# A MODIFIED MEAN-VARIANCE-CONDITIONAL VALUE AT RISK MODEL OF MULTI-OBJECTIVE PORTFOLIO OPTIMIZATION WITH AN APPLICATION IN FINANCE

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*To my beloved Parents, Family and my respected Supervisor*

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# ABSTRACT

This research focuses on the development of a portfolio optimization model based on the classic optimization method and a meta-heuristic algorithm. The main goal of a portfolio optimization model is to achieve maximum return with minimum investment risk by allocating capital based on a set of existing assets. Recently, mean-variance models have been improved to mean-variance-CVaR (MVC) model as a multi-objective portfolio optimization (MPO) problem which is difficult to be solved directly and optimally. In this work, a modified MVC model of portfolio optimization is constructed using the weighted sum method (WSM). In this method, each objective function of MVC model is given a weight. The optimization problem is then minimized as a weighted sum of the objective functions. The implementation of WSM enables the MVC model to be transformed from a multi-objective function to one with a single objective function. The modified MVC model is then solved using ant colony optimization (ACO) algorithm. This algorithm solves the MVC model by the number of ant colonies and the number of pheromone, a chemical creating trails for others to follow. The modified MVC model can be used in managing diverse investment portfolio, including stocks on the stock market and currency exchange. The applicability and effectiveness of the proposed method are demonstrated by solving a benchmark problem and a practical investment problem as examples. The data of practical examples are collected from the foreign currency exchange of Bank Negara Malaysia for the years 2012 and 2013. In conclusion, this thesis presented a hybrid optimization algorithm which utilizes a classical approach, WSM and a meta-heuristic approach, ACO to solve an MVC model of portfolio optimization.

# ABSTRAK

Kajian ini memberi tumpuan kepada pembangunan model pengoptimuman portfolio berdasarkan kaedah pengoptimuman klasik dan algoritma meta-heuristik. Matlamat utama model pengoptimuman portfolio adalah untuk mencapai pulangan maksimum, dengan risiko pelaburan minimum dengan memperuntukkan modal berdasarkan satu set aset yang ada. Kebelakangan ini, model min-varians telah diperbaiki kepada model min-varians-CVaR (MVC) sebagai masalah pengoptimuman portfolio pelbagai-tujuan (MPO) yang sukar untuk diselesaikan secara langsung dan secara optimum. Dalam penyelidikan ini, satu model MVC terubahsuai untuk pengoptimuman portfolio dibina menggunakan kaedah hasiltambah wajaran (WSM). Dalam kaedah ini, setiap fungsi objektif pada model MVC diberikan satu pemberat. Masalah pengoptimuman tersebut kemudiannya diminimumkan sebagai hasiltambah wajaran fungsi objektif. Pelaksanaan WSM membolehkan model MVC ditukarkan dari sebuah fungsi pelbagai-tujuan kepada satu fungsi objektif tunggal. Model MVC terubahsuai kemudiannya diselesaikan dengan menggunakan algoritma pengoptimuman koloni semut (ACO). Algoritma ini menyelesaikan model MVC dengan jumlah koloni semut dan jumlah pheromone, bahan kimia yang menghasilkan jejak supaya dapat diikuti oleh yang lain. Model MVC terubahsuai boleh digunakan dalam menguruskan portfolio pelaburan pelbagai, termasuk saham di pasaran saham dan pertukaran mata wang. Kepenggunaan dan keberkesanan kaedah yang dicadangkan telah ditunjukkan dengan menyelesaikan satu masalah penanda aras dan masalah pelaburan praktikal sebagai contoh. Data dari contoh-contoh praktikal dikumpulkan dari pertukaran mata wang asing Bank Negara Malaysia bagi tahun 2012 dan 2013. Kesimpulannya, tesis ini membentangkan satu algoritma pengoptimuman hibrid yang menggunakan pendekatan klasik, WSM dan pendekatan meta-heuristik, ACO untuk menyelesaikan model MVC pengoptimuman portfolio.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# LIST OF ABBREVIATIONS

| | | |
|---|---|---|
| *ACO* | - | Ant Colony Optimization |
| *ACOR* | - | Ant Colony Optimization in continues space |
| *MV* | - | Mean Variance |
| *WSM* | - | Weighted Sum Method |
| *MPO* | - | Multi-Objective Portfolio Optimization |
| *VaR* | - | Value at Risk |
| *CVaR* | - | Conditional Value at Risk |
| *DM* | - | Decision Maker |
| *PBO* | - | Permutation Based Optimization |
| *AWS* | - | Adaptive Weighted Sum |
| *BNM* | - | Bank Negara Malaysia |
| *RM* | - | Ringgit Malaysia |
| *MVSK* | - | Mean-Variance-Skewness-Kurtosis |
| *MVS* | - | Mean-Variance-Skewness |
| *MVC* | - | Mean-Variance-CVaR |
| *EDA* | - | Estimation of Distribution Algorithm |

# LIST OF SYMBOLS

| | | |
|---|---|---|
| $N$ | - | The number of assets that are available |
| $R_x$ | - | Return depending on a decision vector $x$ that belongs to a feasible set $A$ |
| $x_i$ | - | Proportion of investment in $i^{th}$ asset |
| $\Omega$ | - | The feasible set of solutions |
| $X$ | - | A solution of problem |
| $\mu_i$ | - | The expected mean of the $i^{th}$ asset |
| $\sigma^2(R_x)$ | - | The variance belonging to $R_x$ |
| $K$ | - | The number of assets to invest |
| $\delta_{ij}$ | - | Covariance among the return of assets |
| $\delta$ | - | Covariance matrix |
| $\alpha$ | - | Confidence level |
| $\varphi$ | - | Overall maximum |
| $x^*$ | - | Best solution |
| $f(x^*)$ | - | Pareto optimal |
| $\omega_i$ | - | Weight of objective functions |

# LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 Overview

In finance, a portfolio refers to a collection of investments. Usually a person with a certain amount of fund wants to obtain higher income than interests paid by saving accounts or fixed deposits. The investor tries to choose various assets to purchase with the hope of getting a better return. These assets are commonly shares on the stocks markets. They may also be commodities (gold, iron, aluminum, cements, petroleum, coffee, palm oil, etc.), fixed properties (houses, shops, apartments, and condominiums), share holiday resorts and many others. In recent years, due to fast movement of money electronically and high rate of fluctuations among currencies, they have also become popular assets for investments.

An investor is usually interested in getting good return of his/her investment. He/she may be very ambitious, wanting top returns, and not concerned about possible losses. However, few investments can be assured of gains without possibilities of losses. Thus the investor tries to select assets which will probably yield the best returns with the minimum of losses. In general, assets with higher returns are usually speculative, and they tend to have higher risks of losses. On the other hand, assets which are considered safe in the sense that they are quite sure to gain values are usually less likely to gain much. Depending on the personality of the investors, some choose to go for high returns, even though the risk of losses is higher. However, many prefer to choose a safer route and are not willing to suffer losses.

The unwillingness to lose on investment is called risk aversion. The profile of the investor in terms of intended amount of gain and risk aversion determines the types of assets to invest. This profile sets the tone for selection of assets and apportioning the fund into each asset.

Many investors create their own portfolio based on their own judgments and inclinations. But serious investors with big funds usually seek the help of experts. The usual scenario is the investor (customer) engages a financial institution which plans out the portfolio. It is the institution's responsibility to form the portfolio with maximum return, taking into consideration the customer's profile of intended gain and risk aversion level. Traditionally, the portfolio was created rather arbitrarily, based on the previous knowledge and experience on the assets. Normally the institution trusted with investment will try to present the customer with different situations to select options related to their risk aversion (Mavrotas, 2009). Each version may have different forecasts for expected returns and risks.

It is necessary for a more formalized method to optimize the portfolio returns. Portfolio optimization was first introduced by Markowitz (1952) via a framework of return/ variance risk (Yu *et al.*, 2011). The main problem is in finding a best solution to distribute a given fund on a set of existing assets. Maximization of return and minimizing risks are the two main aims. The user's risk aversion has a direct effect on the best solution. Two criteria are necessary for optimization of the portfolio: First is the set of solution to the portfolio optimization problem called "efficient frontier", or "pareto optimal". Second is the measure against the risk of the portfolio.

Markowitz's approach for portfolio optimization is based on the covariance measure, in which he proposed two criteria. The first is minimization of the risk, and the second is maximization of the expected return. These can be defined as follows:

$$\mu = E(R) = max \sum_{i=1}^{M} \mu_i x_i \,,$$

$$\sigma^2 = \text{var(R)} = \min \sum_{i=1}^{M} \sum_{j=1}^{M} \delta_{ij} x_i x_j,$$

$$\text{s.t.} \quad \sum_{i=1}^{M} x_i = 1, x_i \geq 0, i = 1, \dots, M$$

where, $M$ is the number of existing assets to invest, $x_i$ is the proportion of budget invested in asset $i \in \{1, \dots, M\}$; $\boldsymbol{x} = [x_1, \dots, x_M]^T \in \mathbb{R}^M$ is the solution vector of $M$ dimensions; $\mu_i$ denotes the expected return of asset $i \in \{1, \dots, M\}$; $\sum_{i=1}^{M} \sum_{j=1}^{M} \delta_{ij} x_i x_j$ is the variance among the returns of assets $i, j \in \{1, \dots, M\}$, and $\boldsymbol{\delta} = (\delta_{ij})_{i=1,\dots,M, j=1,\dots,M}$ shows the corresponding $M \times M$ covariance matrix (Ehrgott *et al.*, 2004).

## 1.2 Background of Problem

Numerous types of risk measures for portfolio optimization have been introduced since Markowitz's theory was proposed. The main aim of portfolio optimization model is maximum return on investment with lower risk. Two most popular types of risk measures for portfolio optimization are value at risk (VaR) and conditional value-at-risk (CVaR). VaR refers to the maximum of expected loss on an investment, related to specific time period and particular level of confidence. CVaR represents the expected loss conditional on exceeding a VaR threshold. These measures have been shown to be efficient for the models of optimization (Chen *et al.*, 2012).

The original method of modeling the risk for portfolio optimization is based on mean variance (MV). The expected value of returns and the value of a risk are measured as two statistical quantities that are computable via MV model. This is a practical model for decision making in finance (Aboulaich *et al.*, 2010).

Yu *et al*. (2011) used the weighted sum method (WSM) to model the mean-CVaR model of portfolio optimization. In practice, their model was considered as one of the multi-objective problem. The mean and CVaR are two objective functions of their model. They used multi-objective fuzzy programming (MFP) to solve their proposed model.

Actually using three objective functions in MVC to minimize the risk and to maximize the return of the portfolio selection increases the performance of their model. Also, the MVC model which includes three objective financial functions is useful for decision making in finance. The MV conditional value at risk (MVC) model added a parameter, the CVaR to the objective function. The MVC contains three parameters (Aboulaich *et al*., 2010):

1) the expected returns ($E$),
2) the variance ($\sigma^2$) and
3) the CVaR at a specified confidence level $\alpha \in (0,1)$

The MVC model based on Aboulaich *et al*. (2010) is as follows:

$$min\{CVaR, -E, var\},$$
$$s.t \quad x \in \left\{ (x_1, \dots, x_n) \middle| \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j \in \{1, \dots, n\} \right\}.$$

This model has one multi-objective system for portfolio optimization that consists of three objective functions. The first objective function is to minimize the CVaR of portfolio optimization. The second objective function is to maximize the expected value of return of the portfolio. The final objective function is to minimize the variance of returns. However, as a non-linear portfolio model, MVC is based on a rather complicated quadratic structure which is NP-hard problem. The MVC model was solved by using simulated annealing (SA).

## 1.3 Problem Statement

The main goal of this study is to modify the current model and simplify it by using WSM. The MVC model is defined in terms of mathematical structure which is used to explain the procedure via a recursive algorithm.

However, no attempts have been made to use meta-heuristic approaches to solve the MVC model of multi-objective portfolio optimization, as we intend to do here, particularly ACO. Hence, it gives us the motivation to develop a model of portfolio optimization based on the ACO method.

In this approach, each MVC model is first defined by a mathematical structure. Next, the simulated pseudo code of that model is presented based on ACO procedure, in which the pseudo code is transformed into ACO algorithm to solve the problem. The aim of implementing ACO algorithm consists of finding the pareto solutions based on the constraints of MVC model. This approach helps to increase the efficiency in finding the best solution.

## 1.4 Objectives of Study

The following are the objectives of this research:

1. To identify the variables of portfolio optimization model related to investment.
2. To develop a MVC model for multi-objective portfolio optimization via weighted sum method (WSM).
3. To solve the new model via ACO.
4. To apply the modified MVC model of portfolio optimization to investment.

## 1.5 Scope of the Study

This study presents the identification, analysis, and improvement of the MVC model. We first carry out a review on MVC model of portfolio optimization based on multi-objective approach.

Next, we employ WSM to MVC model of portfolio optimization in multi-objective approach to transform the model to one linear combination with specific coefficients. Then, the permutation based optimization (PBO) is used to solve the improved model.

In this study, the algorithm is run with MATLAB software version 2009. The test data is taken from two datasets. The first dataset is taken from Malaysian Exchange during 2012 and 2013. The second one is taken from China stoke market during 2010 and 2011.

Minimizing the CVaR, maximizing the return and minimizing the variance are done by choosing the best coefficients through PBO. ACO, as one of the meta-heuristic methods, is also utilized.

In addition, the thesis also includes

- Presentation of WSM as a classic optimization methods to develop the modified MVC model
- Application of ACO algorithm to solve the model and evaluating the proposed modified model
- Implementation of MATLAB simulation to evaluate the results of proposed model

## 1.6 Significance of Study

This study promotes the mathematical modeling and applications of MVC model of MPO. MVC model of portfolio optimization (which employs mean, variance and CVaR) is shown to give better results than current risk measure of portfolio (see Chapter 4 and 6). In addition, an important contribution of the research is the presentation of ACO as an alternative procedure in solving of the MPO (Chapter 5 and 6). The modified model is found to be suitable for currencies investment as shown in the examples presented later.

In general, there are two main purposes of this research, which are of theoretical and practical significance.

### 1.6.1 Contribution to Theory

- This study will add to the body of knowledge and the advancement of solution of MVC problem by using the WSM.

- This research develops a hybrid method of WSM and ACO to solve MVC in the multi-objective optimization approach.

### 1.6.2 Contribution to Practice

- This study modifies the mathematical model of MVC model for MPO.

- This research proposes a new solution method that is derived through the integration of WSM and ACO approach.

- The improved current model of portfolio optimization model can help extend financial activity, which helps to create the optimal set of assets (currencies) to investment.

## 1.7 Thesis Outline

This research is arranged into seven chapters. Chapter 2 discusses the current models and theoretical results, covered in the literature review of MPO in conventional finance. Chapter 3 presents the methodology of this research. This chapter also includes the overall research design and research framework of this research. Chapter 4 discusses the development of modified MVC model of portfolio through WSM. To achieve this aim, each objective function of MVC model is assigned a weighted coefficient. The optimization problem is then minimized as a weighted sum of the objectives. The application of WSM enables the MVC model to be transformed from a multi-objective function to a single objective function. Then the developed MVC model is solved via PBO. Also, Chapter 4 presents the comparison between the results of the MVC model based on PBO and LINGO software. Chapter 5 discusses the procedure of obtaining the solution for the new model via ACOR. This algorithm parameterizes the MVC model by the number of ant colonies and the number of pheromone trails. The solution is then compared with the result from LINGO. Chapter 6 presents the application, and comparison between the results of the MVC model based on PBO, LINGO and ACOR via illustrative examples. A practical example of investment in the stock market as a benchmark problem is also considered. The last chapter that is Chapter 7, presents the main contributions of the thesis, limitation of study and direction for future researches.

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