FINITE ELEMENT FORMULATION OF ONE-DIMENSIONAL UNCOUPLED AND COUPLED CONSOLIDATION

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FINITE ELEMENT FORMULATION OF ONE-DIMENSIONAL UNCOUPLED AND COUPLED CONSOLIDATION

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Engineering (Geotechnics)

Faculty of Civil Engineering
Universiti Teknologi Malaysia

JUNE 2013

I begin with the name of Allah the most merciful and the most kind, peace and blessing be upon beloved Prophet (S.A.W) All praise is for Allah.

This thesis is dedicated to my lovely mom and dad. Also to my siblings and my supportive friends.

ACKNOWLEGEMENTS

In the name of Allah, the Most Gracious, the Most Merciful. I am praised to Allah, Lord of the universe for making me able to undertake this research work.

Thanks to King of Malaysia and Public Service Department of Malaysia, for providing the financial support throughout my post-graduate studies.

I take this opportunity to thank my parents, Mr. Wan Hassan and Mrs. Zawahir who has been a colleague and is a constant source of help and great ideas. She is always closest to my joys and sorrows and always standing by me. Thanks for being my close friend in my life. Thanks also to my sibling because always be patient to me whenever my mood swings.

Thanks to my supervisor, Dr. Hisham Mohamad and Dr. Ahmad Kueh Beng Hong for his supervision and support in seeing this work through to completion.

I also would like to thank Prof.Vaughan Griffiths, Dr.John Andrew Abbo and Mr.Ahad Ouria for discussions over the numerical solutions. Needless to say, I am grateful to all of my colleagues at STC, UTM. I am especially thankful to all my friends in UTM for sharing my joys and frustrations.

ABSTRACT

This thesis presents the formulation of finite element method for one-dimensional consolidation problem based on uncoupled and coupled consolidation theories. The consolidation of cohesive soils as a result of dissipation of the excess pore pressures generated by external loading is a problem of considerable concern amongst geotechnical engineers. Theories of consolidation fall into two main categories which are uncoupled and coupled theories. The uncoupled theory only considers the interaction of the pore fluid with the porous media ensuring the continuity equation of the pore fluid whereas the coupled theory is formulated on the basis of solid-fluid interaction which is more realistic but is more difficult to solve. In this study, both theories are solved using finite element method using the formulation of Galerkin weighted residual method and variational approach. The derived formulation is used to develop a computer program for uncoupled and coupled analysis. This was written using the MATLAB® programming code. The proposed finite element code was firstly verified and results were comparable with Terzaghi consolidation theory in both cases. For the case one-dimensional problem of homogenous soil, the numerical results showed very good agreement between the uncoupled and coupled consolidation analyses with difference less than 5%. The criteria for selecting preferable method were based on several factors such as accuracy and stability. From the result, the relative discrepancy for the uncoupled theory is not much significant compared to the coupled theory by giving less than 10% value of difference. Consequently, the uncoupled 1-D consolidation theory was adopted for the subsequent analysis for a layer and multi-layered soil. A case study of multi-layered consolidation problem was compared and the results demonstrated that the layered characteristics of soils have significant influences on the overall consolidation settlement performance.

ABSTRAK

Tesis ini membentangkan formulasi kaedah unsur terhingga untuk masalah satu dimensi pengukuhan berdasarkan teori pengukuhan (*consolidation*) *uncoupled* dan *coupled*. Pengukuhan tanah yang jelekit sebagai hasil penyusupan keluar tekanan air lebihan yang dihasilkan oleh beban luaran adalah masalah yang menjadi kebimbangan di kalangan jurutera geoteknikal. Teori pengukuhan terbahagi kepada dua kategori utama iaitu teori *uncoupled* and *coupled*. Teori *uncoupled* hanya menganggap interaksi bendalir liang dengan media berliang memastikan persamaan keselanjaran bendalir liang manakala teori *coupled* dirumuskan berdasarkan interaksi pepejal- cecair yang lebih realistik tetapi sukar untuk diselesaikan. Dalam kajian ini, kedua-dua teori diselesaikan dengan menggunakan kaedah unsur terhingga menggunakan perumusan kaedah *Galerkin weighted residual* dan pendekatan *variational*. Formulasi yang diterbitkan digunakan untuk membangunkan satu program komputer untuk analisis bagi teori *uncoupled* dan *coupled*. Ini telah ditulis menggunakan kod pengaturcaraan MATLAB ®. Kod menggunakan kaedah unsur terhingga yang dicadangkan pertamanya telah disahkan dan keputusan analisis bagi kedua-dua teori dibandingkan dengan teori pengukuhan Terzaghi. Bagi masalah kes satu dimensi tanah sejenis, keputusan kiraan berangka menunjukkan satu perbandingan yang baik di antara analisis pengukuhan *uncoupled* dan *coupled* dengan perbezaan kurang daripada 5%. Kriteria untuk memilih kaedah yang lebih baik berdasarkan beberapa faktor seperti ketepatan dan kestabilan. Daripada keputusan ini, perbezaan relatif bagi teori terlerai tidak begitu ketara berbanding dengan teori ditambah dengan dengan memberikan nilai perbezaan kurang daripada 10%. Oleh itu, teori penyatuan terlerai 1-dimensi telah diguna pakai untuk analisis berikutnya iaitu analisis untuk tanah selapis dan tanah berlapis-lapis. Satu kes kajian untuk masalah pengukuhan tanah berlapis-lapis telah dibandingkan dan keputusan menunjukkan bahawa ciri-ciri tanah jenis berlapis mempunyai pengaruh yang besar ke atas prestasi penyelesaian pengukuhan keseluruhan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOLS:

E_0 - Stiffness
γ_w - Unit weight of water
h_e - Excess head
t - Time
z - Depth
u, p, p_f - Pore water pressure
c_v - Coefficient of consolidation
H - Drainage path
σ - Normal stress
τ - Shear stress
γ - Strain
E' - Effective Young modulus
v' - Effective Poisson ratio
σ' - Effective stress
v - fluid velocities
$\dot{\varepsilon}$ - Strain rate
θ - Time integration parameter
k - Coefficient of permeability
u, v, w - Deformations
$[N]$ - Matrix of shape functions
$[B]$ - Derivatives of the shape functions
$[D]$ - constitutive matrix
μ - Poisson’s ratio
T - Surface traction

E - total potential energy

W - strain energy

L - potential load energy

$[K_E]$ - Element stiffness matrix

$\{\Delta R_E\}$ - load vector

ϕ_j - trial functions

R - residual

W_m - weight functions

m_v - Coefficient of volume compressibility

ω - Galerkin weight function

$[M]$ - Mass matrix

D - Maximum drainage path

q - Total load

N_p - Matrix shape functions for pore pressures

N_u - Matrix shape functions for deformation

Ks - Solid stiffness matrix

C - Coupling matrix

q - Outward flow per unit area

n - Vector of direction cosines for the unit normal

K_f - Fluid stiffness matrix

P - Uniform surface load

U - Degree of consolidation

S - Settlement

ABBREVIATION:

FEM - Finite element method
MATLAB - Matrix laboratory
FDM - Finite difference method
WRM - Weighted residual method
LBB - Ladyzhenskaya-Babuska-Brezzi

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Research Background

Consolidation can be defined as a process by which soils decrease in volume and it occurs when stress is applied to a soil that causes the soil particle to pack together more tightly, therefore reducing volume. When this occurs in a soil that is saturated with water, water will be squeezed out of the soil (Menéndez et al, 2008). This process is one of the primary reasons of ground settlement. Consolidation settlement will result, for example, if a structure is built over a layer of saturated clay or if the water table is lowered permanently in a stratum overlying a clay layer (Lambe and Whitman, 2000). If a problem arising from the settlement is not kept to a tolerable limit, the desired use of the structure may be impaired and the design life of the structure may be reduced (Desai and Christian, 1977).

The process of consolidation under one-dimensional was first investigated by Terzaghi (1923). Then, in 1963, Rendulic developed a pseudo three dimensional theory of consolidation which is like Terzaghi's theory. However, due to some fundamental assumptions that were made about the behaviour of the stresses, this model failed to coupled the magnitude and rate of deformations properly for two and three dimensions. To overcome the drawback of Rendulic's formulation and provides compatibility between the deformations and pore water pressures, Biot in 1941 was developed a theory which can reasonably describe the complicated relationship between the stress in an elastic body and the water flow in pores.

The theories of consolidation fall into two categories which are uncoupled theory and coupled theory. In the uncoupled theory, it is assumed that the total stress remains constant throughout the consolidation process and the strain are caused only by the change of pore water pressure (Osman, 2010). In addition, in this system there is only one degree of freedom in the governing equation which is the pore water pressure. For any complete consolidation analysis requires not only establishing the relation between the pore water pressure and the consolidation time, but also the variation of deformations with time. Meaning that, procedure to determine the transient component of deformations is from the pore water pressure.

For the coupled theory which was first mathematically described by Biot (1941), the elastic equilibrium equations must be solved simultaneously with a continuity equation under appropriate boundary and initial condition. In Biot's theory, soil is considered to be a porous skeleton filled with water. The porous skeleton is assumed to be an isotropic elastic medium governed by the constitutive relationship and the total normal stress components are equal to the sum of the respective effective normal stress components and the pore water pressure. These equilibrium equations are coupled to a flow continuity equation based on Darcy's law. By solving all equations simultaneously both deformation and pore water pressure changes can be obtained.

For almost five decades, there has been an increasing interest in the use of finite element methods in geotechnical engineering in an effort to overcome the shortcomings found in difference methods. The early use of finite element discretization was in the field of solid mechanics, where the boundary value problems of interest could be cast in an appropriate variational form for direct use of the method. Use of finite element methods was later extended to other areas of mathematical physics when it was realize that Galerkin’s method or more generally the method of weighted residual could be used in place of a variational statement of the problem (Sandhu and Wilson, 1969).

Based on brief discussions of the theories of consolidation and the numerical techniques, the finite element method will be implemented with the time integration procedure to solve consolidation problem. Attention is focused primarily on the development and verification of a computer program for analysis of one dimensional consolidation which cover the uncoupled and coupled consolidation theory.

1.2 Problem Statement

Consolidation plays an important role in many soil mechanics problems. This is evidenced by the lots of literature highlighted to the solution of consolidation problems since the work of Terzaghi in 1923. The term consolidation is used to describe a process whereby fluid is squeezed out of the void spaces in a soil to allow the soil to decrease in volume. The term also used in general way to include swelling as well as compression. Because soils are not infinitely permeable, time is needed for escape of pore fluid, thus, consolidation is a time-dependent process.

There are two main areas where consolidation analysis is extensively applied. In the first area the consideration of physical loading of soil layers and second, considers the change of hydraulic equilibrium in a system comprising aquifers. In early practice, deformations were calculated in most cases using Terzaghi's one dimensional consolidation theory. More recently Biot's three-dimensional theory has been used, based on the linear stress-strain constitutive relationship and Darcy's flow rule. The deformation especially vertical deformation, called settlement of soil is the most important requirement for the safety of structures and should not be excessive and must be within tolerable or permissible limits.

Consolidation analyses are usually treated either by means of Terzaghi’s uncoupled theory or Biot’s consolidation theory. In this thesis, the problem of consolidation displacements for one dimensional layered soil was considered. The potential applications of the finite element solution presented in this thesis are not restricted to geomechanics; it has the potential to be applied to other fluid-saturated porous media. Applications can be found in diverse areas such as geothermal energy extraction, petroleum engineering, chemical engineering, agricultural engineering, pavement engineering, hazardous waste management and biomechanics.

Due to the availability of high speed large storage computers, a numerical method known as finite element method became popular due to its versatility and is widely used for solution of engineering problems. Conventional finite element method has proved to be an extremely powerful analytical tool for the solution of many geotechnical engineering problems. Although analysis of one-dimensional consolidation problems is considered routine, the study based on the formulation of the finite element method for time-dependent problem is far more demanding because it can be applied to more complicated problems. Indeed, to solve uncoupled and coupled consolidation problems with any degree of confidence using finite element method, it is usually necessary to have a detailed understanding of the approximations finite element solution strategies.

1.3 The Objectives of the Study

The aim of this study is to developed a finite element code program for solving uncoupled and coupled consolidation problem and validating its. The research shall undertake the following key objectives:

1) To develop a computer program for consolidation problem using uncoupled and coupled consolidation theory.

2) To validate the derived formulation with analytical solution.

3) To verify the derived formulation with analytical solution.

1.4 Research Scope

The study is limited to the finite element formulation method for Terzaghi’s uncoupled and Biot’s coupled theory of one-dimensional consolidation. Under various assumptions, consolidation in a semi-infinite soil mass can be approximated as one-dimensional. This approximation provides useful engineering solutions for many practical situations such as vertical settlements of foundations and embankments. A finite element formulation based on the one-dimensional idealization is developed to provide acceptable solutions with simplicity and economy of computational and formulation efforts.

For each theory of consolidation, the assumptions have been made and will be discussed while presenting the governing equations. For the extended area of these theories is not considered in this study. It should also be noted that the formulation of finite element for this problem is assumed that the soil is fully saturated. If partial saturation is to be considered, extra terms must be added in the process of formulation. A computer program, MATLAB is developed based on finite element method to solve the consolidation problem which assumed the soil is linear elastic under appropriate boundary and initial condition.

As brief before, this study describes the development a computer program for analysis of one dimensional consolidation of multi -layered soil profiles. In multi -layered soil, the soil properties will be different from layer to layer, which create a complicated geotechnical problem. To further illustrate the solution of finite element method in the analysis of consolidation of a layered soil, a case study of multi-layered soil is investigated in this study.

1.5 The Significance of the Study

Consolidation is one of the most classic and well-studied topics in soil mechanics. Even this study focuses on one-dimensional problem, there are several interesting issues to be addressed in this uncoupled and coupled consolidation problem. The theory of consolidation has several important roles in several areas like the oil industry. In addition, geomechanics is crucial in such problems as production-induced compaction, borehole stability, and hydraulic fracturing. The study here will focus on the mathematical structure of theories of consolidation problems, and the development of computer program based on the finite element formulation. Although the scope of research is limited to the linear elastic problems, this study a suitable platform for understanding the finite element method and the concept of the uncoupled and coupled consolidation theories. Consequently, nonlinear problem can be solved without too much additional effort. In other word, by using computer-implemented mathematical models, one can simulate and analyze complicated systems in science and engineering. This reduces the need for expensive and timeconsuming experimental testing and makes it possible to compare many different alternatives for optimization.

Development of MATLAB programming based on finite element formulation for the consolidation problem very important because the generation of excess pore water pressures can have important implications in both structure and soil. During the construction of geotechnical structures, considerable movement of structures and soil can occur due to pore pressure generation and dissipation. Moreover, pore pressure effects also can influence the development of stresses which may lead to hydraulic fracturing. It is useful to well understand the behaviour of consolidation process because it provides useful engineering solution for many practical situations such as the ultimate settlement for any loaded soil medium, how quickly it will occur or in particular how much settlement will have occurred after certain period of time that causing the damage to the structure. (Duncan and Schaefer, 1988).

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