ELASTIC AND PLASTIC BENDING OF THE BEAMS BY FINITE DIFFERENCE METHOD (FDM)

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ABSTRACT

The Euler-Bernoulli beam model has a wide range of applications to the real life; such as nano electro mechanical system switches in small scale up to the Eifel tower in large scale. Advantages of FDM like simpler mathematical concept and easier programming have made scientist to choose this numerical method to solve many state-of-the-art physical problems of partial differential equations (PDE). There are researches done by using this method in solving many problems; while, how the nodes at the boundaries can be treated in the best way is still unclear. Therefore, this study is subjected to obtaining the beam behavior with the material in two ranges of elastic and ideal plasticity. Firstly, different schemes of FDM are applied to the PDE of the beam in the elastic range for six cases. Afterwards, loading increases that the material goes to the ideal plastic range. In both ranges, validity of the results by comparing with the analytical solutions need to be studied. Finally, the best FD scheme to implement the boundary conditions are determined. Effect of the point load on FDM is investigated. Moreover, optimum value of the vital parameters like number of nodes, layers and load increments are extracted. Advantages of FDM like simpler mathematical concept and easier programming have made scientist to choose this numerical method to solve many state-of-the-art physical problems of PDE. There are researches done by using this method in solving many problems; while, how the nodes at the boundaries can be treated in the best way is still unclear.

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ABSTRAK

Kaedah pembezaan terhingga merupakan salah satu pilihan klasik daripada kaedah berangka kerana ia adalah kaedah pertama yang telah dibangunkan untuk penyelesaian hampiran. Terdapat banyak penyelidikan yang telah dijalankan dalam menyelesaikan pelbagai masalah dengan menggunakan kaedah ini, akan tetapi adalah masih tidak jelas mengenai nod yang harus digunakan di sempadan. Oleh itu, projek ini bertujuan membentangkan pemodelan rasuk dengan menggunakn kaedah pembezaan terhingga dengan keadaan sempadan yang sesuai bagi dua jenis rasuk: rasuk 'Euler-Bernoulli' dan rasuk 'Timoshenko' yang digunakan pada situasi dalam keadaan anjal/elastik. Bagi keadaan elastik dan plastik, terdapat dua situasi yang dikaji iaitu rasuk sokongan momen seragam dan rasuk julujur dengan beban teragih dan kedua-duanya hanya diapplikasikan pada rasuk 'Euler-Bernoulli'. Hasilnya menunjukkan bahawa aspek beban dan nod palsu yang terdapat semasa analisis adalah penting bagi pemilihan skim yang tepat. Selain itu, ia juga menunjukkan bahawa ketepatan meningkat dengan bilangn nod yang digunakan pada rasuk. Hasil kajian menunjukkan bagi pemodelan rasuk dalam julat anjal, adalah penting dalam membuat pilihan yang tepat pada keadaan sempadan bagi mendapatkan keputusan yang menumpu disebabkan bilangan nod akan mempengaruhi ketepatan keputusan. Dalam usaha untuk menggabungkan kelakuan plastik ke dalam skim pembezaan terhingga, pendekatan bertingkat diperkenalkan dan ia menunjukkan keupayaan untuk mengasingkan zon anjal dan plastik. Selain itu, beban peningkatan atau momen memainkan peranan yang penting dalam analisis julat elastik-plastik.

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LIST OF SYMBOLS

Symbol		Definition
n	-	Number of nodes
m	-	Number of layers
p	-	Number of load increments
Lr	-	Load ratio
ε	-	Acceptable amount for convergence
i	-	Integer for number of nodes
j	-	Integer for number of layers
k	-	Integer for increment of applied load
t	-	Integer for iteration of the solution
M	-	Applied load
M_{lim}^{el}	-	Maximum limit of elastic moment
M_{lim}^{pl}	-	Maximum limit of plastic moment
$h_p\|_i$	-	Height of plastic layer of node I (from the bottom)
Δy	-	Thickness of each layer
Δx	-	Distance between two nodes
h_j	-	Height of layer j (from the bottom)
$cnvg$	-	Convergence criterion
$s(j,i)$	-	Shows whether each layer (j) of a node (i) is in either elastic ($E_e = E$) or plastic ($E_p = 0$) range.
$ss(j,i)$	-	Equivalent to EI for each layer of a node.
$S(i)$	-	Equivalent to summation of EI of all layers of a node (normalized EI of node i)
$[SS]$	-	Normalized stiffness matrix, or the matrix equivalent to EImatrix
$[Un]$	-	Normalized beam deflection matrix for all iterations

CHAPTER 1

INTRODUCTION

1.1 Background of the study

Most of the physical phenomena running into different areas of science, such as electro-magnetic, fluid and solid mechanics, can be modeled mathematically by partial differential equations (PDE) [1]. The grid generation strategies employed in numerically solving nonlinear partial differential equations representative of complex physical phenomena involving complex geometry are discussed. A historical perspective of evolving distinct strategies and methodologies for static and dynamic grid generation in view of increased demand for computational field simulations (CFS) is presented [1].

Obtaining an appropriate mathematical model for beam problems especially for the elastic-plastic case is an important issue. Many researches have been done to solve the partial differential equations for beams which require efficient numerical methods. Numerical solution methods have become one of the most successful methods of solving a PDE particularly for complicated equations or geometries. If the discretization step contains the analysis starting from the mathematical equation which usually is in terms of partial differential equations, this will result in procedures of numerical methods for partial differential equations [2]. The discretization step is solved and this result is representing the original physical problem based on the mathematical equations at the first stage of analysis.

There are some classical choices of numerical approximation methods such as the finite difference method (FDM), the finite element method (FEM), the finite volume method (FVM) and the boundary element method (BEM) [2]. In this study, the focus is on modeling using the finite difference method (FDM), which is one of the best choices in the analysis of many practical problems. As more new algorithms have been produced and the availability of faster computers, all these methods were evolved in a lot of areas such as stress analysis, heat transfer and electromagnetic theory, potential theory, fracture mechanics, fluid mechanics, elasticity, elastostatics and elastodynamics, biological and biomedical cases.

Euler introduced finite difference schemes for the first time (1707-1783) and his equation on beams was applied to the Eifel Tower as a large scale engineering problem for the first time. He had used the method to find the approximation of differential equations and only after 1945 many research activities regarding FDM is been applied and explored. It works by defining the next value in a sequence of numbers in terms of preceding ones. This approach has been utilized in many different fields such as applied mechanics, fluid flow, wave equation etc. FDM is based on Taylor's series expansion and there are three major types of approximation in finite difference methods, namely forward, backward and central finite difference approximations [4] where these approximations apply the function values at a set mesh point and approximate the value of derivative at the left most, right most and the central mesh points, respectively. In addition, Taylor says that the more information of the function at the node, the more order of accuracy in results.

Beams are divided in two major groups, Euler-Bernoulli and Timoshenko beams. Their difference in general is that the Euler-Bernoulli beam theory does not take the transverse shear and through-the-depth normal strains into account [5] which normally would involve slender beams where the length is more than 10 times of the width. This beam works on the assumption that the straight lines normal to the mid-plane before and after deformation is remained [6]. However, the Timoshenko beam theory includes the shear deformation effect and the shear correction factor is been introduced to factor the shear force [6-8]. The Timoshenko beam is a second order theory which means it can predict two types of vibration which are the flexible shear motion and the thickness shear motion [9].

In this project, the beam investigations are divided into two cases which are the elastic range and plastic range. The elastic deformation is defined as when a stress versus strain is plotted, the linear function will show the elastic region whereby the material can still assume to its original shape [10]. Plastic deformation defines as a shape of solid body change permanently as a result from the load or stress applied beyond the elastic limit [11].

Beams with a simple structure with a simple geometry and boundary conditions have been investigated through this project. The analytical solution has been used for a comparison with finite difference schemes. At first, the analysis is focused on the elastic behavior and extended to plastic behavior which is the more complicated objective of the study.

1.2 Objectives of the study

The objectives of this research are to identify the best FDM approach for bending problems of elastic and elastic-plastic that result in the desired accuracy. Afterwards, the obtained results from FDM need to be compared with the analytical solutions. Then, the optimum number of each parameter such as number of nodes, layers, load increments need to be investigated. On the basis of achievements, there would be recommendations for the best way to implement boundary conditions.

1.3 Scopes of the study

Scopes of this project are:

1. Elastic Bending Problems
   - A. Analytical solutions
   - B. FD Solution
   - C. Evaluation of the results
2. Elastic-Plastic bending problems
   - A. Analytical solutions
   - B. FD Solution
   - C. Evaluation of the results
3. Presentation and Documentation

1.4 Structure of the research

Table 1.1 Gantt chart of the project

Task		Semester 2 Session 2011/2012					Semester 1 Session 2012/2013					
		1	2	3	4	5	1	2	3	4	5	6
Elastic	Literature review											
	Analytical solution											
	FDM Solution											
	Evaluation results											
Elastic-plastic	Literature review											
	Analytical solution											
	FDM Solution											
	Evaluation results											
Presentation, Documentation and Publications												

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