

PREDICTION OF MESO-SCALE COMBUSTION USING DIFFERENT
TURBULENCE MODEL

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DEDICATION

To my lovely mother, who gave me endless love, trust, constant encouragement over the years, and for her prayers.

To my Family, for their patience, support, love, and for enduring the ups and downs during the completion of this thesis.

This thesis is dedicated to them.

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ABSTRACT

An investigation on the adequacy of different 2-equations turbulence models to characterize the non-premixed meso-scale swirl flow combustion is presented in this paper. The RAS 2-equations turbulence models studied include the standard $k-\varepsilon$ model, RNG (Renormalization Group) $k-\varepsilon$ and SST (Shear Stress Transport) $k-\omega$ turbulence models. The open source CFD software openFOAM is utilized to characterize high resolution flow feature and to determine simulated results of the turbulence models investigated that best capture the combustion characteristics in terms of temperature prediction at various equivalence ratios and graphical representation of the stoichiometric mixture fractions that can be correlated to the outlet flame feature produced in experimental setup as well as to generate comparison of the temperature and velocity profiles captured along the length of the meso-scale combustor. The examination of the velocity and pressure contour also reveal that the velocity decays along the length of combustor with prediction of adverse velocity in centre axis near the outlet induced by the pressure gradient between the lower and upper half of the combustor denoting one of the main feature of the swirl flow. The simulated results show that SST $k-\omega$ turbulence models produces the highest proximity with the experimental data with the lowest overall percentage error around 4.26% registered while the stoichiometric mixture fraction graphical presentation measured in terms of its development of surface features with increasing equivalence ratio demonstrates that SST $k-\omega$ turbulence model produces the most steady development among the other tested turbulence model against the outlet flame features.

ABSTRAK

Siasatan terhadap kecukupan model 2-persamaan pergolakan yang berbeza untuk mencirikan meso-besaran pembakaran aliran pusaran bukan pracampuran dibentangkan dalam kertas ini. RAS model 2-persamaan pergolakan dikaji termasuk model $k-\epsilon$ standard, RNG (renormalization Group) $k-\epsilon$ dan SST (Shear Stress Pengangkutan) $k-\omega$ model pergolakan. Sumber terbuka CFD perisian openFOAM digunakan untuk mencirikan ciri aliran resolusi tinggi dan untuk menentukan keputusan simulasi model pergolakan disiasat yang menangkap terbaik ciri-ciri pembakaran dari segi ramalan suhu pada pelbagai nisbah kesetaraan dan perwakilan grafik pecahan campuran stoikiometri yang boleh dikaitkan dengan ciri outlet api dihasilkan dalam persediaan eksperimen dan juga untuk menjana perbandingan suhu dan halaju profil ditangkap bersama-sama panjang pembakar meso-besaran. Pemeriksaan halaju dan tekanan kontur juga mendedahkan bahawa halaju mereput di sepanjang pembakar dengan ramalan halaju buruk dalam paksi pusat berdekatan dengan salur keluar yang disebabkan oleh kecerunan tekanan antara bahagian bawah dan atas pembakar menandakan salah satu daripada ciri-ciri utama aliran pusaran. Keputusan simulasi menunjukkan bahawa SST model $k-\omega$ pergolakan menghasilkan jarak tertinggi dengan data eksperimen dengan ralat peratusan keseluruhan yang paling rendah sekitar 4.26% berdaftar manakala campuran pecahan persembahan grafik stoikiometri diukur dari segi pembangunan berprestij permukaan dengan nisbah setara yang semakin meningkat menunjukkan yang SST model $k-\omega$ pergolakan menghasilkan pembangunan yang paling mantap di kalangan model gelora menguji lain terhadap ciri-outlet api.

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LIST OF SYMBOLS

L	-	Linear system characteristic
D	-	Diameter
E	-	Energy related to static enthalpy (J)
h	-	Enthalpy (J)
T	-	Temperature (K)
P	-	Pressure
λ	-	Mean free path
δ	-	Flame thickness
Pr	-	Prandtl number
ρ	-	Density (kg/m ³)
δ_{ij}	-	Kronecker delta
k	-	Turbulence kinematic energy (J/kg)
ε	-	Dissipation rate (J/kg.s)
μ	-	Laminar viscosity [kg/m.s]
μ_t	-	Turbulent viscosity [kg/m.s]
μ_{eff}	-	Effective viscosity [kg/m.s]
ν	-	Kinematic viscosity
σ_k	-	Turbulent Prandtl number in k

σ_ε	-	Turbulent Prandtl number in ε
σ_ω	-	Turbulent Prandtl number in ω
ω	-	Specific dissipation rate of k/ε
i	-	In x direction
j	-	In y direction
k	-	In z direction

LIST OF ABBREVIATIONS

FOAM	-	Field Operation And Manipulation
CFD	-	Computational Fluid Dynamics
MEM's	-	Micro Electrical Mechanical system
RANS	-	Reynolds Averaged Navier Stokes
RNG	-	Renormalization Group
SST	-	Shear Stress Transport

CHAPTER 1

INTRODUCTION

1.1 Research background

Combustions at small scale are collecting growing attention nowadays along with the comprehensive potential developments of various applications including that of electrical power, heat generation and mechanical power sources [1,2,3,4]. For example the possible applications are actuators, sensors, portable electrical devices, rovers, robots, thrusters, unmanned air vehicles, mechanical back-up power source for air conditioning system in hybrid vehicles, industrial heating devices as well as freight transportation. These, micro and mesoscale combustors are designed to cater for the power generations of miniature devices. Thus, the concept of the applications mentioned is based on the higher densities hydrocarbon fuels than the existing contemporary batteries which only exhibiting energy densities of about 0.20Kw/Kg that can only support a few hours of notebook computers and video cameras. Moreover, modern batteries will require several hours to be fully recharged while exhibiting limiting rechargeable cycles. Besides the increasing demands for micro and mesoscale combustion devices, the working principles of combustion at that level also gathers interests through the technical aspects of the combustion to solve problems such as quenching issue due to large surface to volume ratio of small scale devices. Thus, accountable towards the general approach, the flame thickness should be reduced so that scale of the combustor reduces. In order to fulfil these objectives, several experimental techniques such as intensification of the pressure for reducing the molecular distance, application of special types of fuel or oxidants used for

intensifying burning rate, and the application of catalytic reactions for preventing termination of the chemical chain reaction have been implemented [5,6] In addition good chemical and thermal stability managements are needed to produce stable combustion at the level of meso or microscale combustion. Hence, swirl flow combustion was introduced and study in-depth promote better air/fuel turbulent mixing in these micro-combustors. Closed-system swirling flows are widely used in industrial combustion applications. In these applications, the internal recirculation zone created by the negative pressure gradient and vortex dissipation of the swirling flows can further escalate the mixing of air and fuel, while harbouring the flame within the combustion zone.

1.2 Problem statement

In previous studies [7,8] the experimental work carried out on stable flame region results in the production of different combination of equivalence ratio and their respective air mass flow rate ranging from 40 mg/s to 170mg/s. With the knowledge of stable flame region in the investigated asymmetric meso-scale combustor, important findings such as the heat loss and generation according to different air mass flow rate, measurement of different outlet flame temperature with varying equivalence ratio and relationship of swirl flame at different equivalence ratio are reported. The computational work carried out using the viscous model encompassing standard $k-\varepsilon$ and RNG $k-\varepsilon$ lack the argument regarding the accuracy of the turbulence model used to characterize the meso-scale combustion. Other CFD models employed in previous research also include the discrete ordinate in radiation model and eddy dissipation turbulence model which is part of the reaction model in Ansys, in which all the documented results do not show comparison in terms of high resolution of combustion characterization as well as the conclusion on the turbulence model that best describes the combustion behaviour in meso-scale swirl flow. The computational resources of reaction and radiation model in large computations are expensive in commercial softwares, hence high fidelity simulation requires the expense of large parallelization of the computation which is costly using commercial software. In addition, no simplification on CFD approach while retaining the relevancy in relation to

the experimental results have produced in previous study as the reaction mechanism includes detailed chemistry which can be costly commutatively.

It is noted that previous studies [7,8] focused on the chemical efficiency of combustion by direct measure of the exhaust product via experimental procedure but combustion is also a chemical process in which study of the interactions between the species are important especially in meso-scale combustion where the residence time of species is relatively shorter than that of chemical time. In addition, the chemical kinetics aspect of the meso-scale combustion is not addressed in the previous studies [7,8]. Thus, it is crucial to establish the chemical mass species distribution profile throughout the combustion process along with the rate of combustion changes within the meso-scale combustor in order to obtain the behaviour of the combustion species inside the combustor.

It should be highlighted that the computational approach in the previous studies [7,8] involved the usage of commercial computational fluid dynamics software which is not cost free in terms of licencing. Hence, open source computational approach is important for future extended computational research on meso-scale combustor intended for any engineering professionals at zero cost since there is no licencing cost imposed on the openFOAM. Furthermore, previous studies lack computational approach in investigating the fuel-air mixing in meso-scale combustor as well no work documented for the proposed turbulence models that best describes the combustion in meso-scale combustor.

1.3 Project objectives

The following are the project objectives of the present study entitled “Prediction of Meso scale combustion using different turbulence model”

1. To analyze the combustion characteristics of a meso-scale combustor using CFD.
2. To study the effects of turbulence model on the prediction of combustion characteristics.

1.4 Scope of the project

The scope of the project defined below in this section below set the domain of the results expected from the project objectives in order to achieve the declared objectives.

1. Validation of prerequisite characteristics of the meso-scale combustor by referring to reference publications [7,8] which include the relationship between temperature, species fraction profile, equivalence ratio and thermal output.
2. Investigate the differences in terms of combustion characteristic of the meso-scale combustion utilizing the standard k- ϵ , RANS k- ϵ and SST k- ω turbulence models with turbulence properties comparison included.

It should be highlighted that the CFD simulation of the meso-scale combustor is within adiabatic temperature without modelling the heat loss via the wall and the meshing algorithm will not be investigated in depth thus the simulation will adopt the simplest mesh generation method using gmsh software.

1.5 Thesis outline

The thesis will cover 5 chapters. Chapter 2 and 3 will entail the literature review and methodology respectively. Chapter 2 will provide insights towards the experimental and computational knowledge associated the current work while chapter 3 will outline the computational approach taken to conduct the CFD simulation via openFOAM. Chapter 4 will comprise of the results and discussions section where high resolution CFD results will be discussed to show the features in swirl flow combustion computatively as well as providing the validation results and tested turbulence models comparison results followed by engineering meaningful discussion. In chapter 5, the whole chapter will be segmented into 2 sections namely the major conclusions drawn from the master project and the future works that should be the sequential research focusing on improving the current work in this master project.

REFERENCES

1. Fernandez-Pello, A.C., 2002. Micropower generation using combustion: issues and approaches. *Proceedings of the Combustion Institute*, 29(1), pp.883-899.
2. N. Chigier, T. Gemci, in: 41st Aerospace Sciences Meeting and Exhibit, AIAA-2003-670, 2003.
3. D. Dunn-Rankin, E.M. Leal, D.C. Walther, *Progress in Energy and Combustion Science* 31 (2005) 422–465.
4. Y. Ju, K. Maruta, *Progress in Energy and Combustion Science*, in press.
5. Okamasa, T., Lee, G. G., Suzuki, Y., Kasagi, N., & Matsuda, S. (2006). Development of a micro catalytic combustor using high-precision ceramic tape casting. *Journal of Micromechanics and Micro-engineering*, 16(9), S198.
6. Ahn, J., Eastwood, C., Sitzki, L., & Ronney, P. D. (2005). Gas-phase and catalytic combustion in heat-recirculating burners. *Proceedings of the Combustion Institute*, 30(2), 2463-2472.
7. Khaleghi, M., Hosseini, S.E. and Wahid, M.A., 2015. Vortex combustion and heat transfer in meso-scale with thermal recuperation. *International Communications in Heat and Mass Transfer*, 66, pp.250-258.
8. Khaleghi, M., Hosseini, S.E. and Wahid, M.A., 2015. Investigations of asymmetric non-premixed meso-scale vortex combustion. *Applied Thermal Engineering*, 81, pp.140-153.
9. Jacobson, S.A. and Epstein, A.H., 2003. An informal survey of power MEMS. *Proc. ISMME K*, 18.
10. Ahmed, M., Shirsat, V., Choudhuri, A. and Gupta, A., 2008. An Investigation on the Dynamics of Mesocombustors. In 46th AIAA Aerospace Sciences Meeting and Exhibit (p. 1138).

11. Chuah, K.H., K. Kuwana and K. Saito, Modeling a fire whirl generated over a 5-cm-diameter methanol pool fire. *Combustion and Flame*, 2009. 156(9): p.1828–1833.
12. Sitzki, L., Borer, K., Schuster, E., Ronney, P.D. and Wussow, S., 2001, June. Combustion in microscale heat-recirculating burners. In *The Third Asia-Pacific Conference on Combustion* (Vol. 6, pp. 11-14). Seoul, Korea.
13. Sher, I., Levinzon-Sher, D. and Sher, E., 2009. Miniaturization limitations of HCCI internal combustion engines. *Applied Thermal Engineering*, 29(2), pp.400-411.
14. Jones, A.R., Lloyd, S.A. and Weinberg, F.J., 1978, March. Combustion in heat exchangers. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* (Vol. 360, No. 1700, pp. 97-115). The Royal Society.
15. Mishra, D.P., *Fundamentals of combustion*. 2007.
16. Okamasa, T., Lee, G. G., Suzuki, Y., Kasagi, N., & Matsuda, S. (2006). Development of a micro catalytic combustor using high-precision ceramic tape casting. *Journal of Micromechanics and Microengineering*, 16(9), S198.
17. Leach, T.T., C.P. Cadou and G.S. Jackson, Effect of structural conduction and heat loss on combustion in micro-channels. *Combustion Theory and Modelling*, 2006. 10(1): p.85–103.
18. Ju, Yiguang, and Bo Xu. "Theoretical and experimental studies on mesoscale flame propagation and extinction." *Proceedings of the Combustion Institute* 30.2 (2005): 2445-2453.
19. Li, Y.H., Lien, Y.S., Chao, Y.C. and Dunn-Rankin, D., 2009. Performance of a mesoscale liquid fuel-film combustion-driven TPV power system. *Progress in Photovoltaics: Research and Applications*, 17(5), pp.327-336.
20. Cozzi, F., Coghe, A., D'Angelo, Y., Renou, B. and Boukhalfa, M., 2009. Experimental study of performances and internal flow field of a meso-scale vortex-combustor. In *Proceedings of the 4th European Combustion Meeting*.
21. Norton, D. G., & Vlachos, D. G. (2004). A CFD study of propane/air microflame stability. *Combustion and Flame*, 138(1), 97-107.
22. Chigier, N.A. and Beer, J.M., 1964. The flow region near the nozzle in double concentric jets. *Journal of Basic Engineering*, 86(4), pp.797-804.

23. Beér and Chigier, N.A., 1972. *Combustion aerodynamics*. New York., J.M.
24. Fiorina, B., Vicquelin, R., Auzillon, P., Darabiha, N., Gicquel, O. and Veynante, D., 2010. A filtered tabulated chemistry model for LES of premixed combustion. *Combustion and Flame*, 157(3), pp.465-475.
25. Bourgooin, J.F., Durox, D., Moeck, J.P., Schuller, T. and Candel, S., 2013, June. Self-sustained instabilities in an annular combustor coupled by azimuthal and longitudinal acoustic modes. In *ASME Turbo Expo 2013: Turbine Technical Conference and Exposition* (pp. V01BT04A007-V01BT04A007). American Society of Mechanical Engineers.
26. Minotti, A., & Sciubba, E. (2010). LES of a meso combustion chamber with a detailed chemistry model: Comparison between the flamelet and EDC models. *Energies*, 3(12), 1943-1959.
27. Wu, M.H., Wang, Y., Yang, V. and Yetter, R.A., 2007. Combustion in meso-scale vortex chambers. *Proceedings of the Combustion Institute*, 31(2), pp.3235-3242.
28. ITO, M. and KIMURA, H., 1979. Boiling heat transfer and pressure drop in internal spiral-grooved tubes. *Bulletin of JSME*, 22(171), pp.1251-1257.
29. Sayma, A., 2009. *Computational fluid dynamics 1st edition*, bookboon
30. Versteeg, H.K. & Malalasekera, W., 2007. *An Introduction to Computational Fluid Dynamics - The Finite Volume Method 2nd ed.*, Edinburgh, England: Pearson.
31. Tu, J., Yeoh, G.-H. & Liu, C., 2013. *Computational Fluid Dynamics, Second Edition 2nd edition*, Elsevier Ltd
32. Guide, F. U. S. (2006). Release 6.3. 26. *Fluent Incorporated* (2005-01-06).
33. Wilcox, D.C., 1998. *Turbulence modeling for CFD* (Vol. 2, pp. 103-217). La Canada, CA: DCW industries.
34. Liu, Y., Tang, H., Tian, Z., & Zheng, H. (2015). CFD simulations of turbulent flows in a Twin Swirl combustor by RANS and hybrid RANS/LES methods. *Energy Procedia*, 66, 329-332.
35. Nanduri, J. R., Celik, I. B., Strakey, P. A., & Parsons, D. R. (2007). *Assessment of RANS-Based Turbulent Combustion Models for Prediction of Gas Turbine Emissions: Turbulence Model and Reaction Mechanism Effects. Assessment.*

36. Williams, T. C., Schefer, R. W., Oefelein, J. C., and Shaddix, C. R. (2007). Review of Scientific Instruments, Vol. 78- 3, pp. 035114-035114-9.
37. Radwan, A., Ibrahim, K. A., Hanafy, A., & Saqr, K. M. (2015). On RANS Modeling of Unconfined Swirl Flow. *CFD Letters*, 6(4), 159-174.
38. German, A. E., & Mahmud, T. (2005). Modelling of non-premixed swirl burner flows using a Reynolds-stress turbulence closure. *Fuel*, 84(5), 583-594.
39. Syred, N., Chigier, N. A., & Beer, J. M. (1971, December). Flame stabilization in recirculation zones of jets with swirl. In *Symposium (International) on Combustion* (Vol. 13, No. 1, pp. 617-624). Elsevier.
40. Jufar, S. R., Huang, R. F., & Hsu, C. M. (2013). Effects of swirl on flow and mixing of acoustically excited swirling double-concentric jets. *Experimental Thermal and Fluid Science*, 49, 40-50.
41. Valera-Medina, A., Syred, N., Bowen, P., & Crayford, A. (2011). Studies of swirl burner characteristics, flame lengths and relative pressure amplitudes. *Journal of Fluids Engineering*, 133(10), 101302.
42. Al-Abdeli, Yasir M., and Assaad R. Masri. "Recirculation and flowfield regimes of unconfined non-reacting swirling flows." *Experimental thermal and fluid science* 27.5 (2003): 655-665.
43. Kalt, P. A., Al-Abdell, Y. M., Masri, A. R., & Barlow, R. S. (2002). Swirling turbulent non-premixed flames of methane: flow field and compositional structure. *Proceedings of the Combustion Institute*, 29(2), 1913-1919.
44. Paik, J., & Sotiropoulos, F. (2010). Numerical simulation of strongly swirling turbulent flows through an abrupt expansion. *International Journal of Heat and Fluid Flow*, 31(3), 390-400.
45. Shanbhogue, S. J., Husain, S., & Lieuwen, T. (2009). Lean blowoff of bluff body stabilized flames: Scaling and dynamics. *Progress in Energy and Combustion Science*, 35(1), 98-120.
46. McAllister, S., Chen, J.Y. and Fernandez-Pello, A.C., 2011. *Fundamentals of combustion processes*. Springer.

47. Bhave, A., & Kraft, M. (2004). Partially stirred reactor model: Analytical solutions and numerical convergence study of a PDF/Monte Carlo method. *SIAM Journal on Scientific Computing*, 25(5), 1798-1823.
48. Chen, J. Y. (1997). Stochastic modeling of partially stirred reactors. *Combustion Science and Technology*, 122(1-6), 63-94.
49. Bartolucci, L., Hamed, N., & Nilsson, H. (2014). EngineFoam: implementation of a different combustion model and the new Janaf thermo equations.
50. Greenshields, C.J., 2015. OpenFOAM. The Open Source CFD Toolbox User Guide
51. Stone, C. P. (2003). Large-eddy simulation of combustion dynamics in swirling flows.