

PLASTIC ANALYSIS OF STEEL FRAME WITH RIGID AND SEMI RIGID  
CONNECTION

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To my beloved family and friends

To my respected supervisor

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## ABSTRACT

Fully plastic condition is defined as one at which a sufficient number of plastic hinges are formed to transform the structure into a mechanism, then the structure is geometrically unstable. On the other hand the actual behavior of beam to column connections in steel frames is seldom fully rigid or fully pinned. But in actual, the connection is behaved most likely between these two conditions. The connection is called semi-rigid. This paper presents rigid and semi rigid steel frame under nonlinear plastic analysis. One of the important issues in the study of steel frames is to find the ultimate load due to lateral load and besides a suitable formulation for semi-rigid connections. This study is focused on plastic analysis of steel frame structure in three steps. Firstly, elastic analysis of rigid frame based on the stiffness method was developed using software called MATLAB to simulate the behaviour of steel frame under rigid connection. The most important thing in this step was to reach a fully plastic condition with increasing the applied load until yielding occurs according to yield locus. Secondly plastic analysis of steel frame under rigid connection and then semi rigid connections considerations. From the result of this study, it is found that by increasing the applied lateral load in elastic step we can find the ultimate load to reach to plastic condition. This is based on yield locus diagram. In addition this study clearly shows the comparison of displacement between rigid and semi rigid connection.

## ABSTRAK

Keadaan plastik sepenuhnya ditakrifkan sebagai salah satu keadaan di mana bilangan engsel plastik yang mencukupi dibentuk untuk mengubah struktur kepada mekanisme, dengan ini struktur geometri adalah tidak stabil. Sebaliknya kelakuan sebenar sambungan rasuk tiang dalam kerangka keluli adalah samada tegar sepenuhnya atau disemat sepenuhnya. Tetapi dalam keadaan sebenar kelakuan sambungan kemungkinan besar antara kedua-dua sambungan. Sambungan ini dipanggil separa tegar. Kertas kerja ini membentangkan kerangka keluli tegar dan separa tegar di bawah analisis plastic tak linear. Salah satu daripada isu penting dalam kajian kerangka keluli adalah untuk mencari beban muktamad kerangka keluli akibat beban sisi selain formulasi yang sesuai untuk sambungan separa tegar. Kajian ini memberi tumpuan kepada analisis plastik struktur kerangka keluli dalam tiga langkah. Pertama, analisis anjal kerangka tegar dijalankan berdasarkan kaedah kekukuhan yang dibangunkan dengan menggunakan perisian yang dipanggil MATLAB untuk menganggar tingkah laku sambungan tegar. Perkara yang paling penting dalam langkah ini adalah untuk mencapai keadaan plastik sepenuhnya dengan peningkatan beban yang dikenakan sehingga menghasilkan alhan menggunakan lokus alah. Kedua analisis plastic kerangka kelulj dengan anggapan sambungan tegar dan sambungan separuh tegar dijalankan. Dari hasil kajian ini, didapati bahawa dengan meningkatkan beban sisi dalam tahap anjal kita boleh mencari beban muktamad untuk mencapai keadaan plastik. Ini adalah berdasarkan kepada gambarajah lokus alah. Di samping itu kajian ini jelas menunjukkan perbandingan anjakan antara sambungan tegar dan separa tegar.

## TABLE OF CONTENTS

<b>CHAPTER</b>	<b>TITLE</b>	<b>PAGE</b>
	<b>DECLARATION</b>	ii
	<b>DEDICATION</b>	iii
	<b>ACKNOWLEDGMENTS</b>	iv
	<b>ABSTRACT</b>	v
	<b>ABSTRAK</b>	vi
	<b>TABLE OF CONTENTS</b>	vii
	<b>LIST OF TABLES</b>	x
	<b>LIST OF FIGURES</b>	xi
	<b>LIST OF SYMBOLS</b>	xiii
<b>1</b>	<b>INTRODUCTION</b>	<b>1</b>
	1.1 Introduction	1
	1.2 Problem Statement	2
	1.3 Objectives	3
	1.4 Research Scope	4
	1.5 Significant of the Study	4
<b>2</b>	<b>LITERATURE REVIEW</b>	<b>5</b>
	2.1 Introduction	5
	2.2 Types of Connection	6
	2.2.1 Simple Connection	6
	2.2.2 Partial Restraint Connection	7
	2.2.3 Fully Restraint Connection	7
	2.3 Plastic Analysis of Frames	8
	2.3.1 History	8

	2.3.2	Introduction	10
	2.3.3	Advantages of Plastic Analysis	10
	2.3.4	Assumptions in Plastic Bnding	11
2.4		Characteristic of Semi-Rigid Frames	11
2.5		Benefits of Semi Rigid Connection	13
	2.5.1	Economic Advantages of Semi-Rigid Connection	13
2.6		Analysis of Frames	14
2.7		Types of Frames Analysis	16
	2.7.1	Linear and Non Linear Analysis	16
	2.7.2	Theoretical Formula of Non Linear Analysis	17
	2.7.3	Linear and Non Linear Analysis of Frame Structure	19
2.8		Material Non-Linear Analysis	21
	2.8.1	Plastic Moment Capacity	21
	2.8.2	Elastic Stage	21
	2.8.3	Plastic Stage	22
	2.8.4	Plastic Hinge Formation-Failure Mechanism	23
	2.8.5	Theory of Plasticity	23
<b>3</b>		<b>METHODOLOGY</b>	<b>26</b>
	3.1	Introduction	26
	3.2	Step 1: Elastic Analysis for Rigid Connection	26
	3.3	Step 2: Plastic Analysis for Rigid Connection	32
	3.3.1	Yield Locus	32
	3.3.2	Plastic Reduction Matrix [km]	33
	3.4	Step 3: Plastic Analysis for Semi-Rigid Connection	37
<b>4</b>		<b>RESULTS AND DISCUSION</b>	<b>43</b>
	4.1	Introduction	43
	4.2	Step 1: Elastic Analysis of Rigid Frame	44
	4.2.1	H=30 KN	45
	4.2.2	H=40 KN	49
	4.2.3	H=50 KN	51

4.2.4	H=60 KN	52
4.2.5	H=70 KN	54
4.2.6	H=75 KN	55
4.2.7	H=76 KN	57
4.2.8	H=77 KN	58
4.2.9	Comparison Between Lateral Load and Displacement	60
4.2	Step 2: Plastic Analysis of Rigid Frame	61
4.3	Step 3: Plastic Analysis of Semi Rigid Frame	65
4.3.1	Coefficient $C_{\theta j}=10^8$ and $C_{\theta k}=10^8$	65
4.3.2	Coefficient $C_{\theta j}=10^7$ and $C_{\theta k}=10^7$	68
4.4	Comparison Between Displacement and Connections	72
<b>5</b>	<b>CONCLUSIONS AND RECOMMENDATIONS</b>	<b>73</b>
5.1	Conclusions	73
5.2	Recommendations	74
	<b>REFERENCES</b>	<b>75</b>



**LIST OF TABLES**

<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE</b>
4.1	Comparison the displacement by increasing the load (h)	60
4.2	Comparison the displacement between different connection and coefficient	72

## LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
2.1	Moment rotation diagrams (m- $\theta$ ) curves	6
2.2	Comparison between m- $\phi$ curves between rigid, semi rigid and simple or flexible connection	7
2.3	Load-deflection curves for a two-storey frame tested at abington	9
2.4	Load-deflection curves for a pitched roof frame tested at abington	9
2.5	Stress-strain curve	10
2.6	Connection moment-rotation curves (chen and lui, 1991)	12
2.7	Rotational deformation of a connection	16
2.8	Pictorial representation of bar with second order effect	17
2.9	Load deformation curves of a frame	20
2.10	Plastic & elastic	21
2.11	Plastic hinge formation-failure mechanism	23
2.12	Elastic-plastic material	24
2.13	Moment-curvature relationship for perfectly elastic-plastic material	24
2.14	Load-deflection curve for perfectly elastic-plastic material	25
3.1	Steps of the project	27
3.2	Arrangement of number for calculation at a trial concrete frame structure	28
3.3	Major axis	32
3.4	Yield locus	32
3.5	Wide flange section yield surface	33
3.6	Concentrated plasticity (plastic hinge) element	34

3.7	Yield surface, force increments and plastic deformation	34
3.8	Semi rigid frame model	38
3.9	Displacements of a semi-rigid frame	39
3.10	Degrees-of-freedom	41
3.11	The reduced displacements	41
4.1	Arrangement of number for calculation at a trial concrete frame structure	45
4.2	Yield locus	48
4.3	Yield locus diagram in $h = 30$ KN	48
4.4	Yield locus diagram in $h = 30$ KN	49
4.5	Yield locus diagram in $h = 40$ KN	50
4.6	Yield locus diagram in $h = 40$ KN	50
4.7	Yield locus diagram in $h = 50$ KN	51
4.8	Yield locus diagram in $h = 50$ KN	52
4.9	Yield locus diagram in $h = 60$ KN	53
4.10	Yield locus diagram in $h = 60$ KN	53
4.11	Yield locus diagram in $h = 70$ KN	54
4.12	Yield locus diagram in $h = 70$ KN	55
4.13	Yield locus diagram in $h = 75$ KN	56
4.14	Yield locus diagram in $h = 75$ KN	56
4.15	Yield locus diagram in $h = 76$ KN	57
4.16	Yield locus diagram in $h = 76$ KN	58
4.17	Yield locus diagram in $h = 77$ KN	59
4.18	Yield locus diagram in $h = 77$ KN	59
4.19	Comparison the displacement by increasing the load (h)	60
4.20	Comparison the displacement by increasing the load (h)	61
4.21	Comparison between lateral load and displacement	61

## LIST OF SYMBOLS

$A$	-	Area of the section
$I$	-	Moment of inertia
$L$	-	Length
$E$	-	Modulus of elasticity
$P$	-	Vertical load
$H$	-	Horizontal load
$\Phi$	-	Angle
$k_e$	-	Local linear elastic stiffness matrix
$K_e$	-	Global linear elastic stiffness matrix
$T$	-	Transformation matrix
$K$	-	The assembled stiffness matrix
$U$	-	Member displacement
$F$	-	Member force
$k_m$	-	Plastic reduction matrix
$P_y$	-	Squash load
$M_y$	-	Plastic moment
$\sigma_y$	-	Stress
$G$	-	Gradient
$C_{\theta j}$	-	Spring coefficient
$C_{\theta k}$	-	Spring coefficient
$M_{if}$	-	Flexural moment
$M_{kf}$	-	Flexural moment
$\phi_j$	-	Rotation
$\phi_k$	-	Rotation
$R_j$	-	Rigidity index
$R_k$	-	Rigidity index
$\phi_{jyr}$	-	Rotal rotation
$\phi_{kyr}$	-	Rotal rotation

$K_{yr}^t$	-	Stiffness matrix relating rigidity index
$K_{s1}$	-	Stiffness matrix of a semi-rigid column element
$K_{s2}$	-	Stiffness matrix of a semi-rigid beam element
$\Delta$	-	Lateral displacement
$\alpha_r$	-	Proportion coefficient
$Z_e$	-	Elastic section modulus
$T_e$	-	Force in elastic stage
$T_p$	-	Force in plastic stage
$M_e$	-	Moment in elastic stage
$M_p$	-	Moment in plastic stage
$Z_p$	-	Plastic section modulus
$\varepsilon_e$	-	Elastic strain
$\varepsilon_p$	-	Plastic strain
$d\Delta_e$	-	Elastic displacement vector
$d\Delta_p$	-	Plastic displacement vector
$d\Delta$	-	Total displacement vector
$M - \theta$	-	Moment rotation

## CHAPTER 1

### INTRODUCTION

#### 1.1 Introduction

The structural frame system mainly consist components of beams, columns and connections. Among these three components, the connection between beam to column play important role to the effect of load distribution, strength, stability and constructability of the structure. It also well known that the connections show a variation of behavior in term of strength and stiffness. Usually in conventional method of analysis, the connection behaves either as a pin transferring only nominal moment or they are function as a rigid and maintain full moment continuity.

The two common assumptions as to the behavior of a building frame are that its beams are free to rotate at their connections or that its members are so connected that the angles they make with each other do not change under load .

Generally, the frame analysis assumes that beam-to-column connections are rigid or pinned. Rigid connections, where no relative rotations occur between the connected members, transfer not only a significant amount of bending moments, but also shear and axial forces. On the other extreme, pinned connections are characterized by almost free rotation movement between the connected elements that prevent the bending moment transmission. Despite these facts, it is largely recognized that great majority of joints do not exhibit such idealized behavior. These connections are called semi-rigid, and their analysis should be performed according to their actual structural behavior.

In addition, in the simple plastic hinge method, the element stiffness matrix is modified to account for the presence of plastic hinges developed suddenly from an elastic state to a fully plastic state. After a hinge has formed in a member, the section is replaced by a real hinge with a constant moment,  $M_p$ , and the incremental equations are adjusted to reflect the change in the member stiffness. Thus, the simple plastic hinge method may over predict the real limit loads of steel frames due to neglecting the effect of partial yield in members. A modified stiffness method is proposed which can take into account the effect of partial plasticity in members. The limit loads and load-deflection responses can be predicted reasonably by the proposed method.

## 1.2 Problem Statement

Conventional or traditional analysis of frames basically based on assumption that the connections are either fully rigid or ideally pinned (simple connections). Fully rigid assumption makes it clear that no relative rotation of the connection occurs and the end moment of the beam is completely transfer to the adjacent column. On the other hand, pinned connection implies that no restraint for connection exists and the end moment at the connection is assumed zero (Chen *et al.*, 1996). However, the actual behavior of the connections used in current practice possesses some stiffness that fall between the two extreme cases of fully rigid and ideally pinned.

On the other hand, although there are numerous research reported about the method and advantages of semi-rigid connection, but there is still no orderly absorption by structural designer due to lack of confidence about its behaviour, Burns (2002). According to Ahmed (1996), the semi-rigid nature of the connection affects the frame behavior in that the distribution of internal forces and moments in the beams and columns are different from those of the standardized curves. Needless to say, frame analysis neglecting the true behavior of the connection will result in unreliable prediction of frame response.

In addition, to compared with the static analysis, the research on plastic analysis of rigid and semi-rigid jointed frames is relatively limited. Although the analysis of frames has been the subject of research for several decades, it was not until recently that investigators started including the effects of the partial rigidity of connections in their analyses. This recent trend in analysis acknowledges the fact that most connections used in steel constructions are neither fully rigid nor completely flexible.

Besides a linear analysis is more prefer as compare to non linear analysis in structural design. This is because it can be simplify by design, time and cost saving, and at the same time it does not require the use of computer software for non linear case. Furthermore, a non linear analysis is more complicated than linear analysis in structural problem solving.

### **1.3 Objectives**

The main objective of this study can be described as follow:

1. Reaching a fully plastic condition by increasing the applied load until yielding according to yield locus.
2. To find the first hinge according to ultimate load.
3. To study the plastic behavior of rigid connection.
4. The foremost objective of this research is to plastic analysis of semi-rigid frames.
5. To investigate the stiffness matrix of semi-rigid connection in 3 steps.



## **1.4 Research Scope**

The behavior of frame structures is highly influenced by the connections. The analysis of such frames, whether elastic or plastic, can only be performed with accuracy if the correct joint behavior is incorporated in the analysis. A computer program (MATLAB) was developed to analyze frames by taking into account the effect of internal and external rotations of the connection based on the stiffness method analysis.

This study was limited to the use of linear analysis and non-linear analysis by considering different stiffnesses in each stage of analysis.

## **1.5 Significant of the Study**

Typically, the behavior of semi-rigid connections relates to the performance on sub-assembly frame of beam-to-column connection. In semi-continuous construction design, semi-rigid connection developed an end restraint leading to reduction on beam moment which resulted to lighter beam in many cases. The amount of restraint developed from the semi-rigid connections depends on the stiffness of the connection. The term stiffness in each connection nodes can be either modeled as pinned, rigid or semi-rigid case. This leads to the simplicity and effectiveness of the structural analysis.

The use of semi-rigid connections in building construction has reduced material usage leading to more effective and quicker construction. Studies conducted on semi-rigid connections have proven the savings in material usage while achieving required strength.

On the other hand, in the simple plastic hinge method, the element stiffness matrix is modified to account for the presence of plastic hinges developed suddenly from an elastic state to a fully plastic state.

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