered.

Chapter 6 studies conjugate of heat and mass transfer and rotating effects of unsteady MHD free convection flow of viscous and second grade fluids past an infinite inclined plate in a porous medium with heat generation/absorption. This problem is solved by isothermal plate.

Chapter 7 concludes the present work. A number of recommendations for future research is also presented.

presented from Figures 6.3 to 6.21. Meanwhile, the numerical results for skin-frictions were shown in tabular forms, Table 6.1 for the case of viscous fluids, and second grade fluids case in Table 6.2. From table 6.1, it was found that skin-friction increases for each increasing of $\Omega$, $\phi$, $M$, Pr and $Sc$ whereas decreases for $Gm$, $Gr$, $K$, $\theta$, and $t$. However, skin-friction increases when $\phi$, $M$, Pr and $Sc$ increases in the imaginary part cases. For the real part of second grade fluids, skin-friction decreases for the increasing value of $\alpha$, $\phi$, $M$, Pr, $\theta$ and $Sc$, whereas $\Omega$, $Gm$, $Gr$, $K$ and $t$ were found opposite directions to that $\alpha$, $\phi$, $M$, Pr, $\theta$ and $Sc$. For the imaginary part, it is observed that skin-friction increases with increasing $\Omega$, $Gm$, $M$, Pr and $\theta$. All of these interesting facts were shown in Table 6.2. Table 6.3 provides numerical results of Nusselt number for different Pr, $\theta$ and $t$. Nusselt number increases with increasing Pr and $\theta$ but decreases when $t$ increased.

## 7.2 Suggestion for Future Research

In this study, viscous and second grade fluids with ramped wall temperature and isothermal plate are considered. Hence, there are so many aspects can be considered for the future to extend this study. Some recommendations can be done are:

(i) Use another physical geometries such as cylindrical and spherical coordinates.

(ii) Add more parameters and use another initial and boundary conditions.

(iii) Extend to the slip boundary condition.

(iv) Extend to others subclasses of non-Newtonian fluids such as third grade fluid, Burger fluid, Maxwell fluid, and so on.

(v) Solve subclasses of non-Newtonian fluids using Laplace transform technique.

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