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OPERATORS, SYMBOLS AND ABBREVIATIONS

I. Operators

$\{*\}^{-1}$: Inverse

$\{*\}^{T}$: Transpose

II. Symbols

$A_i(z^{-1})$: Denominator of the plant model

$B_i(z^{-1})$: Numerator of the plant model

$C_i(z^{-1})$: Denominator of the PI controller

$D_i(z^{-1})$: Numerator of the PI controller

$y_i(k)$: Output of agent *i* at time *k*

$y_j(k)$: Output of agent *j* at time *k*

$y_i(k+p|k)$: *p*-step ahead agent *i* output prediction at time *k*

$y_j(k+p|k)$: *p*-step ahead agent *j* output prediction at time *k*

N_i : Neighbour(s) of agent *i*

$u_i(k)$: Control input of agent *i* at time *k*

$u_j(k)$: Control input of agent *j* at time *k*

$u_i(k+p|k)$: *p*-step ahead agent *i* control input prediction at time *k*

$u_j(k+p|k)$: *p*-step ahead agent *j* control input prediction at time *k*

τ : Constant output network delay

τ_{ij} : Constant output network delay during information transmission from agent *j* to agent *i*

$\tau_{ij,min}$: Minimum constant output network delay exists during information transmission from agent *j* to agent *i* in NMAS

$\tau_{ij,max}$: Maximum constant output network delay exists during information transmission from agent *j* to agent *i* in NMAS

$\tau_{ij}(k)$: Bounded random output network delay for information transmission from agent *j* to agent *i* at time *k*

$\bar{\tau}$: The upper bound of random output network delay at time *k*

K_i : Coupling gain at each link of agent *i*

K_r : Coupling gain at link between external reference input and Agent 1

K_{ij} : Coupling gain at link between agent *i* and agent *j*

n : Number of agent in NMAS

$R(k)$: External reference input

p : Prediction step

p_{max} : Maximum step of prediction sequence

L : Laplacian matrix

I : Identity matrix

$soe_i(k)$: Sum of output error between agent *i* and received output from its neighbouring agent(s) with delay compensation at time *k*

0_* : Zero matrix for dimension $*$

cm : centimetres

V : volts

III. Abbreviations

NMAS : Networked multi-agent system

PI : Proportional-integral

UDP: User datagram protocol

TCP : Transmission control protocol

LAN : Local area network

C-MEx : C-Matlab Executable

UAV : Unmanned autonomous vehicle

A/D : Analogue-to-digital conversion

D/A : Digital-to-analogue conversion

SISO : Single-input single-output

RPC : Recursive prediction control

DC : Delay compensator

LC : Loss compensator

ARX : Auto-regressive exogeneous

DMSG1 : Delay measure signal generator 1

DMSG2 : Delay measure signal generator 2

CDL : Consecutive data losses

GER : Gain error ratio

NPCA-GER : Networked predictive controller based GER formula

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ABSTRACT

This thesis is concerned with the design and application of the prediction method in the NMAS (networked multi-agent system) external consensus problem. The prediction method has been popular in networked single agent systems due to its capability of actively compensating for network-related constraints. This characteristic has motivated researchers to apply the prediction method to closed-loop multi-process controls over network systems. This thesis conducts an in-depth analysis of the suitability of the prediction method for the control of NMAS.

In the external consensus problem, NMAS agents must achieve a common output (e.g. water level) that corresponds to the designed consensus protocol. The output is determined by the external reference input, which is provided to only one agent in the NMAS. This agreement is achieved through data exchanges between agents over network communications. In the presence of a network, the existence of network delay and data loss is inevitable. The main challenge in this thesis is thus to design an external consensus protocol with an efficient capability for network constraints compensation.

The main contribution of this thesis is the enhancement of the prediction algorithm's capability in NMAS applications. The external consensus protocol is presented for heterogeneous NMAS with four types of network constraints by utilising the developed prediction algorithm. The considered network constraints are constant network delay, asymmetric constant network delay, bounded random network delay, and large consecutive data losses.

In the first case, this thesis presents the designed algorithm, which is able to compensate for uniform constant network delay in linear heterogeneous NMAS. The result is accompanied by stability criteria of the whole NMAS, an optimal coupling gains selection analysis, and empirical data from the experimental results. 'Uniform network delay' in this context refers to a situation in which the agent experiences a delay in accessing its own information, which is identical to the delay in data transfer from its neighbouring agent(s) in the network.

In the second case, this thesis presents an extension of the designed algorithm in the previous chapter, with the enhanced capability of compensating for asymmetric constant network delay in the NMAS. In contrast with the first case—which required the same prediction length as each neighbouring agent, subject to the same values of constant network delay—this case imposed varied constant network delays between agents, which required multi-prediction lengths for each agent. Thus, to simplify the computation, we selected a single prediction length for all agents and determined the possible maximum value of the constant network delay that existed in the NMAS. We tested the designed control algorithm on three heterogeneous pilot-scale test rig setups.

In the third case, we present a further enhancement of the designed control algorithm, which includes the capability of compensating for bounded random network delay in the NMAS. We achieve this by adding delay measurement signal generator within each agent control system. In this work, the network delay is considered to be half of the measured total delay in the network loop, which can be measured using a ramp signal. This method assumes that the duration for each agent to receive data from its neighbouring agent is equal to the time for the agent's own transmitted data to be received by its neighbouring agent(s).

In the final case, we propose a novel strategy for combining the predictive control with a new gain error ratio (GER) formula. This strategy is not only capable of compensating for a large number of consecutive data losses (CDLs) in the external consensus problem; it can also compensate for network constraints without affecting the consensus convergence time of the whole system. Thus, this strategy is not only able to solve the external consensus problem but is also robust to the number of CDL occurrences in NMAS.

In each case, the designed control algorithm is compared with a Proportional-Integral (PI) controller. The evaluation of the NMAS output performance is conducted for each by simulations, analytical calculations, and practical experiments. In this thesis, the research work is accomplished through the integration of basic blocks and a bespoke Networked Control toolbox in MATLAB Simulink, together with NetController hardware.

1 INTRODUCTION

As an introduction to the thesis, a brief overview and survey covering important aspects in the NMAS is given in this chapter. First, the rationale behind the application of multi-agent control structures in this research is discussed in Section 1.1. In Sections 1.2 to 1.4, the background of the consensus and the reviews on the existing related works to the consensus problems are presented. The conceptual idea of implementing the prediction strategy for multi-agent system is laid out in Section 1.6. In Section 1.5, the motivations of the research are described followed by the general aim and objective in Section 1.7. This chapter is concluded with the description of the thesis layout.

1.1 Networked multi-agent system (NMAS) control structure

A networked multi-agent system (NMAS) is a control system that is composed of multiple subsystems (called 'agents') that are physically separated. In this control structure, all agents communicate with each other (depending on the system's topology) via their local controllers over the network.

Based on the numbers of published research findings within the control community, NMAS-related studies have been noticeably active over the last few decades. The NMAS structure in particular has sparked the curiosity of researchers who are interested in finding a reliable control strategy, especially for large-scale process control systems that cover large geographical areas; these systems consist of many subsystems, sensors, and actuators. This structure increases the flexibility and scalability of the control system by allowing relatively easy and cheap expansion of the physical structure of the system.

In terms of controller performance, the NMAS also helps to simplify large systems' operations and makes them simpler and faster to accomplish. In other words, because it is a large system, performing a cooperative mission among a number of agents (rather than a solo mission) in handling the workload should theoretically help to ease the burden on the controller. Completing the task cooperatively is much more feasible and reliable this way, as the work is split into smaller components that are much easier to complete. Accordingly, with these smaller tasks, the system's responsiveness is increased, which directly improves overall system efficiency (Negenborn 2007). All of these factors have motivated the utilisation of the NMAS structure in multi-disciplinary applications such as multi-robot systems (Shaw et al. 2010), power networks (Negenborn 2007), and district heating systems (Wernstedt 2005).

The basic structure of the NMAS is illustrated in Figure 1–1.

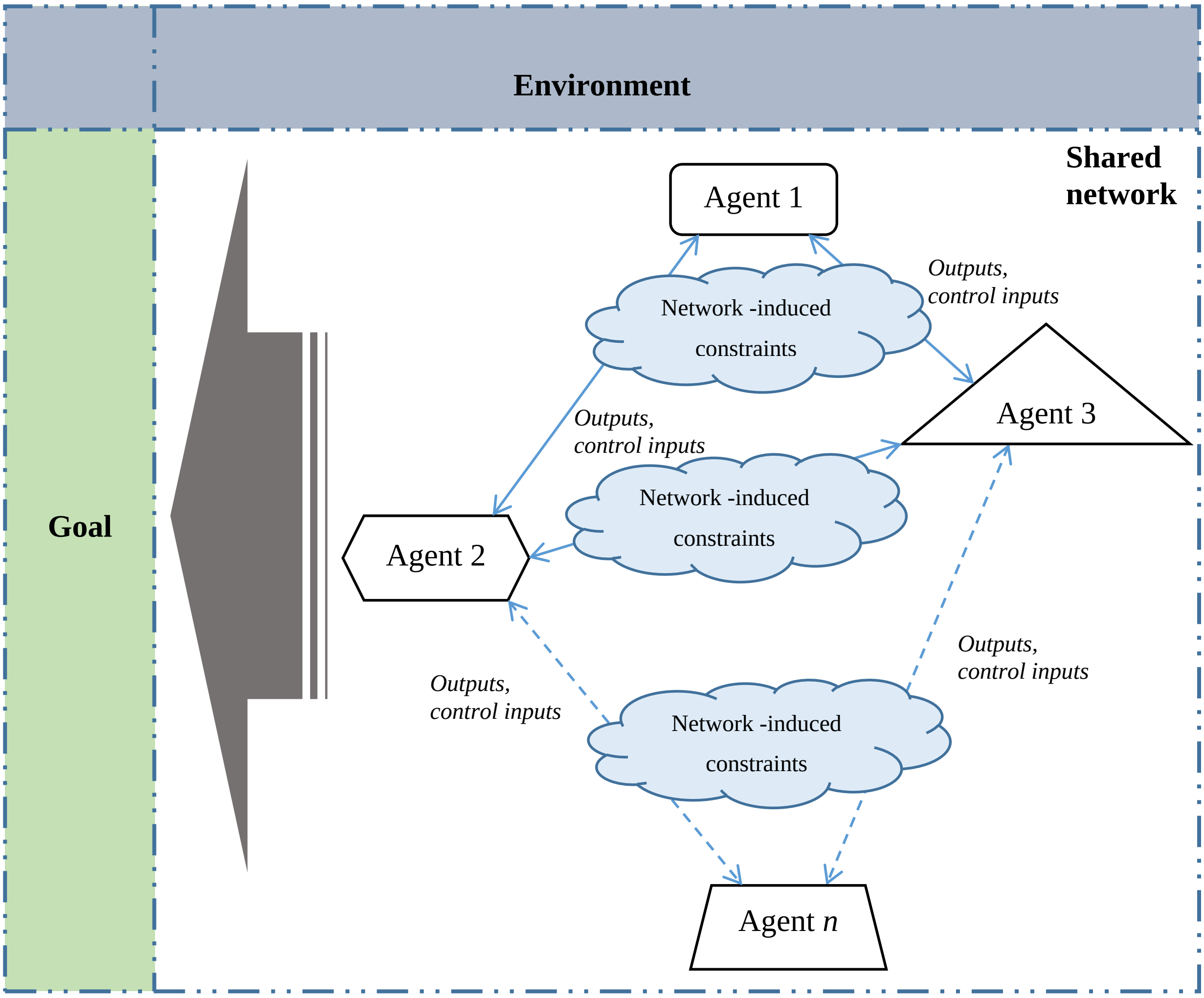

Figure 1–1 The networked multi-agent system (NMAS) control structure. The dissimilar shapes of the agents represent the heterogeneous dynamic of NMAS agents.

Figure 1–1 shows that the exchanged data usually consists of agents' outputs and control inputs' information. The agents communicate and cooperate amongst themselves under the same network in a particular environment in order to achieve their common goals.

The utilisation of a communication network in the NMAS, however, inevitably introduces constraints caused by limited bandwidth or overhead in the network or in the communication nodes. The presence of network constraints (such as persistent network delay and/or data or packet loss) will lead to significant deteriorations of controller performance if either one or both network constraints are not properly managed. Accordingly, an effective mechanism has to be designed to ensure that the chosen control method is able to compensate for these constraints without sacrificing NMAS performance. Thus, researchers have put significant effort into understanding and developing an effective solution for network constraints in NMAS applications.

1.2 Consensus of NMAS

One of the most common cooperative behaviours in the NMAS is ‘consensus’. Research on this consensus has become mainstream within the control system research community due to its desirable capacity within the distributed control system application.

The term ‘consensus’ originally comes from the Latin word *consentire,* meaning an agreement. In the broad control system perspective, consensus represents the cooperating behaviour of the NMAS’s agents to converge to a consensus value by considering the state of every agent via information exchange through shared network communication. Various types of consensus are known in the NMAS, including ‘average’, ‘max-min’, ‘function’, and ‘external’. Each represents the mechanism whereby the NMAS agents determine the consensus value.

In the NMAS, each agent has its own local controller for executing any given task. In the NMAS consensus problem, agents have to identify the target trajectory and consensus value from their neighbouring agent(s) via data that is exchanged through the network. Each agent then updates its current value or state accordingly. The consensus problem thus is solved cooperatively among the agents, using both individual and neighbouring agents’ information.

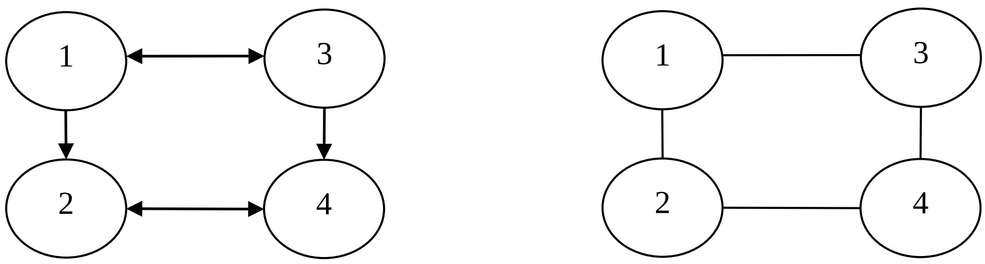

Figure 1-2 Directed (left) and undirected (right) agents’ connections.

The communication graph in Figure 1-2 illustrates (on the left) the unidirectional/bidirectional flow of sending and receiving information between NMAS agents in general, as well as (on the right) undirected flow between agents. The NMAS agents are represented by circles (nodes) labelled 1 to 4. The arrows indicate the direction of the information flow. In the directed graph on the left, the NMAS has bidirectional arrows between agents 1 and 3 and agents 2 and 4, which signifies that agents are able to send and receive data amongst themselves. The unidirectional information flow is illustrated between agents 1 and 2 and agents 3 and 4. Agents 1 and 3 can only *send* information, while agents 2 and 4 can only *receive* information.

In contrast, the NMAS can also be represented by an undirected flow graph, as shown by the figure on the right. There are no arrow symbols at the edge of the connecting lines between agents, which indicates that all of the participating agents can send and receive information amongst themselves.

Mathematically, the above directed graph is refer to be $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ for $i \in \{1, 2, 3, 4\}$ with four nodes or vertex $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$ and edges $\mathcal{E} = \{(v_1, v_3), (v_3, v_1), (v_1, v_2), (v_3, v_4), (v_2, v_4), (v_4, v_2)\}$. The node refer to agents while the edges refer to the information flow paths. For the undirected graph, an edge can be represented as $(v_i, v_j) \leftrightarrow (v_j, v_i)$ indicating that information from *i* to *j* and *j* to *i* has no exact direction. Label $\mathcal{A}$ refers to an adjacency matrix or also known as the connectivity matrix. This matrix provides a numerical representation of agents' relationship and can be either a weighted or unweighted adjacency matrix. In this thesis, only undirected graph is considered for each proposed work. All edges have positive unweighted adjacency matrix $a_{ij} = a_{ji} = 1$ for all *i*, *j*. No self-loop is allowed, hence $a_{ii} = a_{jj} = 0$. The set of neighbours of agent *i* is denoted by $N_i = \{j \in \mathcal{V}: (j, i) \in \mathcal{E}\}$. The Laplacian matrix L with respect to undirected graph $\mathcal{G}$ can be simply obtained as

$$L = [l_{ij}]_{n \times n}$$

where

$$l_{ij} = \begin{cases} |N_i|, & i = j \\ -1, & i \neq j \text{ } and \text{ } v_i \text{ } is \text{ } adjacent \text{ } to \text{ } v_j \\ 0, & otherwise \end{cases}$$

However, in this work, the Laplacian matrix L has non-zero elements because the agents are assumed to be interconnected to one another. Clearly, all the row-sums of L are zero. Therefore, L always has a zero eigenvalue $\lambda_1 = 0$ and the second smallest eigenvalue of L is

$\lambda_2 > 0$ if and only if $\mathcal{G}$ is connected and has a spanning tree (Ren et al. 2007). Thus, the eigenvalues of L can be ordered as $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Further explanation regarding this theory that related to the consensus problem can be found in (Lewis et al. 2014).

NMAS is said to achieve the consensus if $\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0$ for $i, j \in \{1, 2, \dots, n\}, i \neq j$ where $y_i(t)$ and $y_j(t)$ are the output of agent i and j respectively. Thus, in order to satisfy that condition, the consensus protocol $u_i(t)$ is designed to meet that condition by using available information collected by each agent.

1.3 Communication constraints

Due to the application of network communication between agents within a system, networked control systems suffer from multi-network constraints during data transmission (Heemels et al. 2010). Communications among agents throughout the network exhibit network-induced delay every time data is exchanged. The delay is worst with the occurrence of network data loss, which is marked by prolonged network-induced delay that exceeds a predetermined network delay threshold.

Network delay and data loss are elemental ‘conditions’ in any control system that utilises network communication; even though they have been minimised with the introduction of superior hardware performance, a flawless network communication system is still absent in practical applications. A reliable and practical solution for network delay and data loss problems therefore has to be implemented in the NMAS in order to limit the effects of such problems whenever they occur. A few types of network delay characteristics are considered in this thesis, as listed in Table 1-1.

Table 1-1 Network delay characteristics

Network delay characteristics	Description
Uniform	Identical delay present for agent to access its own output and for exchanged output data from its neighbours.
Asymmetric	Different delay for each communication link due to different passing routes.
Random / Time-varying	Non-identical delay present for agent to access its own output and for exchanged output data from its neighbours.

The terms used to describe these different data loss situations are listed in Table 1-2.

Table 1-2 Network data losses

Term	Description
Consecutive	Series of losses
Maximum consecutive	Total number of losses (maximum) during any particular time period
Uniform consecutive	Series of regular losses occurs at regular intervals
Random consecutive	Multi-series of losses occurs at irregular intervals

1.4 Literature review on NMAS consensus

This section reviews significant works that have been published on NMAS consensus. Interest in consensus problems for multi-agent systems through network communication has been growing in recent years, as seen by the number of research outcomes around the world. These works have yielded various kinds of analysis, depending on the authors' interests in solving the consensus problem. According to the cited literature, the considerations or factors that are usually taken into account in analysing the consensus problem can be listed as follows Table 1-3.

Table 1-3 Categories of NMAS characteristics reported in the existing literature

NMAS characteristics	**Category considered**			
Communication constraint	Constant network delay	Random network delay	Data loss / dropout	
System dynamic	Linear	Non-linear	Homogeneous / identical	Heterogeneous / non-identical
Order of the system	Single - integrator (1st order)	Double - integrator (2nd order)	*n*-order system	
Network topology	Fixed	Switching	Undirected	Directed
Type of consensus	Average	Max-Min	Function	External
Consensus behaviour	Leader - follower	Leaderless	Tracking	
Consensus performance	Convergence speed, time, rate	Coupling strength / gain		

Various studies (Cao et al. 2013; Chen et al. 2015; Ren et al. 2005; Wang et al. 2014) provide a comprehensive overview of recent progress that has been made in the NMAS consensus research area, a basic consensus analysis with related theories, and a summary of theoretical development in the NMAS consensus problem.

A few relevant earlier works address the consensus problem, with a focus on network topology and network constraints. For example, Olfati-Saber has published an extensive series of fundamental works on NMAS consensus that cover various aspects and considerations that underpin the development of this field. In 2003, Olfati-Saber and Murray conducted an in-depth analysis in particular at directed graph and switching topology for integrator NMAS (Olfati-Saber & Murray 2003). The authors described the relationship between the second smallest eigenvalue of a Laplacian matrix and convergence speed for a directed graph to solve the average consensus problem. With the same dynamics, the following year the same authors investigated linear and nonlinear protocols with and without constant network delay in fixed/switching network topologies (Olfati-Saber & Murray 2004). In 2007, Olfati-Saber, Fax, and Murray extended this analysis to investigate the relationship between the second smallest eigenvalue of an associated Laplacian matrix and consensus convergence speed in both discrete-time and continuous-time (Olfati-Saber et al. 2007).

The general linear consensus protocol (without the consideration of network communication constraints) is obtained as follows:

$$u_i(k) = \sum_{j \in N_i} a_{ij}\left(x_j(k) - x_i(k)\right), \qquad i, j \in \{1,2, \dots, n\}, \tag{1.1}$$

where $x_i(k)$ is the state of agent *i* at time *k*.

All significant related literature corresponding to these authors' concerning about solving the NMAS consensus problem are summarised in the following subsections.

1.4.1 Type of consensus value (Average, Max-Min, Function, and External)

Consensus is achieved through the implementation of the consensus protocol that is designed; the consensus value is determined by the operation that is included within the designed algorithm. Different consensus values create different types of tasks for all agents within the NMAS. The most common consensus problem discussed in the literatures is 'average' consensus (Olfati-Saber & Murray 2003; Olfati-Saber et al. 2007; 2004; Wu & Shi 2012; J. Liu et al. 2013; Yu et al. 2014; Kecai et al. 2011; Huang 2012; Sakurama & Nakano 2011; Wang & Elia 2009; Xie & Wang 2007; Zhou & Xiao 2013). The consensus value for average consensus problem is obtained through the equation:

$$consensus\ value, \alpha = Ave\left(x_i(0)\right) = \frac{1}{n}\sum\nolimits_{i=1}^{n} x_i, \qquad x_i(0) = x_i \tag{1.2}$$

Another type of consensus algorithm that is often found in the literature is 'min-max' consensus (Mulla et al. 2014). For example, Nejad, Attia, and Raisch provide the application of max-consensus for the case of minimum time rendezvous or leader election (Nejad et al. 2009). They describe the decision value for max-consensus and min-consensus problems as follows:

$$\begin{aligned} consensus\ value, \alpha &= max_i x_i \\ consensus\ value, \alpha &= min_i x_i \end{aligned} \tag{1.3}$$

Another type of consensus protocol that has been reported in the literature (Sayyaadi & Doostmohammadian 2011) is 'consensus function', where the consensus value is determined through the calculation of a unique equation or function. Cortés (Cortés 2008) models the weighted power mean consensus function by a weighted and directed graph in switching topology. For example, Q. Li et al. (Li et al. 2014) vary the consensus value depending on the value of the weighting coefficient of agent i ε_i, in which the consensus function is formulated as:

$$consensus\ value, \alpha = \frac{\sum_{i=1}^{n} x_i(0)\frac{1}{\varepsilon_i}}{\sum_{i=1}^{n}\frac{1}{\varepsilon_i}} \tag{1.4}$$

Another type of consensus protocol that does not have any constraints on the initial value of its agent is called 'external consensus'. S. Li et al. (S. Li et al. 2011) propose a consensus framework with an external input for a general (homogeneous or inhomogeneous) linear NMAS with a transfer function model. With the designed consensus protocol, the consensus value is independent of the initial value of the agents' states or outputs. All agents must track the external set point or reference input that is given to one or more agents in the NMAS. In the S. Li et al. study, however, the derived stability criteria for each agent were examined independently, which unfortunately cannot guarantee the stabilisation of the NMAS as a whole. This problem was solved by Yang and Xu (Yang & Xu 2012), however, who studied and proved the whole NMAS stability by demonstrating a proposed algorithm with two heterogeneous systems under the proportional-integral (PI) controller. Their designed consensus protocol with external reference input $R_i(s) = R(s)$ was given as:

$$\begin{gathered} U_i(s) = -\sum_{j\in N_i} a_{ij}\left(X_i(s) - X_j(s)\right) - X_i(s) + R(s) \\ consensus\ value, \alpha = R(s) \end{gathered} \tag{1.5}$$

Yet, Yang and Xu's solution still has a few obvious limitations. The NMAS stabilisation is guaranteed if the denominator of the designed controller has external reference signal terms. As a result, the designed controller is dependent upon the type of external reference input that is given to the NMAS, which limits the controller's capability. In addition, these results cannot be directly applied to the NMAS with network-induced constraints.

Such limitations are the basis of our motivation to propose a new, more generic external consensus protocol in this thesis.

1.4.2 Consensus behaviour (Leader-follower & Leaderless)

Various works in the literature have considered two common types of agent converging behaviour for reaching consensus. One is called 'leader-follower' behaviour, where the consensus value is achieved by all agents through direction from one or more leader agents (Ran & Wu 2013b; Ni et al. 2013; Ding et al. 2013; Ferrari-Trecate et al. 2009; Gong et al. 2013a; Li et al. 2009; Liu et al. 2015). In this type of consensus behaviour, leader agents can only transmit data to their neighbours. The other agents that are not connected directly to the leader agent will communicate and exchange data amongst themselves in order to track the leader's direction. In (Ran & Wu 2013a), for example, Ran and Wu design the output feedback consensus protocol with leader-follower behaviour as:

$$u_i(k) = K\left[\sum_{j\in N_i}\left(y_j(k) - y_i(k)\right) - d_i(y_i(k) - y_0(k))\right] + u_0, \qquad i,j \in \{1,2,\dots,n\} \tag{1.6}$$

where $d_i > 0$ if agent i is a neighbour of a leader agent labelled as 0. In (Wang & Hu 2010), Wang and Hu design an observer-based leader-follower consensus protocol as:

$$u^i = -K\left[\sum_{j\in N_i} a_{ij}\left(x^i - x^j\right) + \alpha_i(x^i - x^l)\right] + \hat{u}_l^i, \qquad i,j \in \{1,2,\dots,n\} \tag{1.7}$$

where u_l is not available for agent i; thus, each agent i has to estimate u_l, which is denoted as $\hat{u}_l^i$.

Another type of consensus behaviour considered in the published literature is 'leaderless' consensus behaviour. In this type, data is exchanged between agents in the absence of a formal leader and converges to a common consensus value based on the designed consensus protocol. Examples of this type of consensus behaviour can be found in many works in the literature, such as (He & Cao 2011a; Wu & Shi 2012; Hu & Lin 2010; Hu et al. 2008). Leader-follower and leaderless behaviours can be seen in the average, max-min, and function types of consensus. A handful of studies cover both consensus behaviours (Liu et al. 2014; Kim et al. 2014; Su & Huang 2012).

Unlike the consensus behaviour described earlier, in external consensus, all agents track the external reference input that is given to one or more agents in the NMAS. Any agents that are connected directly to the external reference input can both transmit and receive data from their neighbouring agents.

1.4.3 Types of NMAS

In addition to consensus behaviour, many studies have also examined agents' dynamic variation. Most related studies have presented their work in the context of integrator dynamics, i.e. single- or first-order integrators (Kecai et al. 2011; X.-L. Feng et al. 2013; Yu et al. 2014; Wu & Shi 2011; Wu & Shi 2012; Li et al. 2012; Li et al. 2014; J. Liu et al. 2013; Sun & Wang 2009; Jin & Zheng 2009; Shida & Ohmori 2011; Sakurama & Nakano 2011; Olfati-Saber et al. 2007; Olfati-Saber & Murray 2004; Olfati-Saber & Murray 2003; Andreasson et al. 2013; Chang et al. 2011; Cao et al. 2013; Ferrari-Trecate et al. 2009; Wang et al. 2014; Wei et al. 2010; Liu & Tian 2007; Seuret et al. 2008) and double- or second-order integrators (Eichler & Werner 2014; Hu & Lin 2010; X.-L. Feng et al. 2013; Gong et al. 2013a; Liu & Liu 2010; Xie & Wang 2012; Pan et al. 2014; Ferrari-Trecate et al. 2009; Andreasson et al. 2013; Ding et al. 2013; B. Liu et al. 2013; Cheng et al. 2013; Zhang & Tian 2012a), which can be represented by continuous-time models, as in (1.8) and (1.9), respectively:

$$\dot{x}_i(t) = u_i(t) \tag{1.8}$$

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= u_i(t) \end{aligned} \tag{1.9}$$

Or agents with discrete-time models, as in (1.10) and (1.11), respectively:

$$x_i(k+1) = x_i(k) + \epsilon u_i(k), \qquad \epsilon = \text{sampling period} > 0 \tag{1.10}$$

$$\begin{aligned} x_i(k+1) &= x_i(k) + \epsilon v_i(k), \\ v_i(k+1) &= v_i(k) + \epsilon u_i(k), \end{aligned} \tag{1.11}$$

In real-world applications, however, an agent's dynamic is usually more complex than an integrator dynamic; researchers have thus devoted significant effort to enhancing the practicability of NMAS with a more general framework. For example, several studies (Su & Huang 2011; Zhang & Duan 2011; Namerikawa & Yoshioka 2008a; Tan & Liu 2013; Tan & Liu 2012; Tan & Liu 2011; Yang & Xu 2012; Liu et al. 2014; S. Li et al. 2011; Münz et al. 2011; Tian & Zhang 2012; Ni et al. 2013; Mulla et al. 2014; Zhang & Tian 2012a; Zhao et al. 2011; Zhongkui Li et al. 2011; Li et al. 2009; Li et al. 2010; Wang et al. 2008; Zhou & Xiao 2013; He & Cao 2011a; Ran & Wu 2013b; Z. Li et al. 2011; Wang et al. 2011; Wieland et al.

2008; Xu et al. 2013; Shida & Ohmori 2011; Wen et al. 2013; Zeng & Hu 2013; Zhang & Tian 2009) have presented a more complex form with any order or higher-order general linear dynamical system, in either state-space or transfer function forms. NMAS agents with a linear time-invariant dynamic can be described as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \qquad i = 1,2,\dots,n \tag{1.12}$$

or in a discrete-time model:

$$x_i(k+1) = Ax_i(k) + Bu_i(k) \tag{1.13}$$

where A and B are constant system matrices.

In addition, in real-life applications, NMAS systems with homogeneous agents rarely exist, especially in process control manufacturing industries; these are relatively large in size in most cases, and consist of many subsystems with multi-purpose tasks. For practical reasons, heterogeneous or inhomogeneous (non-uniform or non-identical) agents' dynamics should therefore be considered in NMAS consensus problems (Ren et al. 2005; Wang et al. 2014). This element, however, will add a degree of difficulty and complexity in the process of designing the consensus protocol and developing the stability and consensus analysis, as well as in implementing it on a real-world NMAS platform. The overall complexity of the developed control system may be even greater if network-induced constraints are also taken into consideration.

The external consensus problem for distributed heterogeneous NMAS is studied in (S. Li et al. 2011; Yang & Xu 2012) within a similar framework, with a transfer function model as follows:

$$G_i(s) = \frac{a}{bs^m + cs^{m-1} + \cdots + ds + e}\,, \qquad i = 1,2,\dots,n \tag{1.14}$$

where $a, b, c, d,$ and e are the polynomial coefficients, and m is the highest order of the polynomial denominator.

In (C. Tan & Liu, 2012, 2013), consensus analysis is developed with discrete-time heterogeneous linear NMAS, in which the agents' dynamics are represented as follows:

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t)$$
$$y_i(t) = C_i x_i(t) \tag{1.15}$$

where A_i, B_i, and C_i are constant matrices of agent i.

In (Wang & Sun 2015), by using the linear matrix inequality (LMI) technique, Wang and Sun solve both consensus and the H_∞ consensus problem for heterogeneous NMAS with time-varying delay. The heterogeneous agents' dynamics are encompassed with single and double integrators into NMAS. The authors studied the implementation of the prediction in the consensus protocol to compensate for the constant network delay.

We conduct our simulation in this thesis by considering the NMAS agents as an approximated model of a real-laboratory test rig, which is represented in a transfer function form. This form has a higher degree of complexity compared to the integrator form, and was obtained from an identification method based on experimental data. The method also provides a generic form of the approximated agents' dynamics, which can be used to model any actual processes in real life. Furthermore, we also consider the heterogeneity of the agents' dynamics of the NMAS, represented by three water level process controls with different dynamics. These considerations are important to ensure that the works are applicable to a general framework of NMAS, whilst still being practical.

1.4.4 Network constraint (Network delay)

Establishing the validity of the consensus study in the NMAS without considering the network constraints is unrealistic. The constraints and challenges in control systems' practical applications need to be taken into account in order to produce a sensible control protocol that is capable of performing well when implemented in a real-world environment (Wang et al. 2014). As a result, many researchers, motivated to produce a practical and realistic NMAS consensus protocol, have considered various types of network-induced constraints within their proposed solutions to the consensus problem.

Although network-communication-induced problems in networked control system applications come in many forms, two main types of network-induced problems that have gained a great deal of attention are network-induced delay and data or packet loss (i.e. dropout). In order to tackle network delay problems in a simplified manner, researchers often choose to assume network delay to be constant (Pan et al. 2014; Sayyaadi & Doostmohammadian 2011;

Liu & Tian 2007; Wei et al. 2010; Münz et al. 2007; Münz et al. 2011; Liu & Liu 2010). This constant value is usually determined by the upper bound of the measured time-varying network delay, or the maximum tolerable network delay (Tan & Liu 2013; Namerikawa & Yoshioka 2008a). In (Olfati-Saber & Murray 2004), for example, Olfati-Saber and Murray present an analysis of determining the maximum allowable constant network delay from the largest eigenvalue of a Laplacian matrix for average consensus; the authors show that the robustness of the NMAS towards network delay is explicitly dependent upon the network topology in the case of undirected and static connection topologies. When they considered network-induced delay, the authors used the following linear consensus protocol:

$$u_i(k) = \sum_{j \in N_i} a_{ij}(x_j(k - \tau_{ij}) - x_i(k - \tau_{ij})), \qquad i, j \in \{1,2, \dots, n\}, \tag{1.16}$$

where $\tau_{ij} > 0$. If network delay is assumed to be a uniform and constant value, the above protocol can be simplified as:

$$u_i(k) = \sum_{j \in N_i} a_{ij}(x_j(k - \tau) - x_i(k - \tau)), \qquad i, j \in \{1,2, \dots, n\}, \tag{1.17}$$

In order to achieve consensus, the value of τ must be within its tolerable range, as follows:

$$0 \leq \tau < \frac{\pi}{2\lambda_n}$$

where $\lambda_n$is the largest eigenvalues of a Laplacian matrix.

In (Tan & Liu 2012), Tan and Liu study discrete-time NMAS with a uniform constant network delay under directed and static topology. ‘Uniform delay’ refers to a situation in which the agent experiences a delay in accessing its own information, which is identical to the delay in data transfer from its neighbouring agent(s) in the network. The authors designed the developed consensus protocol based on the relative outputs and aggregate relative state predictions between agents. They derived both sufficient and necessary conditions to clearly show the effectiveness of the designed consensus protocol in compensating for network delay. Their designed consensus protocol can be expressed as:

$$u_i(t) = K_i \hat{\xi}_i(t | t - \tau) \tag{1.18}$$

where $\hat{\xi}_i$ denotes the aggregate relative states’ prediction sequence of agent *i*.

For the case of non-uniform network delay, heterogeneous delay is considered during information exchanges between agents and within the agent itself, as follows:

$$u_i(t) = -\sum_{v_j \in N_i(t)} \frac{a_{ij}}{d_i} (y_i(t - T_{ji}) - y_j(t - \tau_{ji})), \qquad i, j \in \{1,2,\dots,n\}, \tag{1.19}$$

If the delay within the agent itself is negligible, then the consensus protocol can be designed as:

$$u_i(t) = -\sum_{v_j \in N_i(t)} \frac{a_{ij}}{d_i} (y_i(t) - y_j(t - \tau_{ij})), \qquad i, j \in \{1,2,\dots,n\}, \tag{1.20}$$

These protocols are discussed in (Ulrich et al. 2011; Shida & Ohmori 2011; Sakurama & Nakano 2011; Seuret et al. 2008); the impact of the neighbouring agents' delay on the convergence rate is also discussed in (Seuret et al. 2008).

More complex network delay conditions have been studied by considering NMAS with time-varying network delay. For example, in (Wu & Shi 2012), Wu and Shi design the consensus protocol with the effect of both bounded time-varying delay and data loss as:

$$u_i(k) = -\gamma_c \sum_{j \in N_i} \gamma_{ij}(k) a_{ij} (x_i(k - d(k)) - x_j(k - d(k))), \qquad i, j \in \{1,2,\dots,n\}, \tag{1.21}$$

where $\gamma_{ij}(k) = 1$ if no data loss occurs (and vice versa), $d(k)$ is the transmission delay at time k, and γ_c is the control gain; the value γ_c is determined by testing the feasibility of the derived LMI. Other solutions for this particular problem can be found in (Wang & Sun 2015; Tian & Zhang 2012; Zhang & Tian 2010; Blondel et al. 2005; Shida & Ohmori 2011; Lin & Jia 2009; J. Liu et al. 2013; Sun & Wang 2009; Wu & Shi 2011; Wu & Shi 2012; Hu & Lin 2010; Ding et al. 2013).

1.4.5 Network constraint (Data loss)

If the cooperation among NMAS agents is performed through a shared communication network, it is hard to ensure that all transmitted data will be successfully received by the neighbouring agent(s). In practical applications, because the network communication condition is highly dependent upon the rate of usage, it is not continuously stable, and the network can sometimes be inadvertently disabled for a few moments. It is thus common for transmitted data to be lost while it is being transmitted. When information is delayed for a long period of time (exceeding the maximum allowable network delay), it is considered to be a situation of data loss or data dropout.

Wang et al. consider the average consensus problem with uniform data loss (where all network connections within the system intermittently fail) in (S. Wang et al. 2010). They investigate consensus performance by comparing two consensus protocols: memory and memoryless consensus protocol. Their study proved that the memory consensus protocol yields better results (in terms of convergence rates) compared to the memoryless protocol. Kim, Choi, and Park propose the same approach in (Kim et al. 2014), in which they solve the consensus problem for heterogeneous NMAS with first- and second-order integrators with random link failures by utilising the latest stored previous value. They designed the consensus protocol as:

$$\begin{aligned} u_i(k) = -k_1 \sum_{j=1}^{n} \big[\{ \alpha . a_{ij} x_j(k) + (1-\alpha) . a_{ij} x_j(k-1) \} \\ - a_{ij} x_i(k) \big], \quad for\ first\ order\ agent \end{aligned} \tag{1.22}$$

In (Zhou & Xiao 2013), Zhou and Xiao derive a new disagreement vector in order to solve the average consensus problem for linear NMAS with random packet loss. The authors prove the stability through the presentation of necessary and sufficient conditions where the mean-square consensus could be achieved if the union graph were connected. Two studies (Shaw et al. 2010; Klein et al. 2008) present the experimental results of the consensus problem with data loss for multi-robots and an unmanned autonomous vehicle (UAV), respectively. Differing from the approach of studies (Li et al. 2014; Guinaldo et al. 2012), these two studies introduce an event-based consensus control to solve the consensus problem with the occurrence of data loss. Other related work can be found in (Li et al. 2012; Zhang & Tian 2010; Zhang & Tian 2012a; Gong et al. 2013b; Wu & Shi 2012).

Most studies have solved the network-induced problem by considering the homogeneous NMAS agents or the integrator dynamic agents; exceptions include (Kim et al. 2014; Tan & Liu 2011; Tan & Liu 2013; Tan & Liu 2012), which consider a heterogeneous linear NMAS. In addition, Zhang and Tian (Tian & Zhang 2012) present an analytic solution for heterogeneous high-order NMAS, although the numerical simulation of the solution, where only homogeneous NMAS with integrator dynamics is presented, does not provide similar considerations. As such, the NMAS consensus problem with heterogeneous and non-integrator dynamics agents remains relatively unexplored, and is worthy of further investigation.

1.4.6 Network topology

The data loss problem is caused by intermittent network failure, which frequently occurs in a random manner. In order to analyze this condition, many researchers have considered the NMAS consensus problem by using the switching connection topology. For example, in (Zhang & Duan 2011), Zhang and Duan consider both the static and switching interaction topologies with the implementation of a predictive algorithm by considering the agents' input constraints in the consensus protocol. In (Wang et al. 2008), Wang, Cheng, and Hu study the switching topology case for any order of identical NMAS with local information–based decentralised controls. In (Olfati-Saber & Murray 2004; Olfati-Saber & Murray 2003), Olfati-Saber and Murray develop the disagreement function, using simple Lyapunov theory stability analysis, in order to investigate the possibility of mobile networks in directed switching topology to reach average consensus.

A great deal of consensus analysis in switching topology can also be found in (Ferrari-Trecate et al. 2009; Münz et al. 2007; Sun & Wang 2009; Cortés 2008; Ghadami 2012; He & Cao 2011a; Lin & Jia 2009; Ni et al. 2013; Olfati-Saber & Murray 2003; Olfati-Saber & Murray 2004; Sayyaadi & Doostmohammadian 2011; Shida & Ohmori 2011; Su & Huang 2011; Su & Huang 2012; Wang & Elia 2009; Wang et al. 2008; Wang et al. 2013; Wen et al. 2013; Xie & Wang 2007; Zhan & Li 2013; Zhao et al. 2014; Zhou & Xiao 2013; Zhu & Yuan 2014). In these publications, the researchers focus their analyses on deriving an appropriately sufficient condition for a specific topology, rather than developing a proper controller to solve the consensus problem. Even though this method is the most convenient for researchers in terms of proving the control theory, it is not the most effective way to pursue a practical solution for this particular situation, as failure occurrence is rarely predictable (Yu et al. 2014; Wang et al. 2014; Wu & Shi 2012).

In this thesis, we develop a distributed consensus algorithm that has the capability of compensating data loss within different topologies. We find this solution to be more practical than previous efforts in this area, since the algorithm's capability of compensating for the data loss problem is not restricted to any one type of NMAS topology.

1.4.7 Consensus performance (Convergence speed, Coupling weight, and Gain)

In the consensus problem, the length of time for each agent to reach its consensus value (known as 'speed of convergence', 'convergence time', or 'convergence rate') is an essential consensus performance indicator (Wang et al. 2014). The shortest convergence time or highest convergence speed (rate) is important in order to achieve desirable NMAS consensus performance. Unfortunately, among the detrimental effects of network delay and data loss in the NMAS are the increments that are used in convergence time, which is certain to degrade the consensus performance significantly. Convergence performance has thus become a focal point of many NMAS-related studies.

Olfati-Saber and Murray establish the relationship between convergence speed and algebraic connectivity (the second smallest eigenvalues of a Laplacian matrix) in (Olfati-Saber & Murray 2004); the authors also discuss the trade-off between convergence speed and the consensus protocol robustness to constant network delay. Tan, Liu, and Guan apply the same approach in (Tan et al. 2009), where the maximum network delay is determined in order to maintain convergence performance. In (Eichler & Werner 2014), Eichler and Werner find the optimisation of convergence speed for double-integrator NMAS by determining the proper value for three parameters: two parameters that were obtained from the analytic solution (which depended on the largest and smallest eigenvalues of a Laplacian matrix) and a third parameter that was derived from LMI conditions.

Other authors take a different approach (in (Liu et al. 2014; Z. Li et al. 2011)); these studies analyse the relationship between convergence speed and the largest negative real part of the eigenvalues of the closed loop system. The consensus protocol based on relative outputs between agents is designed as follows:

$$u_i = cK \sum_{j \in N_i} a_{ij}\left(y_j - y_i\right), \quad i, j \in \{1,2,\dots,n\} \tag{1.23}$$

From the protocol, by designing a suitable feedback control gain matrix K and coupling gain $c > 0$ that conforms to the developed relationship, the consensus of the system can be reached with the initially prescribed convergence speed.

In (Clark et al. 2013), Clark, Bushnell, and Poovendran propose a unifying framework for encompassing both leader-follower and leaderless systems in order to minimise converging time. The authors achieve this by choosing a suitable leader and finding an optimal weight for each of the interaction links; they prove this technique to be a good combination in minimising convergence time. In (Pan et al. 2014; Yu et al. 2014), they make the consensus convergence comparison between the common linear consensus protocol and the consensus protocol based on second-order neighbours' information. By utilising the second-order neighbours' information, the authors prove that the convergence speed can be accelerated. Their consensus protocol is designed as:

$$u_i(t) = \sum_{j \in N(G,i)} a_{ij} \left[\left(x_j(t) - x_i(t)\right) + \sum_{k \in N^2(G,i)} a_{ik}\left(x_k(t) - x_i(t)\right) \right] \tag{1.24}$$

where $N^2(G,i)$ denotes the second-order neighbours of agent i.

Another common approach to achieving a higher convergence rate is through the optimisation of the weight or gain at the communication link between agents (Sheng & Ding 2013; Zhu & Yuan 2014). In (Zhu et al. 2009), Zhu, Tian, and Kuang introduce coupling gains into the second-order consensus protocol to maximise the convergence speed; they determine suitable values for the gains from the largest and smallest eigenvalues of a Laplacian matrix. In (Zhang & Tian 2012b; Zhang & Tian 2009), Zhang and Tian suggest that in discrete-time NMAS, the coupling weight should be treated as a control parameter, where the static state feedback consensus protocol is designed as:

$$u_i(t) = K \sum_{j \in N_i(t)} a_{ij}(t) w_{ij} \left(x_j(t) - x_i(t)\right), \quad i,j \in \{1,2,\dots,n\} \tag{1.25}$$

where K is the static state feedback gain and w_{ij} is the weight. These parameters must be properly designed in order to achieve consensus. In (Ning et al. 2012), Ning, Ren, and Zhao study the modified Vicsek model with a proposed time-varying coupling weight; they find that improvements in convergence time can be obtained by adjusting the weight between agents, which minimises the agents' state differences.

From the abovementioned studies, we can see that most works have successfully improved the consensus convergence to various degrees, depending on the connectivity of the network topology (Xie & Wang 2007). Yu, Zhang, and Sun clearly show in (Yu et al. 2010) that higher connectivity leads to faster convergence rates.

In (Wei et al. 2010; Y. Feng et al. 2013; He & Cao 2011a), the authors use the selection of appropriate controller parameters or a suitable control method for consensus protocol to improve convergence speed. For example, in (Wei et al. 2010), Wei, Fang, and Wang use the proportional-derivative (PD) terms to design a consensus protocol that can improve the convergence performance; the protocol is designed as:

$$u_i(t) = \sum_{v_j \in N_i} a_{ij}(t)\left(x_j(t-\tau_{ij}) - x_i(t-\tau) + \eta \dot{x}_i(t-\tau)\right), \quad i,j \in \{1,2,\dots,n\} \tag{1.26}$$

where $\eta > 0$ denotes the PD feedback density.

Other than optimisation of the coupling weight between agents and the controller's parameters, the consensus algorithm with memory has also been proven to improve the consensus convergence (X.-L. Feng et al. 2013). Feng et al.'s designed consensus protocol can be presented as:

$$u_i(t) = a_{ij}(t) \sum_{j \in N_i(t)} \left(x_j(t) - x_i(t)\right) - x_i(t-\tau) + x_i(t), \quad for \;\; t \in [-\tau, 0) \quad i,j \in \{1,2,\dots,n\} \tag{1.27}$$

Adding an opposite sign to the last two terms in the above protocol, however, could lead to a slower convergence speed.

In (Gao et al. 2015), Gao et al. study the combination of proportional-integral (PI) controller with a two-hop network. Based on the presented analytical and numerical results, this combination provides the control system with a fast convergence speed. Shang uses a difference approach in (Shang 2014), where a non-uniform external control input is introduced in the consensus protocol to obtain a fast convergence speed. Other related work on convergence studies can be found in (Xiao et al. 2014; Blondel et al. 2005; Sayyaadi & Doostmohammadian 2011). It is worth mentioning that optimal convergence for general linear NMAS is still an open problem (Cao et al. 2013).

1.4.8 Existing control strategies

Based on the cited literature, most studies have focussed on deriving the sufficient and necessary conditions for the NMAS to achieve consensus with or without network-induced constraints. These conditions are derived mainly to provide information about the maximum allowable network delay or data loss probability for the designed consensus protocol to achieve consensus.

While a significant number of studies have concentrated on providing a theorem or criterion in choosing the suitable coupling strength, gain, or weight parameter that can be used to improve or adjust consensus performance, very few studies have designed an appropriate controller that can drive the NMAS to achieve consensus value. Below are a few significant publications that have provided a designed controller, depending on the authors' chief focus:

- A controller based on state or output feedback is designed based on the designed feedback gain matrix, observer gain matrix, or controller parameters. Several authors have developed and derived theorems or conditions to design the feedback gain matrix, observer gain matrix, and controller parameters for the consensus protocol. The chosen values of gain matrices and parameters have to satisfy the developed conditions in order for the NMAS to reach consensus. Common related theories include Riccati's algebra equation, the Lyapunov function, and linear matrix inequality (Ding et al. 2013; Namerikawa & Yoshioka 2008b; Z. Li et al. 2011; Liu et al. 2014; Yu et al. 2014; Ni et al. 2013; Liu & Liu 2010; He & Cao 2011b; Zhou & Xiao 2013; Su & Huang 2011; Zhang & Tian 2009; Hu et al. 2015; Zhao et al. 2011; Wu & Shi 2012; Zhang & Tian 2012a).
- The controller is designed to consist of second-order neighbours' information in order to accelerate the consensus convergence speed (M. Yu et al., 2014).
- The output-feedback gain matrices are designed through the pole-assignment method (Wang et al. 2011; Ran & Wu 2013c).
- The controller is designed based on the predictive method; the observer-based predictive controller is designed using states and relative states prediction (Tan & Liu 2011; Tan & Liu 2013; Tan & Liu 2012; Tan et al. 2015; Yang & Liu 2014; Zhang & Zhang 2014).
- The controller transfer function is designed to contain a reference input term at the denominator of the controller's transfer function. The proposed controller is thus fully dependent upon the reference input (Yang & Xu 2012).
- The controller is designed to impose proportional-derivative (PD) terms represented by the scalar value called 'PD feedback density'. This value is used to improve the consensus convergence speed (Wei et al. 2010).
- The controller is designed based on an event-triggered control strategy (Hu et al. 2015; Li et al. 2014; Guinaldo et al. 2012)
- The controller is designed based on the adaptive control strategy (Yucelen et al. 2015; Radenkovic & Tadi 2015)

We can conclude from this that the consensus problem with an optimal control method is still worthy of investigation.

Over the last few decades, several studies have reported that the application of a prediction strategy in the networked single-agent structure shows a promising capability of compensating for network-induced constraints (Liu 2009; Liu 2005; Liu, Xia, et al. 2005a; Liu, Xia, et al. 2005b; Liu et al. 2005; Liu et al. 2005; Liu et al. 2004; Onat et al. 2008; Srinivasagupta et al. 2004; Xia & Fu 2010; Zhan & Li 2013; Mill et al. 2008; Liu, Xia, Chen, et al. 2007; Ulusoy et al. 2007; Liu 2010; Caruntu & Lazar 2011; Chai et al. 2008; Miklovicova & Mrosko 2012; Istanbul 2006; Zhe et al. 2006; Xueyan & Zhongjie 2010). This scenario has thus motivated us to adopt this strategy in this thesis in the context of NMAS application; we therefore explore the application of prediction strategy to the external consensus problem.

The prediction strategy algorithm for external consensus protocol developed in this thesis is inspired by simple single agent prediction strategy founded by Chai et al. 2008. The predicition strategy algorithm capability has been fundamentally enhanced to enable solving the multi-agents NMAS within external consensus problem. The proposed algorithm involves relatively simple mathematical equations, which helps reduce the burden of hardware requirements in large-scale applications, and increases the probability for the control system to be implemented in real-world practical applications. The details of the proposed prediction algorithm is presented in Chapter 3 to 6.

1.5 Motivations and contributions

An analysis of the consensus reached in related studies indicates that nearly all studies have focussed on theoretical frameworks, which usually consist of highly complex equations that are unfeasible to be embedded in real-world hardware platforms. In addition, the results and the designed consensus algorithms are generally only tested and demonstrated through mathematical analyses, and are validated using numerical simulations. The actual capability and performance of NMAS consensus in real-world practical applications is therefore still questionable, since (to the best of our knowledge) the studies have provided experimental verification. Simulation results through mathematical models can only predict the outcome of the control system; they will never be able to provide an exact picture of actual practical conditions. Only a handful of papers have investigated the consensus problem with a practical implementation (Klein et al. 2008; Namerikawa & Yoshioka 2008a; Shaw et al. 2010). From the beginning, the work in this thesis has thus been consciously steer ed towards producing practical and feasible solutions for the NMAS in a real-world application.

In this thesis, the main motivation for this research is thus to provide a practical solution to the NMAS external consensus problem in the presence of network-induced constraints, with comprehensive validation through practical experimentation. By presenting the empirical results from the experiments to support the designed solutions, we hope that it will become a solid foundation for NMAS technology to be integrated into real-world practical applications.

In this thesis, the works presented are related to the development of a distributed cooperative algorithm for the external consensus problem in heterogeneous linear NMAS with network-induced constraints. The proposed consensus protocol is designed to be capable of managing complex conditions such as network constraints, heterogeneity, and complexity in agents' dynamics, while being theoretically proven and practically feasible. To the best of our knowledge, no analysis or practical work has been done on this subject to date.

It is a monumental challenge to combine several related theoretical works with experimental implementations on each designed control system within one general research study such as this. Every stage of the research demands multifaceted knowledge. Extensive theoretical understanding is essential on topics that include overall control methodology, communication technology (network topology and intranet-based communication technology), and related mathematical analyses. In addition, working knowledge related to computational software and applications, and general engineering skills in setting up and performing the experimental works, are also vital for ensuring overall research success. The combination of this knowledge is crucial for ensuring that the proper considerations regarding the designed controller practicality, and compatibility to the available experimental hardware, can be incorporated into the development of the theoretical solutions during the early stages of the research process.

Authors of earlier related works have generally preferred to focus on deriving a sufficient condition for NMAS to achieve consensus with or without network-induced constraints, rather than designing a suitable controller to solve the consensus problem. Furthermore, some of the solutions that authors have provided - for example in (Blondel et al. 2005; Wang & Elia 2009) - have only been presented in analytical format, without any corresponding proof via numerical simulation. Thus far, because very limited research has been published on heterogeneous linear NMAS, most of the new developments in the NMAS field with constraints have been confined to academia; there has been little impact on practical applications.

Although various published studies have documented the advantages of prediction strategy for networked controlled single-agent systems, very few studies have explored the application of prediction strategy in NMAS. Methods of solving the consensus problem with prediction strategy are presented in (Droge & Egerstedt 2013; Zhang & Duan 2011; Ferrari-Trecate et al. 2009; Roshany-Yamchi et al. 2011; Tan & Liu 2011; Tan & Liu 2013; Tan & Liu 2012). The application of prediction strategy in NMAS has significant potential, as several studies have proved its capability in compensating for network-induced constraints, such as delay and packet loss for a single-agent systems (Liu, Xia, Rees, et al. 2007; Liu, Xia, Chen, et al. 2007; Liu 2010; Chai et al. 2008; R. Wang et al. 2010). While several varieties of prediction approaches have been developed for single-agent systems, many are too mathematically demanding to be implemented within NMAS. Thus, in this thesis, we prove that by carefully selecting a suitable prediction algorithm, the prediction strategy can be successfully applied in the NMAS with manageable mathematical complexity, which allows for comprehensive empirical testing.

We achieve further improvement of the proposed prediction algorithm's performance through the application of a suitable designed coupling gain, which is also one of our main contributions in this thesis. Motivated by our observations, we put forward a novel prediction-based distributed external consensus protocol by modifying and enhancing the predictive controller that Chai et al. present in (Chai et al. 2008).

Most of the presented works in the cited literature have the obvious limitation of considering the integrator dynamic in the NMAS consensus problem. The integrator dynamic represents a simple agent's dynamic, which is rarely present in real-world practical systems; thus it cannot provide a general solution that can truly represent the characteristics of NMAS agents in most real-world applications. Bearing in mind that our research objective is to produce a practical solution, we developed the NMAS agent models used in this thesis using a system identification process through off-line empirical data obtained via open-loop experimentation. In addition, we propose a general framework for an external consensus protocol by considering the heterogeneous linear dynamics of NMAS agents. This general framework has the flexibility to be expanded further in order to represent homogeneous linear NMAS.

When dealing with network communications, it is obvious that network-induced constraints are inevitable. A consideration of network constraints in the designed consensus protocol is thus vital to ensure that the control protocol performs well in practical applications. This thesis therefore considers three types of network delay. First, we consider a uniform constant network delay, where identical delay is present within the agent itself, both when it accesses its own data and during data transmission from its neighbour(s). Second, we consider an asymmetric constant network delay, where identical delay is present within the agent itself when it accesses its own data, as well as during data transmission from its neighbouring agent(s), but the value of the delay varies for each passing route where data is transmitted. Finally, we consider bounded random network delay, where a random value of delay is present within an allowable range at every network link between the agents. The three cases are reported in Chapter 3, 4, and 5, respectively.

For the case of data or packet loss, earlier published works have focussed on data loss probability percentage, which usually occurs only during optimal operating conditions (Zhang et al. 2006). In this thesis, however, we consider the worst-case scenario by analysing a large number of consecutive data losses (CDL), presented in Chapter 6. We propose a prediction algorithm combined with a coupling gain–based 'gain error ratio' (GER) formula to solve this problem. The proposed method has not only solved the external consensus problem but has also significantly reduced the consensus convergence time.

By taking into account all of the practical considerations at every stage of the research work, this thesis successfully provides solutions for a general framework for external consensus protocol for linear NMAS with network-induced constraints. All of the proposed solutions are proven through mathematical analyses, substantiated through numerical simulations, and demonstrated in practical experiments.

1.6 The proposed external consensus protocol

The distributed cooperative protocol is proposed to solve the external consensus problem for linear heterogeneous NMAS in the presence of two types of network-induced constraints: network delay and data loss. External consensus is defined as a group of agents that cooperate through data exchange in order to achieve their common target, which is the external reference input (consensus value). The external consensus protocol is represented by a distributed cooperative algorithm that directs all agents in the NMAS to converge to an external reference input that is given to only one of the agents (Agent 1) via a local communication network. In cooperative control systems, agents share information about their common objectives along a trajectory to the consensus value, which requires each agent to communicate and update its own and its neighbours' states or outputs in order to reach consensus. As a result, even though the agents are physically separated, the agents' behaviour is coupled with that of neighbouring agents in order to accomplish their tasks.

External consensus is unique when compared to the other consensus type, as it does not depend on the initial condition of any of the participating agents. All of the other consensus types, in contrast, do rely heavily on the initial condition of the participating agents, since the NMAS consensus values are obtained based on the agents' initial values or states.

In this thesis, the consensus value is water level (in centimetres [cm]). Each agent in the NMAS is represented by a unique water level process control test rig. Within the scope of this thesis, the general task for the agents is thus to attain the given water level set point introduced by the operator. As the system is set as external consensus, the set point value is given only to one agent which is selected by the operator and represents Agent1. Other agents have to communicate with Agent1 and their neighbouring agents concurrently in order to complete their tasks.

The main aspiration of this thesis is to further unleash the potential that NMAS technology has for practical applications. As such, it is vital that the proposed solution must be not only theoretically strong and well proven: it also must be feasible in practice.

Several criteria need to be taken into account when developing a practical solution for the NMAS with network-induced constraints:

- Complex mathematical equations must be avoided, as these will be untenable with a large number of agents in practical applications;
- A simple and practical algorithm should be implemented in the hardware;
- The solution should be highly efficient in compensating for network-induced constraints.

For these criteria, we propose a *model-based* prediction control strategy in this thesis. Model-based prediction means that the NMAS agents use the prediction model to predict the behaviour of the system over a certain prediction length; the prediction value is then used to compensate for the network-induced constraints between the agents.

1.7 Aims and objectives

The aim of this study is to develop a practicable control algorithm that can solve the NMAS external consensus problem with network-induced constraints. In order to achieve this, several integral research objectives have been created:

- A novel external consensus protocol for heterogeneous linear NMAS which is capable of solving the external consensus problem in the context of network-induced constraints.
- Develop corresponding theoretical analysis on the stability of the whole NMAS.
- Develop corresponding theoretical analysis on consensus convergence performance.
- Empirically validate the external consensus protocol.

Based on these aims and objectives, the general aspiration of this research is to bring the vast potential of the theoretical knowledge about NMAS technology into real-world practical applications. With this mind-set, the whole research process has been focused on developing a practical control algorithm for the NMAS that not only has a sound theoretical foundation and is mathematically provable, but that can also be successfully demonstrated in practical applications.

1.8 Thesis outline

The following chapters of this thesis are organised into six main chapters and lists of references and publications. The layout of each chapter is described as follows.

Chapter 2 presents detailed descriptions of the hardware and software that were used in this research. For the experimental test, a unified experimental NMAS framework consisting of three dissimilar water level process control test rigs was set up. The software we used includes MATLAB Simulink and NetConTop. In addition, the chapter also describes the calibration of the test rigs, model identification, and network delay measurement.

In Chapter 3, we solve the external consensus problem for heterogeneous NMAS with uniform constant network delay by utilising the recursive prediction algorithm. In addition, we derive the stability criterion to guarantee the whole NMAS stability. Chapter 3 also presents the novel criterion in selecting the optimal coupling gains. The results of this chapter have been published in (Mohd Subha & Liu 2013; Mohd Subha & Liu 2015).

In Chapter 4, we consider the external consensus problem for heterogeneous NMAS with asymmetric constant network delay. This condition generalised the network delay situation, where the connection link between agents is not subjected to an identical network traffic condition during transmission. The results of this chapter have been published in (Mohd Subha & Liu 2014).

Chapter 5 further expands the scope of the developed control algorithm from Chapter 4. We investigate the NMAS external consensus problem with bounded random network delay by including an additional network delay measurement signal generator into the NMAS structure. We demonstrate the performance of the developed control algorithm through simulation and experimentation on the NMAS with two agents.

Chapter 6 investigates the occurrence of large numbers of consecutive data or packet losses (CDLs) in NMAS. In this chapter, we improve consensus performance by implementing the combination of prediction strategy and the gain error ratio (GER) formula into the designed consensus protocol. The results in this chapter have been provisionally accepted for publication in the journal *IET Control Theory & Applications*.

Chapter 7 concludes this thesis and presenting the list of the contributions in the presented works, as well as brief discussions of possible future research directions.

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