MODIFIED TWO-STEP METHOD FOR STOCHASTIC DIFFERENTIAL EQUATION’S PARAMETER ESTIMATION

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To my beloved father and mother

Md. Lazim Bin Ismail and Zainun Bt. Ahmad

Respected Supervisor and co-Supervisor

Dr. Haliza Abd. Rahman and Dr. Arifah Bahar

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ABSTRACT

A previous study introduced two-step method of Stochastic Differential Equations (SDEs) for estimating the parameters of SDEs models where the selection of optimal knot is required when regression spline is used in the first step of this method. However, the choice of optimal knot is considered only for single optimal knot since it is suitable for a selected case study. Thus, modified two-step method of SDEs as an alternative to the limitation and computational difficulties in choosing optimal knot is proposed. A new non-parametric estimator which is Nadaraya-Watson (NW) kernel regression estimator is applied to replace regression splines in the first step of modified two-step method. The NW kernel regression model is later utilised in the second step to estimate the parameters of one-dimensional linear SDEs models. The outcome indicates a modification of two-step method providing better estimates for SDEs model compared to two-step method. The performance of modified two-step method is compared with the well-known established classical methods, particularly Simulated Maximum Likelihood Estimation (SMLE) and Generalised Method of Moments (GMM) by using simulated data. Results indicate GMM method is the best parameter estimation method since it outperforms other methods in terms of percentage of accuracy and computational times. Nevertheless, the differences of percentage of accuracy were not too great, and therefore, modified two-step method could be considered comparable for practical purposes. The computational time of modified two-step method is faster than SMLE method although not as good as GMM method. This however verifies that modified two-step method serves as a good alternative to the existing classical methods because it excludes the difficulty of finding transition density, moment functions and optimal knot.

ABSTRAK

Kajian sebelum ini memperkenalkan kaedah dua langkah Persamaan Pembezaan Stokastik (SDEs) untuk menganggar parameter model SDEs di mana pemilihan simpulan optimum diperlukan apabila splin regresi digunakan dalam langkah pertama. Walau bagaimanapun, pemilihan simpulan optimum hanya untuk simpulan optimum tunggal kerana ia sesuai untuk kajian kes yang dipilih. Oleh itu, kaedah dua langkah diubahsuai sebagai alternatif kepada had dan kesukaran pengiraan dalam pemilihan simpulan optimum dicadangkan. Satu penganggar tak berparameter baharu iaitu penganggar regresi kernel Nadaraya-Watson (NW) digunakan untuk menggantikan splin regresi dalam langkah pertama kaedah dua langkah diubahsuai. Kemudian model regresi kernel NW digunakan dalam langkah kedua untuk menganggar parameter model SDEs linear satu dimensi. Hasil menunjukkan kaedah dua langkah yang diubahsuai memberikan anggaran yang lebih baik untuk model SDEs dibandingkan dengan kaedah dua langkah. Prestasi kaedah dua langkah diubahsuai dibandingkan dengan kaedah klasik yang terkenal, terutamanya kepada Anggaran Kebolehjadian Maksimum Simulasi (SMLE) dan Kaedah Momen Umum (GMM) menggunakan data simulasi. Keputusan menunjukkan kaedah GMM adalah kaedah penganggaran parameter terbaik kerana ia mengatasi kaedah lain dari segi peratusan ketepatan dan masa pengiraan. Namun, perbezaan peratusan ketepatan tidak terlalu besar, oleh itu kaedah dua langkah diubahsuai boleh dianggap setanding untuk tujuan praktikal. Masa pengiraan menggunakan kaedah dua langkah diubahsuai lebih cepat daripada kaedah SMLE, walaupun tidak sebaik GMM. Ini juga mengesahkan bahawa kaedah dua langkah diubahsuai menjadi alternatif yang baik kepada kaedah klasikal sedia ada kerana ia tidak termasuk kesukaran mencari ketumpatan peralihan, fungsi momen dan simpulan optimum.

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LIST OF SYMBOLS/ ABBREVIATIONS/ NOTATIONS

$E(X)$	–	Expected value of X
f'	–	First Derivative
f''	–	Second Derivative
ε	–	Random Error
m	–	Minutes
ODEs	–	Ordinary Differential Equations
SDEs	–	Stochastic Differential Equations
RMSE	–	Root Mean Square Error
AMISE	–	Asymptotic Mean Integrated Square Error
MLCV	–	Maximum Likelihood Cross Validation
GCV		Generalised Cross Validation
SNPIC		Stochastic Non-Parametric Information Criteria
t	–	Data Points/time
$W(t)$	–	Wiener Process
$f(t, X(t); \theta)$	–	Drift Term
$g(t, X(t); \sigma)$	–	Diffusion Term
θ	–	Drift Coefficient
$\hat{\theta}$	–	Estimated Drift Coefficient
θ^*	–	True Drift Coefficient
σ	–	Diffusion Coefficient
$\hat{\sigma}$	–	Estimated Diffusion Coefficient
σ^*	–	True Diffusion Coefficient
$\hat{m}(x)$	–	Estimated Kernel Regression Estimator
h	–	Kernel Regression Estimator Coefficient
$\prod$	–	Multiplication
$\sum$	–	Summation

Z, z	–	Standard Normal Variable
$\|x\|$	–	Modulus of x
$\|\|x\|\|$	–	Norm of x
x^*	–	True solution of ODEs
$\dot{x}^*$	–	Derivative of true solution of ODEs
$\hat{x}_n$	–	Consistent estimator of true solution of ODEs
$\hat{\dot{x}}_n$	–	Consistent estimator of derivative of true solution of ODEs
$K(u)$		Kernel function
$g_n(\theta)$		Vector of orthogonality
p		Transition density
p^r		Transition density with r^{th} simulations
L		Likelihood function
L^R		Likelihood function with r^{th} simulations
w_j		Bartlet weight
$\hat{S}$		Newey-West estimator
$\hat{\alpha}$		Cronbach Alpha

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CHAPTER 1

INTRODUCTION

1.1 Introduction

For the past thirty years, Stochastic Differential Equations (SDEs) is applied in a variety of fields. One of the largest fields is mathematical finance, where SDEs are also essential for many of modern finance theory and have been broadenly applied in modelling the behavior of main variables, such as asset returns, instantaneous, asset prices and short-term interest rate (Sundaresan, 2000). The good of using SDEs is that the actual states in the model are evaluated from data and this provide the prediction to stay close to data even when the parameters in the model is inaccurate (Leander *et al*., 2014). From an applied perspective, SDEs can be loosely described as a field of mathematics. Most of the mathematical modelling is not considering the existence of noise term that can be described as an Ordinary Differential Equations (ODEs). SDEs come from the necessity to include random noise term and unpredictable factors into ODEs. There are two widely used types of stochastic calculus, It$\hat{o}$ and Stratonovich SDEs. Gardiner (2009) claims that the It$\hat{o}$ SDEs is mathematically the most sufficient, but it is not often the most natural choice physically. While, the Stratonovich SDEs is the natural choice for an interpretation. This study considers only It$\hat{o}$ one-dimensional linear SDEs model and it will be discussed in Chapter 2

.

1.2 Research Background

One of the major applications in SDEs is the parameters estimation of SDEs models. Previously, there are various studies on parameters estimation methods of SDEs called classical parameter estimation methods, involving Simulated Maximum Likelihood Estimation (SMLE), Generalised Method of Moments (GMM), Least Squares Estimation (LSE), non-parametric method, Kalman filtering and others. We need to solve likelihood function to get estimated parameters of SDEs model when using Maximum Likelihood Estimation (MLE) method. Unfortunately, the drawback for the MLE method is obviously more demanding, because the transition density term in likelihood function must be estimated. Usually, the transition density is unknown and difficult to obtain analytically (Hurn, Jeisman and Lindsay, 2006). Thus, there are various numerical approximation methods in estimating the transition density. One of them is SMLE which provides direct approximation of the likelihood function (Lacus, 2009). He mentions that this method is needed to simulate many times trajectories of the process using Monte Carlo method in order to integrate the transition density. This work may take a lot of time if this approximation method has to be applied in obtaining maximum likelihood estimates of the parameters. Furthermore, Picchini (2006) states that SMLE method is computationally intense and poorly accurate.

The disadvantage of the GMM method is the necessity of the moment functions which is infeasible in certain demand SDEs models. There are two cases in GMM, particularly just-identified case and overidentified case. The number of moment functions is same to the number of unknown parameters in just-identified case, while the number of moment functions exceeds the number of unknown parameters for the overidentified case. Imbens (2002) describes that just-identified case normally possible in getting the estimated parameter by putting the sample average of the moments exactly equivalent to zero. Nonetheless, this is not feasible for overidentified case. In order to face this problem, Hansen (1982) proposes alternative approach by setting a linear combination of the sample average of the moment functions equivalent to zero, with the dimensions of the linear combination equivalent to the number of unknown parameters. However, this approach is inefficient to estimate the optimal linear combination. Many researchers have studied the small-sample properties of GMM estimators in too distinct contexts and summarise that this GMM estimators perhaps are extremely biased and extensively dispersed in small samples (Jondeau, Bihan and Gallés, 2004)

In these circumstances, Rahman, Bahar and Rosli (2013) suggest two-step method of SDEs as an alternative to classical parameter estimation methods of SDEs. They use spline technique, primarily regression spline in their method which is considered quite distinct from previous works. Budiantara (2006) prefers non-parametric spline regression than smoothing spline for the selection of optimal knots because this spline method is simpler and can be computed with the optimisation of ordinary least squares. In the first step of two-step method of SDEs, Rahman *et al*. (2013) estimates parameters of regression spline using Bayesian approach where this technique requires selection of the optimal knot. The smallest value of Generalised Cross Validation (GCV) indicates the best number and location of knots of the fitted spline. But, the selection of optimal knot for this two-step method of SDEs is considered only for single optimal knot since it is suitable for the selected case study. For the second step, the estimated drift and diffusion parameters are obtained by using criterion introduced by Varah (1982) and new proposed criterion by (Rahman *et al*., 2013).

1.3 Problem Statement

Most of the SDEs parameter estimation methods do not have analytical solution, and numerical method provides a tool in handling this problem. As mentioned in previous section, classical methods have their own deficiencies. In SMLE method, the transition density is acquired, but difficult to estimate and unavailable for some models. While, the GMM requires moment functions which is infeasible in certain cases. As an alternative to classical parameter estimation methods of SDEs, two-step method of SDEs is suggested where the preliminary proposal of this method applied regression spline with truncated power series basis in the first step. Previous studies are using heuristic selection to choose suitable knots which is based on the visual inspection of scatter plots of the data based on the change of slope or the location of the local maxima, minima or inflection points of the data. Unfortunately, this optimal knot selection technique tends to be subjective, waste of time and can sometimes be tedious and confusing. In two-step method of SDEs, new algorithm is presented to find optimal knot by choosing the least value of Generalised Cross Validation (GCV) as the best number and location of knots of the fitted regression spline. For a chosen case study, the selection of a single optimal knot is found suitable to be applied. However, the case is not suitable for multiple knot. Other than that, the order of spline need to be determined before finding the optimal knot that causes the use of spline regression become more complicated.

Due to the difficulty and limitation in the existing classical parameter estimation methods and two-step method of SDEs with regression spline as a non-parametric estimator, a modified two-step method is introduced in this study. In the first step of modified two-step method of SDEs, we apply Nadaraya-Watson kernel regression estimator as a new non-parametric estimator to replace regression splines with truncated power series basis. Therefore, this new non-parametric estimator is expected to be very beneficial and simpler since it does not involve the computational difficulties encountered by such methods, very straight forward to use, suitable for many cases and can easily be adapted to accommodate different demand models. Thus, the modified two-step method of SDEs is considered as another option in estimating the parameters of SDEs models. The estimated parameters of one-dimensional linear SDEs models are computed using three methods such as modified two-step method, SMLE and GMM and the performances of each method will be compared in terms of percentage of accuracy and computational time

1.4 Research Objectives

This study embarks on the following objectives:

i) to choose the best kernel bandwidth selection method to be applied in Nadaraya-Watson kernel regression estimators when employing in modified two-step method.

ii) to compare the performances of two-step method and modified two-step method of SDEs in terms of percentage of accuracy and computational time using ten number of sample paths.

iii) to compare the performances of modified two-step method, SMLE and GMM in terms of percentage of accuracy and computational time using simulated data.

iv) to assess the reliability of modified two-step method in estimating each one-dimensional linear SDEs model by evaluating Cronbach's alpha.

v) to apply modified two-step method of SDEs to the Johnson & Johnson's stock price data.

1.5 Research Scope

This study focuses on one-dimensional linear It$\hat{o}$ SDEs model namely Bachelier, Black-Scholes and Ornstein-Uhlenbeck models. Besides, drift and diffusion parameters of each model will be estimated using modified two-step method of SDEs and two classical parameter estimation methods, particularly SMLE and GMM. In the first step of modified two-step method, we are choose Nadaraya-Watson (NW) kernel regression estimator as a non-parametric estimator and apply two kernel bandwidth selection methods in NW kernel regression estimator, which are Asymptotic Mean Integrated Square Error (AMISE) for optimal bandwidth and Maximum Likelihood Cross Validation (MLCV) technique. The estimated parameters of one-dimensional linear SDEs model is evaluated using the modified two-step method, SMLE and GMM. The performance of each method is compared in terms of percentage of accuracy and computational time using simulated data. Furthermore, observed data from Johnson and Johnson's stock price data are also applied to the modified two-step method.

1.6 Significance of Research

In this research, modified two-step method is presented where this method reduces the complexity and difficulty of computational aspect of such an approach because it excludes the finding of transition density in SMLE, the derivation of moment functions in GMM and the choosing of the optimal knot in two-step method of SDEs. Modified two-step of SDEs is proposed by considering new non-parametric estimator which is Nadaraya-Watson (NW) kernel regression estimator to replace regression spline in the first step of this method. Beneficially, NW kernel regression estimator avoids the selection of optimal knot in regression spline and improves the limitation of two-step method of SDEs which is only suitable for one optimal knot in selected case study. This study provides comparative analysis of the parameter estimation methods in highlighting the advantages and disadvantages of each parameter estimation method of SDEs, based on the chosen SDEs model.

1.7 Thesis Organisation

Organisation of the thesis is as follows.

Chapter 1: This chapter discusses the introduction, research background, problem statements, research objectives, research scope, significances of research and thesis organisation.

Chapter 2: This chapter highlights the literature reviews of one-dimensional linear SDEs models followed by the past literature reviews of parameter estimation methods of SDEs and application of each method.

Chapter 3: This chapter discusses the existing methodolgy which are two-step method of SDE, Simulated Maximum Likelihood Estimation (SMLE) and Generalised Method of Moments (GMM). Moreover, modified two-step method of SDEs with Nadaraya-Watson (NW) kernel regression estimator as a new nonparametric approach also be included in this chapter.

Chapter 4: This chapter discusses the properties of one-dimensional linear SDEs models used in our study. Comparison between two-step method and modified two-step method are included. Besides, the results of estimated parameters of one-dimensional linear SDEs models by using modified two-step method, SMLE and GMM are provided. Then, all the results obtained will be compared and included in this chapter.

Chapter 5: This chapter discusses the application of Johnson and Johnson's historical stock price data. This data will be applied to one-dimensional linear SDEs model and the parameters will be estimated using modified two-step method.

Chapter 6: This chapter discusses the conclusion of the study. Some suggestions for future research are enclosed.

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