LAMINAR FLOW IN A UNIFORMLY POROUS PIPE

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To my beloved mother, father, and all of my friends

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ABSTRACT

This study focused on fully laminar flow of an incompressible viscous fluid in a uniformly porous pipe with suction and injection. An exact solution of the Navier-Stokes equations is given. The velocity field can be expressed in a series form in terms of modified Bessel function of the first kind of order *n*. The volume flux across a plane normal to the flow, the vorticity and the stress on the boundary are presented. The flow properties depend on the Reynolds number, $U a / \nu$, where U is the suction velocity, a is the radius of the pipe and ν is the kinematic viscosity. It is found that for large values of the cross-Reynolds number, the flow near the region of the suction shows a boundary layer character. In this region the velocity and vorticity vary sharply. Outside the boundary layer, the velocity and the vorticity do not show an appreciable change.

ABSTRAK

Kajian ini fokus pada aliran yang sepenuhnya lamina bendalir likat yang tidak boleh dimampat dalam paip seragam berliang dengan sedutan dan suntikan. Penyelesaian tepat bagi persamaan Navier-Stokes diberikan. Medan halaju boleh diungkapkan dalam bentuk siri dalam sebutan fungsi Bessel terubahsuai jenis yang pertama peringkat *n*. Isipadu fluks merentasi satah normal kepada aliran, vortisiti dan tekanan di sempadan juga ditunjukan. Sifat aliran bergantung kepada nombor Reynolds, $U a/\nu$, dengan U adalah halaju sedutan, a adalah jejari paip dan ν adalah kelikatan kinematik. Didapati bahawa aliran berhampiran kawasan sedutan menunujukan satu lapisan sempadan untuk nombor silang Reynolds yang besar. Halaju dan vortisiti berubah dengan ketara di kawasan ini. Di luar lapisan sempadan, halaju dan vortisiti tidak menunjukkan sebarang perubahan.

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LIST OF ABBREVIATION AND SYMBOLS

I_n	-	Modified Bessel function of the first kind of order *n*
U	-	Suction velocity
a	-	Radius of the pipe
r, θ, z	-	Cylindrical polar coordinate
λ	-	Suction parameter or cross-Reynolds numbers
θ	-	Angle of the pipe
ν	-	Kinematic viscosity
$\omega_r, \omega_\theta, \omega_z$	-	Vorticity component
μ	-	Fluid viscosity

CHAPTER 1

INTRODUCTION

1.1 Introduction

This study is concerned with the fully laminar flow of an incompressible viscous fluid with suction and injection in a uniformly porous pipe. This chapter will discuss about research background, statement of the problem, objectives of the research, scope of the research and significance of the research.

1.2 Research background

The study of fluid mechanics is very important to the engineers especially for those who are involved in the application of fluid mechanics to solve industrial problems. Recently, the engineers and mathematicians seem to show an interest in studying the flow of an incompressible viscous fluid problem through porous channels and pipe. The problems have been applied to the transpiration cooling, gaseous diffusion, drinking water treatment and biomedical engineering. For example, a transpiration cooling problem which is a very effective process in reducing the heat

transfers between the fluid and the boundary layer. Thus, this makes us understand more about the problems of cooling rocket and jet.

The fluid flow can be categorized as external and internal which depend on whether the fluid is forced to flow neither over a plate nor in a conduit. The properties of internal and external flows are not the same. The fluid mechanics concerns the fluids flow which is can be classified as steady or unsteady. The fluid properties such as the velocity, pressure, cross-section and density at any point in the fluid is independent with time so that the flow pattern remains unchanged with the time known as steady flow. Meanwhile, unsteady flow refers to the conditions change with time at any point in fluid where the flow pattern varies with time. In this research, the steady internal flow in a porous pipe will be considered.

The viscous flows are important and will help us to understand about the complex flow such as laminar flow and turbulent flow. Laminar flow has smooth streamlines and highly order motion at small or moderate Reynolds number, whereas the chaotic flow when the velocity is increased and highly disorder motion at large Reynolds numbers is known as turbulent flow. Reynolds number was discovered by Osborne Reynolds in 1880 which is defined as the ratio of inertial forces to viscous forces of fluid (Kundu, 1990). For laminar flow, the viscous forces are prominent and large enough to prevent the rapid fluctuation and to keep the fluid to flow in parallel because of small Reynolds numbers. In contrast, the inertial forces are dominant over the viscous forces for large Reynolds numbers and thus the viscous forces cannot prevent the irregular and rapid velocity fluctuation of the fluid. Apart from that, numerous studied have been made of laminar flow in uniformly porous pipe only with suction or injection and as well as by considering both of them. The porous pipe is the pipe that has holes where the fluid can pass freely and continuously. The suction region is the region where fluid flows out from the pipe meanwhile for the injection region, the fluid enter the pipe.

Moreover, the Navier-Stokes (N-S) equations is non-linear partial differential equations and are derived from the conservation of momentum which is it can be applied to solve the viscous flows problems. N-S equations are differential equations that

describe the motion of fluid in viscous flow and can be used to model flow of water in pipe, air flow around the wing and weather.

In this research, the Navier-Stokes equations were reduced to the governing equation for velocity. Then, the solutions for the the volume flux, the vortocity and the stress on the boundary in the form of series modified Bessel function of the first kind of order *n* were obtained using the method of separation of variable. Finally, the values of the flow properties were obtained by using Mathcad 15 and the analysis was done via graph.

1.3 Statement of the problem

This study investigates the suction and injection on laminar flow of an incompressible viscous fluid in a uniformly porous pipe. The study will explore the following questions. How to reduce the partial differential equations into ordinary differential equation? How to analyses the fluid properties such as behavior of the velocity, the volume flux, the vorticity and the stress on the boundary?

1.4 Objectives of the research

This study focused on laminar flow of an incompressible viscous fluid in a uniformly porous pipe with suction and injection. The flow properties are investigated. For that purpose the objectives of the research are:

To study and understand the concept of internal laminar flow through the porous pipe with suction and injection.

To find the velocity field from Navier-Stokes equation in terms of modified Bessel function of the first kind of order *n* using method of separation of variable.

To analyses the effect of Reynolds number on the behavior of flow properties by studying the velocity, volume flux, the vorticity distribution and the stress on the boundary.

1.5 Scope of the research

There are many methods to solve the governing equation for velocity. The method of separation of variable is used to get an exact solution for velocity which is useful to obtain the vorticity, volume flux and stress on the boundary. The problem of laminar flow in a uniformly porous pipe with suction and injection is investigated based on work completed by Erdoğan and İmrak (2008).

1.6 Significances of the research

This research gives us better understanding about the laminar flow in a uniformly porous pipe with suction and injection. Since this research focused on circular porous pipe, it provides reasonable prediction for flows over more complicated shaped such as triangular cross section. Moreover, this information is useful in designing pipe system.

1.7 Organization of dissertation

This report consists of five chapters counting this introductory chapter. Chapter 1 discussed the research background, statement of the problem, objectives of the research, scope and significances of the research. There are several works done by some past

researcher that will be reviewed in Chapter 2. Their work will support this study with strong evidence from the result published.

The next chapter is the research methodology. In this chapter, the continuity equation and momentum equation in the form of cylindrical polar coordinate were introduced. These equations are used to obtain the governing equation for the velocity. The method of separation of variable is used to reduce the partial differential equation to ordinary differential equation.

In the Chapter 4, the vorticity, the volume flux and the stress on the boundary is presented. These flow properties are obtained by making use of velocity equation that has been introduced in the previous chapter. The flow properties are analyses based on the graph plotted by using Mathcad 15. The effect of Reynolds number on the flow properties in the suction region and injection region will be discussed in this chapter. Finally, the summary of this research will be included in Chapter5. There are some suggestions for future work in this chapter.

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