MULTIVARIATE PROCESS VARIABILITY MONITORING FOR GENERAL SAMPLE SIZE

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MULTIVARIATE PROCESS VARIABILITY MONITORING FOR GENERAL SAMPLE DESIGN

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To my beloved family and friends

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ABSTRACT

In multivariate setting, from time to time, process variability is summarized and numerically represented as a covariance matrix, say ***S***. It is generally measured as a non-negative real valued function of ***S*** such that the more scattered the population, the larger the value of the function and vice versa. In the literature, the three most popular functions are total variance (TV), generalized variance (GV) and vector variance (VV). Algebraically, TV is the sum of all eigenvalues of ***S***, GV is the product of those eigenvalues, and VV is their sum of squares. The last two measures are designed for large sample size. If GV is to detect large shift in covariance matrix, VV is for small shift. Even though these measures can also be used for small sample size but the control limits must be determined based on heuristic approach. In order for those control limits to be related to the probability of false alarm, a control charting procedure that monitors multivariate variability changes is introduced in this thesis. The methodology is by maximizing the summation of conditional variances for all possible permutations or order of variables. This chart can be used to monitor process variability regardless of the sample size. Under normality, its exact distribution is derived. For practical purposes, when the sample size is small, we introduce the use of Solomon and Stephen's approximation to that distribution with adjusted-probability of false alarm. The advantage of the proposed chart is that it could detect very small magnitude of disturbance in correlation structure which cannot be detected by existing charts. Besides that, the order of variables will lead to better diagnostic features. The performance of the proposed chart in terms of average run length (ARL) is very promising. Some industrial application examples are presented to illustrate the advantages of the proposed chart.

ABSTRAK

Dalam aturan multivariat, dari semasa ke semasa, proses kebolehubahan diringkaskan dan diwakili secara berangka sebagai matriks kovarians, katalah ***S***. Ia secara umumnya diukur sebagai fungsi bernilai nyata tak negatif ***S*** demikian sehingga semakin bertaburan sesuatu populasi, semakin besar nilai fungsi dan sebaliknya. Dalam literasi, tiga fungsi yang paling popular adalah jumlah varians (TV), varians umum (GV) dan varians vektor (VV). Secara algebra, TV ialah hasil tambah semua nilai eigen ***S***, GV ialah hasil darab nilai eigen yang berkenaan dan VV ialah jumlah kuasa dua nilai eigen tersebut. Dua pengukuran yang terakhir direka untuk saiz sampel yang besar. Jika GV adalah untuk mengesan anjakan besar dalam matriks kovarians, VV adalah untuk mengesan anjakan yang kecil. Bahkan kedua-dua pengukuran ini boleh juga digunakan untuk saiz sampel yang kecil tetapi had kawalan mesti ditentukan berdasarkan pendekatan heuristik. Untuk membolehkan had kawalan tersebut dikaitkan kepada kebarangkalian isyarat palsu, prosedur carta kawalan yang memantau perubahan kebolehubahan multivariat diperkenalkan dalam tesis ini. Metodologi berkenaan ialah dengan memaksimumkan penjumlahan varians bersyarat bagi semua pilihatur yang mungkin atau aturan pembolehubah. Carta ini boleh digunakan untuk memantau proses kebolehubahan tanpa mengira saiz sampel. Dalam keadaan normal, taburan yang tepat telah diterbitkan. Untuk tujuan praktikal, apabila saiz sampel kecil, kami memperkenalkan penggunaan penghampiran Solomon dan Stephen kepada taburan yang diselaraskan dengan kebarangkalian isyarat palsu. Kelebihan carta yang dicadangkan ialah dapat mengesan magnitud gangguan yang kecil dalam struktur korelasi yang tidak dapat dikesan oleh carta sedia ada. Selain daripada itu, susunan pembolehubah akan dapat memberikan ciri diagnostik yang lebih baik. Prestasi carta yang dicadangkan dari segi purata panjang jalan (ARL) adalah sangat baik. Beberapa contoh pengunaan dalam industri dibentangkan untuk menggambarkan kelebihan carta yang dicadangkan.

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LIST OF ABBREVIATION

GV	-	Generalized variance
VV	-	Vector variance
TV	-	Total variance
IGV	-	Improved generalized variance
OOC	-	Out- of control
MEWMA	-	Multivariate exponentially weighted moving average
MCUSUM	-	Multivariate cumulative sum
PCA	-	Principle component analysis
LCL	-	Lower control limit
UCL	-	Upper control limit
CL	-	Control limit
PPCC	-	Probability plot correlation coefficient
Q-Q plot	-	Quantile-Quantile Plot

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CHAPTER 1

INTRODUCTION

The aim of this chapter is to introduce the importance of this research. In Section 1.1, the background of the problem will be discussed. In section 1.2, the discussion will be on the problem statement which is the sole ingredient for the output of this thesis. The following section will cover the objectives of this research. The scope of the study, thesis organization and significance of the study will be presented in Section 1.4, Section 1.5 and Section 1.6, respectively.

1.1 Background of the Study

In industrial practices, a process is always monitored to make sure that it goes on the right track with the quality that intended. A process can be monitored in terms of its process target and variability. The monitoring process of the latter is more difficult than the former which, in this era, is getting more attention. This is because variability is reciprocal to quality. The smaller the variability of a process the higher the quality of that particular process and conversely. Thus, to ensure the quality of the process, variability must be monitored. In multivariate setting, which means that more than one interrelated quality characteristics need to be monitored simultaneously, the situation becomes more crucial due to the difficulty of defining and measuring process variability.

Montgomery (2005) mentions that monitoring process variability is as important as monitoring the process target. Alt & Smith (1988) give a similar argument in their work. The importance of monitoring multivariate process variability have been discussed in numerous literature, for example, Wierda (1994), Mason et al. (1997), Woodall & Montgomery (1999) until the most recent journal publications Yeh et al. (2006), Patel & Divecha (2011) and Hamed (2014). In multivariate settings, variability is summarized and numerically represented as a covariance matrix, say $\boldsymbol{S}$. The distributional function of $\boldsymbol{S}$ can be found in many literatures such as Anderson (1984), Muirhead (1982), Iwashita & Siotani (1994) and Djauhari et al.(2008). A multivariate variability measure is a non-negative real valued function of a covariance matrix such that the more scatter the population, the larger the value of that function and conversely.

There are some multivariate variability measures that have been proposed. For example, Wilks (1932) introduces the covariance determinant, also called generalized variance (GV), $|\boldsymbol{S}|$, for testing the equality of the covariance matrices. In other words, it is to test whether a new covariance matrix is equal to the in-control covariance matrix. This test is actually the modification of likelihood ratio test which is introduced by Neyman & Pearson (1928). Then, Montgomery and Wardsworth (1972) propose control chart based on GV to monitor multivariate process variability. Djauhari (2005), improves the GV control chart by removing the bias in control limit estimation and names it as improved generalized variance (IGV) chart. GV is the most popular statistic used to monitor the variability. Still, this measure has some limitations or situations where the determinant could not be computed or used. This situations have been pointed out by Alt & Smith (1988).

If GV is the multiplication of all eigenvalues of $\boldsymbol{S}$, Djauhari (2007) introduces another alternative multivariate variability measure which is the sum of squares of all eigenvalues of $\boldsymbol{S}$. This measure is called vector variance (VV). He explains how the limitations in GV have been overcome by using this measure. Then, Herwindiati et al. (2007) introduce robust estimation for covariance matrix based on minimum vector variance (MVV) and Djauhari et al. (2008) introduce a control charting procedure based on VV. Another measure of variability has been proposed by Costa

& Machado (2009). The statistic, named VMAX , is constructed based on the standardized sample variance of p quality characteristics. This statistic is said to be faster in detecting the process changes and better in identifying an out of control signal compared to GV.

Those statistics lead to the construction of control charts for monitoring multivariate process variability. The sole objective of those control charts is to monitor whether the process is in-control. In the literature there are three famous charts which are total variance (TV) chart, GV or IGV chart and VV chart. All these statistics can be viewed as a result of algebraic diagonalization. However, algebraic diagonalization has some limitations; it provides us only with a necessary condition for two covariance matrices to be equal to each other.

TV chart will not be included in the discussion as it involves only the variances and not the covariance among the variables. The latter two charts are more widely used in real life. These measures have their own limitations. Besides that, the exact distribution is difficult to be derived or impractical for $p>2$. The control limits for these charts are constructed based on probability limit or 3 sigma principle. When probability limit is used, the false alarm is known but it is only applicable for large sample size. Meanwhile, 3- sigma principle chart is applicable for both large and small sample size but the disadvantage of this method is that the probability of false alarm is unknown.

If the probability of false alarm is taken into account in the determination of its control limits, then a large sub-group size is needed. However, this is not favorable in manufacturing industry environment because: (i) in this industry data collection is not cheap, it is time consuming, and thus only limited sub-group size is available during monitoring operations, and (ii) fast and accurate decision is the main concern from management. In practice, the available charts are more suitable to detect large shift in the covariance structures and applicable for equal subgroup size.

If the probability of false alarm is taken into account in the determination of its control limits, then a large sub-group size is needed. However, this is not favorable in manufacturing industry environment because: (i) in this industry data collection is not cheap, it is time consuming, and thus only limited sub-group size is available during monitoring operations, and (ii) fast and accurate decision is the main concern from management.

These limitations and challenges lead to the outcome of this thesis. There are three open problems to be tackled. In this thesis, a new measure to monitor multivariate process variability is constructed. This measure is helpful in detecting small shift in covariance structure where the existing charts are not able to do so. The exact distribution of the measure will be derived. The control limits with known false alarm will also be constructed for both large and small sample size. Then, this measure will be extended to be used in all sample size or general sampling design. The control limits of GV and VV chart for general sampling design are also derived. The implementation of the proposed measure will be demonstrated by using user friendly software. The extension of the existing charts and the proposed one will also be developed to cover service industry, where sub - group size cannot be controlled. At the end, some industrial examples based on manufacturing industry and service industry will be presented to illustrate the advantages of the proposed methods.

1.2 Statement of the Problem

Research backround presented in the previous section leads us to the following research problem. In previous works, the three most popular measures of multivariate dispersion, are TV, GV and VV. These measures are a positive, real valued functions of $\boldsymbol{S}$. These functions are defined as an algebraic function. Algebraically, TV is the sum of all eigenvalues of $\boldsymbol{S}$, GV is the product of all eigenvalues, and VV is the sum of squares of all eigenvalues.

These measures pose some drawbacks. Although, they are used widely in practice, the signals in the chart are clueless about the exact root causes. In this situation a measure which can be more effective as variability measure with better diagnostic features is needed. It is worth noting that until present, there exists no comparable charting technique for the case where the subgroup size, *n,* is small and the control limits are defined based on probability limits. The advantage of this technique is that the probablity of false alarm is known. Moreover, it can detect the overall shift in covariance structure, while the existing charts are designed to detect the large shift.

Thus, the main obstacle that need to be tackled in this thesis is to find a control charting method which can detect small shift in covariance structure. On the other hand, the generalization of the existing charts with varying sample size is needed. Therefore, this research is basically a research in mathematical statistics to develop a statistical tool which can overcome the limitations of existing control charts without neglecting its applications.

1.3 Objectives of Study

The intention of this study is to handle the limitations of the existing multivariate process variability control charting procedure and introduce a new one. There are five main objectives in this study.

(i). To construct a new measure of multivariate process variability with better diagnostic features.

(ii). To derive the distribution of the proposed measure to understand its behaviour.

(iii). To develop control chart to monitor multivariate process variability based on probability limit for the case where sample size, *n,* is small.

(iv). To derive the existing control limits of GV and VV chart for equal subgroup size.

(v). To modify the control limits of GV and VV chart for general sampling design.

At the end of this study, author believe that the research achieved all the objectives above.

1.4 Scope of Study

The research is focused on process variability monitoring in multivariate settings. It covers three main aspects; theoretical, computational and practical aspects.

(i) Theoretical aspects

In order to have a better understanding about multivariate process variability monitoring and controlling, author have derived and discussed the readily available control charts, i.e., GV chart and VV chart. The control limits of these charts are then derived for general sampling design. Then, a new multivariate variability measure having better diagnostic features is constructed. The distribution of this measure is derived. In order to monitor the variability, a control chart based on this measure with probability limits is introduced. Its power is investigated in terms of average run length (ARL). ARL is the most commonly used metric to validate the performance of the control chart.

(ii) Computational Aspects

From computational point of view, the scope of study covers the simulation experiments to illustrate the convergence of the new measure towards the asymptotic distributions. An algorithm to compute the proposed measure and a chart construction are discussed too. Finally, simulation experiments on its ARL is carried out to show the power of proposed chart. A user friendly software, constructed in MATLAB system and in MS Excel System, are also developed for computation purpose.

(iii) Practical Aspects

Application in real industrial problems is presented and discussed to show the advantages of the new measure and its corresponding control chart developed in this thesis.

1.5 Contributions of the Study

This study offers few contributions. There exists no comparable control charting technique for the case where sample size, n, is small and the control limits are defined based on probability limits. By defining the control limits based on probability limit users will be aware of the exact false alarm risk. This will be the main contribution of this study. Besides that, this study will explicitly contribute to the statistical process variability control as follows :

I. A new measure of multivariate process variability with more promising diagnostic feature.
II. A new multivariate process variability monitoring based on the proposed measure.
III. Two theorems based on Solomon and Stephens approximation and normal approximation of new measure is discovered
IV. Control chart which is able to detect small shift in the covariance matrix.
V. Algorithm to compute the new measure and construct the corresponding control chart.

VI. Derivation of existing GV and VV chart control limits for the case of one sample and *m* equal samples.

VII. Extend the usage of GV and VV charts to the general sampling design.

VIII. Better understanding about the variance or covariance shift by using the existing charts and the new chart simultaneously.

In brief, this study is believed to give a significant impact on statistical process control in general and multivariate process variability monitoring in particular.

1.6 Limitation of the Study

This study is limited to process variability monitoring in multivariate setting. In this situation, the thesis is only focusing on monitoring process when observations are available as batch data or subgroups. Thus, it is limited to $n>1$. Also, the extended GV chart and VV chart are only applicable when large sample size is available. But, the proposed chart do not require sample size restriction as it can be used for both large and small sample size. The restrictions of the charts in terms performance are listed as below:

(i) GV chart is suitable to be used in the situation where the covariance structure experiencing large shift. It is not sensitive towards small shifts in covariance structure.

(ii) VV chart is more appropriate to be used in the case where covariance structure poses small shift . This chart also can be used for large shift but more appropriate to be used in small shift cases.

(iii) The proposed chart is able to detect the variance shift in both small shift and large shift. But, according to correlation shift it is more appropriate to be used to detect very small shift in correlation and not the large shift .

The limitations have been discussed in terms of availability of sample size as well as the performance. In conclusion, it will be more appropriate to use all the three charts simultaneously to understand the situation in process variability in detail as they react to different type of situations.

1.7 Organization of the Thesis

This thesis is organized into seven chapters. The first chapter is to give an overall idea about the research work. It covers the aim and the need of doing this research. Thus, it includes some backgrounds about the research work together with the problem statement, objectives of research, scope or limitation of research, and also the significant contributions of the study in the statistical quality control area.

The second chapter will cover the literature review which is essential for this particular research activity. It represents the past work of researchers regarding multivariate process variability control. In this chapter, there will be a discussion regarding past works on multivariate process variability, the existing multivariate variability measures and their advantages and also weaknesses. It also includes different approaches in defining control limits of the control charts for monitoring multivariate process variability and how the performance of the chart is being evaluated. In the last part of this chapter, an overall view on the proposed solution will be presented.

Chapter 3 is devoted for the extension of GV and VV charts which make them enable to be used in general sampling design. There will also be detailed derivation of control limits for both charts in all three cases; (i) one sample, (ii) m equal subgroup size and (iii) m unequal subgroup size. In addition, the exact and asymptotic distribution of covariance matrix is also described. Chapter 4 will cover the main objective of this thesis. The new control chart based on the new measure is proposed. Two different approaches in computing the measure will be covered in this chapter. An algorithm to calculate the measure is given too. Besides that, the exact and also asymptotic distribution of the new measure will be discussed. Lastly, the control chart construction for large sample size and small sample size will close the presentation of this chapter.

Next, in Chapter 5 there will be an exploration of asymptotic performance of the proposed chart. First, the power function of the proposed measure will be derived and it is followed by an evaluation of its performance for different number of variables. In the last part of this chapter the advantages and limitations of the proposed measure and chart will be discussed. In Chapter 6, applications of the methods developed in this research will be found, not only in manufacturing industry but also in service industry. The presentation of this thesis will be closed with some conclusions and also some suggestions for the future research works in Chapter 7. The overall thesis is summarized in Figure 1.1.

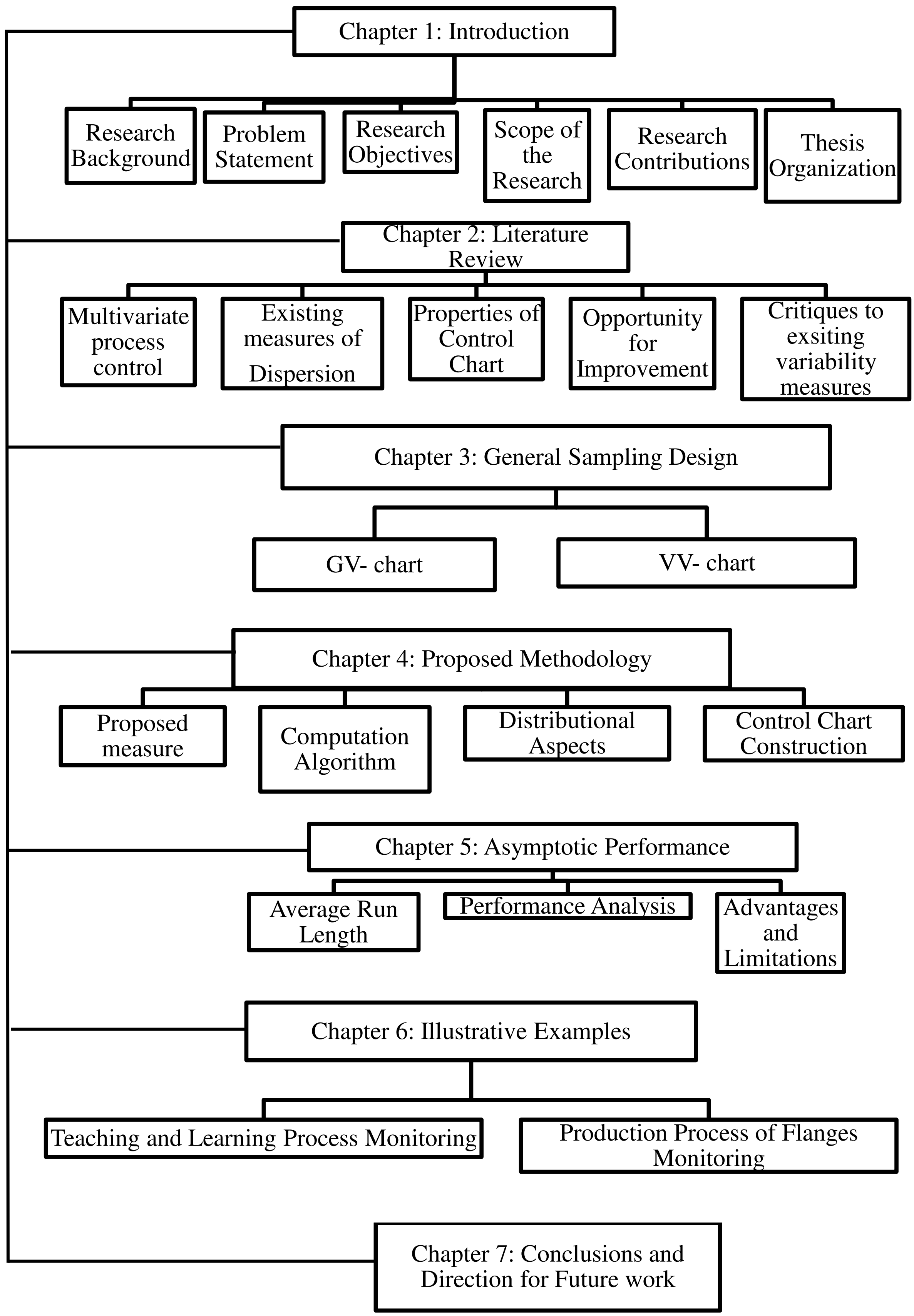

Figure 1.1: Summary of the thesis content

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Appendix A

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