REPRESENTATION OF MULTI-CONNECTED SYSTEM OF FUZZY STATE SPACE MODELING (FSSM) IN POTENTIAL METHOD BASED ON A NETWORK CONTEXT

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For Him who always give me strength

And

To my beloved family...

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All thanks and praise is due to the almighty Allah (SWT). Our creator, the sustainer and the owner of the universe, Master of the day of reckoning. May His bountiful and endless blessing be onto our noble Prophet Muhammad (S.A.W), members of his household, his Sahabas and all that follow his teaching up to the day of Judgement.

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JIBRIL AMINU

ABSTRACT

The complexity of a system of Fuzzy State Space Modeling (FSSM) is the reason that leads to the main objective of this research. A multi-connected system of Fuzzy State Space Model is made up of several components, each of which performs a function. These components are interconnected in some manner and determine how the overall system operates. In this study, we study the concept of graph, network system and network projections which are the requisite knowledge to potential method. Finally, the multi-connected system of FSSM of type A Namely Feeder, Common feeder and greatest common feeder are transformed into potential method using various method of transformation.

ABSTRAK

Pemodelan Ruang Keadaan Kabur (FSSM) adalah sangat kompleks serta membawa kepada objektif utama kajian ini. Suatu sistem Pemodelan Ruang Keadaan Kabur multi-terkait terdiri daripada beberapa komponen. Setiap komponen tersebut mempunyai fungsi yang ditentukan. Komponen ini saling terkait serta menentukan bagaimana keseluruhan sistem itu beroperasi. Dalam penyelidikan ini, konsep graf, sistem rangkaian dan unjuran rangkaian dikaji. Konsep-konsep ini diperlukan bagi memahami kaedah potensi. Yang terakhir, sistem pelbagai sambungan FSSM jernis A yang dinamakan Masukkan, Masukkan Biasa dan Masukkan Terbesar ditukarkan kepada kaedah potensi dengan menggunakan kaedah penukaran yang pelbagai.

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U	-	Universe of Discourse or universal set
$A, B, C, X, \ldots$	-	Arbitrary set
$\mu_A(x)$	-	Membership function for set A
$\{x_1, x_2, x_3, \ldots\}$	-	Set of elements $x_1, x_2, x_3, \ldots$
$a, b, c, \ldots$	-	Elements of arbitrary sets
$Supp(A)$	-	Support of set A
K	-	Real number K
r	-	Least Upper Bound of K
R	-	Relation
$Sup\ K$	-	Supremum of K
$h(A)$	-	Height of set A
$Sup\ A(x)$	-	Supremum of set A
$A \cup B$	-	Union of set A and B
$A \cap B$	-	Intersection of set A and B
$\bar{A}$	-	Complement of A
$\forall$	-	For all
$\in$	-	Element of or belong to
$\notin$	-	Not element of
Min	-	Minimum
Max	-	Maximum
$\tilde{u}(t)$	-	Fuzzified input vector
$\tilde{y}(t)$	-	Fuzzified output vector
A_{PXP}	-	State matrix A
B_{PXn}	-	Input matrix B

C_{mXP}	-	Output matrix C
B_{eXv}	-	Incidence matrix
S_{GF}	-	Collection of interconnected FSSM systems
S_{gF}	-	Common feeder
S_{FF}	-	Greatest common feeder
$X_X X$	-	Cartesian product
$\sim$	-	Equivalence
$a\|b$	-	a divide b
$\mathbb{Z}$	-	The set of integers
$\mathbb{R}$	-	The set of real numbers
G	-	Graph
V	-	Set of vertices (or nodes)
E	-	Set of edges
deg(v)	-	Degree of vertex
w_{ij}	-	Weight between node i and node j
$\sum$	-	Summation
p	-	Number of mutualistic partner between node i and node j
N_p	-	Number of Authors in paper p
α	-	arc of directed graphs
$\mathcal{A}$	-	Set of arcs
$F(\alpha)$	-	Flow component

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

L. A. Zadeh in the early 1960s proposed a Fuzzy set as the general model of uncertainties inherent in engineering and systems (Zadeh, 1965). In his passionate effort, traditional concept was applied in justifying the truth value of membership and true values, respectively. Conversely, this effort has been faced with two major problems during the physical modeling in the real world. Firstly, much of the situations are not fresh and elegant, so they cannot be correctly described. Secondly, a complete description of real systems often requires more detailed data than humans can perceive the same time, process and understand. Therefore, he further stressed the approach to modeling uncertainties arising in the ordinary course of human thought. According to him, the basic limit of our ability to characterize the dynamics of complex systems is described by “principle of incompatibility” (Zadeh, 1973) which states that the more complex the system is, the less our ability to more precisely characterizes the relevant counter. The significance of this assumption is based on fact that we may not be able to access an accurate model at all in the normal sense for complex dynamics systems. In this case, we have to solve the soil moist formulae suitable to represent the models of the system. However, even if we can get the same quantitative models for complex systems, the original offer model often shows much information of systems. This tends to blur the attractive features that are useful for another reason. Hence, it is important that the complex system made just right "resolution" without going into unnecessary details.

A system is a combination of variable components connected with each other and work together to perform a function (Paraskevopoulos, 2002). Lately, various efforts have been undertaken to synthesize and analyses the control systems in order to create a club approach to the system usually with multiple inputs and multiple outputs. One of the ways used is the state space analysis.

A System model is a model that explains the concept and representation of the system. The system can be described by differential equations including mechanical systems, electronic systems, thermodynamic system, hydraulic system or a chemical system, etc. In response to the input and output of the system, the differential equations can be solved and then, the characteristics of the system can be analysed. Model systems are needed to describe and represent all the various views.

System Modeling plays a vital role in different areas of Engineering (Ismail, 2005). The significant feature of modeling a system is the development of an input and output scheme that will describe behaviour of the system on the working range. Control System modeling is an area of specialization, more general to mathematical modeling. In addition, it provides an approach in modeling and operating the system, and determines the relationship between cause and effect of the input and output manipulation measures. In processing and analysing the system, two functions must perform; Modeling the System and solving for the response of the model. The foundation of establishing a model of the system is the physical laws of the elements of the system and the interconnection between them that must be hold.

The implementation and use of mathematical models to explore some of the features of the relationship between cause and effect of the system is called a solution of the model. These could include time analytical information of simple model and the more complicated computer solution of the model.

Modeling is one of the most important tasks in control system design (Ismail, 2005). A model that can express the dynamics of a system correctly will lead to an appropriate controller design. Mathematical models of control systems are mathematical expressions which are describe by the relationships among system inputs, outputs and other inner variables. Establishing a mathematical model in describing a control system is the foundation for analysis and design of control systems. A mathematical model should reflect the dynamics of a control system and be suitable for analysis of the system. Thus, when we construct the model, we should simplify the problem to obtain the approximate model which satisfies the requirements of accuracy.

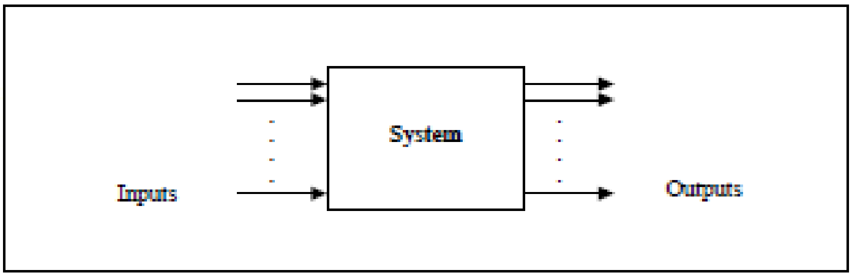

Figure 1.1: Mathematical Model of a System

Mathematical models of control systems can be established by theoretical analysis or practical experiments. The theoretical analysis method is to analyze the system according to physics or chemistry rules (such as Kirchhoff's voltage laws for electrical systems, Newton's laws for mechanical systems and Law of Thermodynamics). The experimental method is to approximate the system by the mathematical model according to the outputs of certain test input signals, which is also called system identification. System identification has been developed into an independent subject.

Researches in recent years proposed several types of mathematical models for the description of systems (Taufiq, 2007). The selection and the identification of appropriate mathematical representations are of central importance in the analysis of systems. The most obvious choice for this mathematical model in a lot of cases is the state space representation since the major part of modern system and control theory such as the design of observers, filters and optimal controllers regards this very efficient and compact representation. The state space models come from the state-space equations. State-space equation or simply state equation is a description in the time domain which may be applied to a very wide category of systems. This model has special theoretical, computational and practical importance for the following main reasons (Paraskevopoulos, 2002):

1. State equations can describe a large category of systems, such as linear and nonlinear systems, time-invariant and time-varying systems, systems with time delays, systems with nonzero initial conditions and others.

2. Due to the fact that state equations are a set of first-order differential equations, they can be easily programmed and simulated on both digital and analog computers.

3. State equations by their very nature greatly facilitate both in formulating and subsequently in investigating a great variety of properties in system theory such as stability, controllability and observability. They also facilitate the study of fundamental control problems, such as pole placement, optimal and stochastic control and adaptive and robust control.

4. State equations provide a more complete description of a system than the other methods such as original differential equations. This is because state equations involve very important additional information about the system namely the state of the system. This information is particularly revealing of the structure of the system.

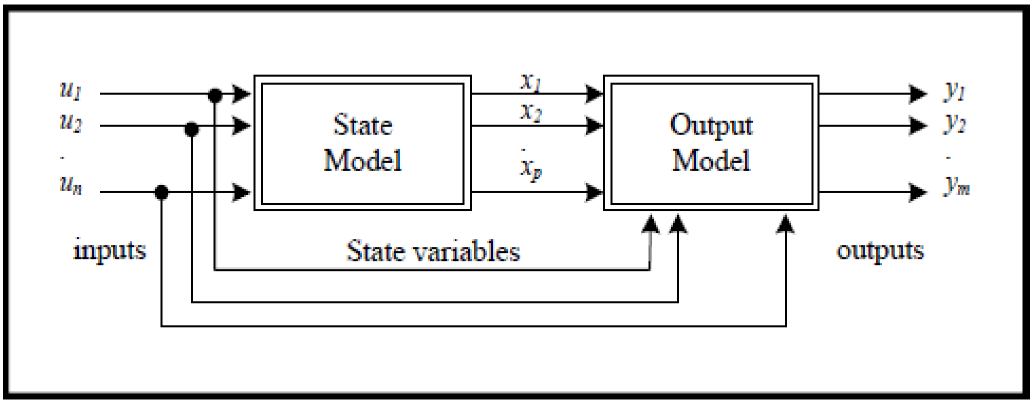

Figure 1.2: State space representation of a system

A dynamic system consisting of a finite number of lumped elements may be described by ordinary differential equations in which time is the independent variable (Ogata, 2010). By use of vector-matrix notation, an nth-order differential equation may be expressed by a first order vector-matrix differential equation. If n elements of the vector are a set of state variables, then the vector-matrix differential equation is a *state* equation.

The development of the algorithm for FSSM is based on three phases of a fuzzy system; Fuzzification, fuzzy environment (Ahmad, 1998) and defuzzification. After which Ismail (2005) developed Fuzzy State Space Modeling FSSM

This research is an extension to the research on Some Aspects of Number Theory Approach on the Multi-connect Systems of Fuzzy State Space Model in Taufiq (2007). In his work, he explores the concept of number theory to describe connections between systems, in particular interconnected systems of Fuzzy State Space Model (FSSM). However, he also presents potential link and adopted it to his work connectivity of FSSM.

Our aim in this research is to study potential method in relation to multi-connected Fuzzy State Space Model using the potential link concept in Taufiq (2007). This will be achieved using network projection as discussed by Pedron *et al* (2011) and Opsahl (2013).

1.2 Problem Statement

The increasing significance of Fuzzy State Space Modeling (FSSM) in the field of mathematics and engineering have captured the research interest of many researchers across the globe. Although, few authors have applied FSSM in various methods. However, the presentation of FSSM to Potential Method (PM) has not been reported. Therefore, the present study aims at presenting Multi-connect Systems of Fuzzy State Space Model in PM based on a Network context.

1.3 Objectives of the Study

The main objectives of this research are:

1. To explore the concepts and some properties of potential method.
2. To present a potential method based on multi-connected Fuzzy State Space Model.

1.4 Scope of this Study

This research will focus on the study of the concepts of potential method and its relation base on multi-connected fuzzy state space systems of type A presented by Taufiq (2007).

1.5 Significances of this Study

The significant of this research is to highlight the complexity of fuzzy state space modeling which is use to present potential method with regard to multi-connected Fuzzy State Space systems.

1.6 Thesis Structure and Organization

This dissertation is organized base on five chapters. Apart from this Chapter One, that discusses the background in fuzzy sets, systems and system models, mathematical modeling of a system as well as the state space modeling. The chapter also discusses the objective, implication and scope of this study.

Chapter Two will gives an overview on the Fuzzy State Space Modeling FSSM. The chapter will start with membership function, fuzzy operation, fuzzy systems as well as fuzzy modeling. It also presents FSSM in multi-connected systems and algebraic view of systems of multi connected FSSM.

Chapter Three will discusses the graph theory, network system and potential method which are the requisite knowledge to this research.

However, Chapter Four covers the main objective of this research. It started with the highlight of the system of multi-connected FSSM, the definition of Potential method as well as brief description of network projection methods. Then, the chapter presents the transformation of multi-connected system of FSSM of type A base on potential method using the Binary projection method.

The last but not the least, Chapter Five will give the concluding remark on this research and recommendation for further study of FSSM hence the references.

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