

Design of Low-Density Parity-Check Codes in Relay Channels

by

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Abstract

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Recent breakthroughs in forward error correction, in the form of low-density parity-check (LDPC) and turbo codes, have seen near Shannon limit performances especially for point-to-point channels. The construction of capacity-achieving codes in relay channels, for LDPC codes in particular, is currently the subject of intense interest in the research and development community. This thesis adds to this field, developing methods and supporting theory in designing capacity-achieving LDPC codes for decode-and-forward (DF) schemes in relay channels.

In the first part of the thesis, new theoretical results toward optimizing the achievable rate of DF scheme in half-duplex relay channels under simplified and pragmatic conditions (equal power or equal time allocation) are developed. We derive the closed-form solutions for the optimum parameters (time or power) that maximize the achievable rates of the DF scheme in the half-duplex relay channel. We also derive the closed-form expression for the DF achievable rates under these simplified and pragmatic conditions.

The second part of the thesis is dedicated to study the problem of designing several classes of capacity-achieving LDPC codes in relay channels. First, a new ensemble of LDPC codes, termed multi-edge-type bilayer-expurgated LDPC (MET-BE-LDPC) codes, is introduced to closely approach the theoretical limit of the DF scheme in the relay channel. We propose two design strategies for optimizing MET-BE-LDPC codes; the *bilayer approach* and the *bilayer approach with intermediate rates*. Second, we address the issue of constructing capacity-achieving distributed LDPC codes in the multiple-access and two-way relay channels, with broadcast transmissions and time-division multiple accesses. We propose a new

methodology to asymptotically optimize the code's degree distribution when different segments within the distributed codeword have been transmitted through separate channels and experienced distinct signal-to-noise ratio in the relay system. Third, we investigate the use of LDPC codes under the soft-decode-and forward (SDF) scheme in the half-duplex relay channel. We introduce the concept of a *K-layer doping matrix* that enables one to design the rate-compatible (RC) LDPC code with a lower triangular parity-check matrix, subsequently allowing the additional parity bits to be linearly and systematically encoded at the relay. We then present the soft-decoding and soft-re-encoding algorithms for the designed RC-LDPC code so that the relay can forward soft messages to the destination when the relay fails to decode the source's messages. Special attention is given to the detection problem of the SDF scheme. We propose a novel method, which we refer to as *soft fading*, to compute the log-likelihood ratio of the received signal at the destination for the SDF scheme.

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Acronyms

AF	Amplify and Forward
APP	<i>A Posteriori</i> Probabilities
AWGN	Additive White Gaussian Noise
BP	Belief Propagation
BER	Bit Error Rate
BMS	Binary Memoryless Symmetry
BSC	Binary Symmetry Channel
BPSK	Binary Phase Shift Keying
CSI	Channel State Information
CF	Compress and Forward
CN	Check Node
DDP	Degree Distribution Polynomials
DF	Decode and Forward
EDV	Edge Degree Vector
EF	Estimate and Forward
FT	Fourier Transform
FFT	Fast Fourier Transform
LDPC	Low-Density Parity-Check Codes
LLR	Log Likelihood Ratio
MAP	Maximum <i>A Posteriori</i>
MARC	Multiple Access Relay Channel

MIMO	Multiple Input Multiple Output
MET	Multi-Edge Type
ML	Maximum Likelihood
pdf	probability distribution function
pmf	probability mass function
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
r.v.	random variable
RDV	Received Degree Vector
SP	Sum-Product
SDF	Soft Decode and Forward
SNR	Signal-to-Noise Ratio
TWRC	Two Way Relay Channel
VN	Variable Node

Introduction

1.1 Motivation

The fields of channel coding, source coding and information theory were founded by Claude Shannon in his most celebrated 1948 paper, “A Mathematical Theory of Communication” [1]. In this paper, Shannon presented his famous channel coding theorem that governs how rapidly information can be reliably transmitted through a noisy channel. Shannon’s channel coding theorem proved the existence of channel codes, which ensure reliable communication between a transmitter and a receiver even in the presence of noise, provided that the information rate for a given code did not exceed the so-called capacity of the channel. Unfortunately Shannon did not explicitly show how such a code can be constructed to approach the capacity limits. As a result, in the first 45 years after Shannon published his channel coding theorem, a large number of ingenious and effective coding systems has been devised, e.g. Hamming code [2], BCH code [3] [4], convolutional code [5], Reed Solomon code [6], and concatenated code [7]. The approach to construct these earliest codes has been overwhelmingly algebraic in nature. Codewords are formed from strings of symbols chosen from a finite field with the best possible distance distribution

and the maximum likelihood decoding is performed, where the valid codeword which is the closest to the corrupted received vector from the channel is chosen. The downside of these early coding systems is that none of these codes had been demonstrated, in a practical settings, to closely approach Shannon's theoretical limit.

The breakthrough came in 1993 with the discovery of turbo codes by Berrou, Glavieux and Thitimajshima. This discovery heralded a fundamental departure from algebraic approaches to code design [8] [9]. Through the use of parallel concatenation of simple constituent codes and a pseudo random block interleaver, turbo codes are able to operate near the capacity limits promised by Shannon with practical and manageable complexity of iterative decoder. The second codes with near Shannon limit performances came in 1996 with the rediscovery of low-density parity-check (LDPC) codes [10]. This rediscovery is established when the coding research community recognized the importance of iterative algorithms operating on codes defined on graphs [11].

For high-performance applications, LDPC codes are seen as serious contenders to turbo codes due to several reasons. First, LDPC codes are capable of outperforming turbo codes for block lengths greater than around 10^5 , and the error floors of LDPC codes at bit error rates below about 10^{-5} are typically much less pronounced than those of turbo codes. For example, the best known error-correction performance on the additive white Gaussian noise (AWGN) channel has been achieved with an LDPC code, albeit one with an impractically long block length and high implementation complexity [12]. Second, the absence of an explicit interleaver, such as those required by their turbo counterparts, leads to highly parallel (and therefore low latency) decoder implementations in application-specific integrated circuits [13]. Third, LDPC codes are also capable of exceptional performance on channels where data is not just corrupted but may be lost entirely, so-called erasure channels. This opens the way to new application domains such as reliable Internet multicasting where whole packets of lost data are reconstructed without the network overhead of retransmission [14]. While there are some advantages of adopting LDPC codes mainly due to their low complexity iterative decoders, turbo codes have also been widely applied in the communication systems thanks to their low encoding complexity, which is one of the shortcomings of LDPC codes.

For point-to-point channels, the design of LDPC codes is now quite mature as the codes have been rediscovered over a decade ago. The capacity-achieving LDPC codes can be designed using an elegant asymptotic analysis technique (where the analysis assumed that the block length of the code goes to infinity), so-called density evolution. This technique helps in guiding code designers to tune the degree distribution of LDPC codes into well-performing (finite-length) irregular LDPC codes. A remarkable progress has also been made in the point-to-point wireless communication technology, not only in the construction of capacity-achieving codes, but also in other forms of technology such as the orthogonal frequency-division multiplexing (OFDM) [15] [16] and code-division multiple access (CDMA) [17] [18]. Communications have even moved beyond the dimensions of time and frequency, e.g., the spatial dimension of multiple-input multiple-output (MIMO) system [19] [20], to deliver higher data rates to the user. Even though there are advancements made in point-to-point communications, technology is still falling behind in the race with the growing demands. To cope with these increased demands, one of the future generation wireless communications that has attracted a significant amount of attention in the research and development community is the relay systems. Currently, almost all modern communication standards provision relaying in one way or another to improve the system's performance, e.g., the 3GPP LTE-Advanced standard [21], and IEEE 802.16j (WiMax) [22]. Moreover, adopting relays also appear to be a promising technique to achieve higher throughput at the cell edge, where path loss and interference significantly degrade the performance [23] [24] [25].

In this thesis, the main idea behind the studied LDPC codes applied for relay channels is that the relay forwards additional parity-check bits to the destination. This idea is known as a parity-forwarding scheme [26], which can be thought of as a generalization of Cover and El Gamal's well-known decode-and-forward strategy for the classic three-terminal relay channel. The main advantages of designing LDPC codes under the parity-forwarding scheme is that not only it achieves the theoretical limit of decode-and-forward strategy for the classic three-terminal relay channel but also the scheme is very flexible and can be easily applied to networks with complex topologies. These two advantages make the parity-forwarding scheme suitable for future application, e.g. the future wireless sensor networks and cellular networks.

1.2 Problem statement

One of the most interesting and challenging research problem is the design of capacity-achieving LDPC codes that are able to operate near the theoretical limits of the coding strategy in relay systems. Traditionally for point-to-point communications, LDPC codes have been pseudo-randomly defined according to their degree distribution, which has proven to be very effective in producing well-performing LDPC codes, particularly for very long codes. However, a set of new constraints arises to devise LDPC codes with sufficient flexibility to cope with the myriad of applications opening up in the relay systems. The first constraint of devising new LDPC codes in relay channels is that the code structures are no longer pseudo random because of the distributed nature of the coding scheme at the source and the relay nodes. Due to this distributed nature, the LDPC codes have to satisfy a predefined structure, which needs to be taken into account when designing the codes. The second constraint is that these new LDPC codes, in general, have to operate at two different rates; the first operating rate is to ensure that the relay can successfully decode the source's LDPC codeword, while the second operating rate is to ensure a successful decoding of the source's LDPC codeword at the destination with the help from the relay. These two constraints have made it a challenge to optimize the degree distribution of LDPC codes into well-performing codes for the application in relay channels. In addition, in many cases, the standard methodology to design LDPC codes for point-to-point communications cannot be applied for relay channels.

A serious shortcoming of LDPC codes is their potentially high encoding complexity, which is in general quadratic in code length. Due to the distributed nature of the coding scheme at the source and the relay nodes, the task of encoding LDPC codes is even more challenging in relay channels. This is because the LDPC codes adopted for relay systems, e.g. the rate-compatible LDPC (RC-LDPC) codes, form a nested sequence of code bits where the parity bits of higher rate codes for the source-to-relay channel are embedded in those of lower rate codes for the source-to-destination. As a result, the encoder at the relay must be systematic. Unfortunately, this systematic encoder cannot be obtained by means of a standard Gaussian elimination method, as there is a predefined structure that must

be satisfied by the RC-LDPC codes for the application in relay channels. This predefined structure disallows the column permutation operation of the Gaussian elimination.

The detection of the transmitted data has always been a complex problem in relay systems. This is due to the fact that in many cases the relay performs non-linear processing on the received signal before re-transmitting it to the destination node. For the integration of coding in relay transmissions, the general approach requires the relay to firstly decode the source's transmitted signals before any processing can be performed. However, this perfect decoding condition cannot always be guaranteed in practice. Additionally, in designing relay coding systems, the detection of the transmitted signals becomes even a tougher problem to tackle given the issue of erroneous decoding at the relay.

Another significant challenge on relaying is the resource allocation problem between the nodes, which maximizes the throughput of the systems. In theory, the maximum throughput can be achieved by tuning all systems parameters, such as the power constraint, the time allocation between nodes, and the source and relay signal correlations. In practice, tuning all these system parameters to achieve the best possible throughput promised by the theory may require a complex and expensive relay system. From a practical perspective, the most attractive schemes when actually deploying real communication networks are those that achieve the optimum (or near optimum) performance with minimal processing complexity at all terminals. There are other considerations for minimizing complexity in the design of the practical system like the location of relays for the purpose of maintenance and most importantly the cost in constructing such relay networks.

With this background in mind the main problem considered in this thesis is the design of low-density parity-check codes for use with sum-product decoding in relay channels. The central idea of this thesis is to apply the framework of multi-edge type (MET) LDPC ensemble to the design of new LDPC codes in relay channels. The potential benefit of considering the code design under the MET framework is that the exact asymptotic analysis predicting the performance of new LDPC codes in the relay channels can be carried out. In addition, the distributed nature of the coding scheme at the source and the relay nodes can also be easily captured under the MET framework. Apart from the main problem of designing LDPC codes, this thesis will also be devoted to tackle

the detection problem when erroneous decoding occurs at the relay, and the resource allocation problem under realistic conditions in relay communications systems.

In the first part of this thesis, the focus is on the resource allocation problem of the sub-optimal relay system under simplified and pragmatic conditions. An achievable rate analysis has been performed, where the closed-form solution of the optimum system parameter that maximizes the achievable rate of relay channels is derived.

The second part of the thesis is dedicated to the design of three LDPC codes in relay channels. Each LDPC code has its own unique code structure that needs to be incorporated in the code design. First, we design the bilayer-expurgated LDPC codes [27]. This code is unique in the sense that it disallows the check node degrees to be concentrated, as a consequence, making it difficult to optimize its degree distribution. We propose two novel design strategies; the *bilayer approach* and the *bilayer with intermediate approach*, to optimize this code so that it achieves the rate limits of decode-and-forward (DF) schemes for a wide range of channel settings. Second, we design the distributed LDPC codes [28] for use over the multiple-access and two-way relay channels. The main challenge of designing this code is its non-standard code design settings, where different segments within the distributed codeword have been transmitted through different channels and experienced distinct SNRs. We show that this non-standard code design setting can be easily formulated under the framework of MET-LDPC codes, subsequently the asymptotic analysis that predict the performance of distributed LDPC codes can be performed. This asymptotic analysis enables us to optimize the degree distribution of distributed LDPC codes into a well-performing codes in the multiple-access and the two-way relay channels. Third, we design the RC-LDPC codes [29] for the use under an advanced soft decode-and-forward (SDF) scheme in half-duplex relay channels. While the SDF scheme is initially proposed to mitigate the problem of error propagation due to the erroneous decoding at the relay [30] [31], the best method in the literature to detect the received signal at the destination node relies on the assumption that the soft-errors at the relay is a Gaussian noise [31], which is not valid in general. Here, we make two main contributions. The first contribution is that we propose a methodology to design linear and systematic LDPC codes in order to enable a systematic encoder for the additional parity bits at the relay. This allows us to derive the soft-decoding and soft-re-encoding algorithms for

the designed RC-LDPC code, which facilitates the relay to forward soft messages to the destination when the relay fails to decode the source's message. The second contribution is that we propose a novel method, known as the *soft fading*, that deals with the detection problem under the SDF scheme.

1.3 Outline of the thesis

In general terms, the main scope of this thesis is designing low-density parity-check codes for the decode-and-forward scheme in relay channels. In particular, we consider three different structures of LDPC codes. These codes are the bilayer-expurgated LDPC codes, the distributed LDPC codes, and the rate-compatible LDPC codes. In addition of designing LDPC codes for the DF scheme in relay channels, this thesis also investigates the resource allocation problem in determining the achievable rate of the DF scheme in half-duplex relay channels. Furthermore, the detection problem of the SDF scheme is also studied.

The outline of each of the chapters is as follows:

Chapter 1 presents the problem statement and the motivation of this thesis. It also presents the outline and lists the contributions of this thesis.

Chapter 2 - This chapter provides the background material of the relay channels and the LDPC codes that will be used extensively throughout this thesis. An introduction to LDPC codes is presented, including an overview of a special class of LDPC codes known as the multi-edge-type LDPC codes. The aim of this chapter is to create a foundation for the reader to have a good understanding about the relay channels and the LDPC codes.

Chapter 3 - This chapter deals with the pragmatic issue of sub-optimal relay systems, where we can only optimize either on the time allocation, or on the power allocation. In particular, an achievable rate analysis for this sub-optimal relay system is performed. We derive the closed-form solutions for the optimum parameters (time or power) that maximize the achievable rates of DF strategy in the half-duplex relay system. We also derive the closed-form expression for the DF achievable rates under these simplified and

pragmatic conditions.

Chapter 4 - In this chapter we focus on the design of bilayer-expurgated low-density parity-check (BE-LDPC) codes as part of a DF strategy for use over the relay channel. A new ensemble of codes, termed multi-edge-type bilayer-expurgated LDPC (MET-BE-LDPC) codes, is introduced where the BE-LDPC code design problem is transformed into the problem of optimizing the multinomials of a multi-edge-type LDPC code. We propose two novel design strategies for optimizing MET-BE-LDPC codes; the *bilayer approach* is preferred when the difference in SNR between the source-to-relay and the source-to-destination channels is small, while the *bilayer approach with intermediate rates* is preferred when this difference is large.

Chapter 5 - Here we study the problem of finding the optimum degree distribution for the distributed LDPC codes in the time-division multiple-access relay channels (TD-MARC) and time-division two-way relay channels (TD-TWRC). We introduce a new ensemble of codes, called distributed multi-edge-type LDPC (DMET-LDPC) codes and a corresponding design methodology to asymptotically optimize the code's ensemble when different segments within the distributed codeword have been transmitted through separate channels and experienced distinct SNRs. This chapter presents a complete framework for the code design of the proposed DMET-LDPC codes; starting with the code design formulation; the detail explanation related to the code structure; the code design strategies and optimization procedure; and finally the numerical results exemplifying the gain of adopting our new code design.

Chapter 6 - In this chapter we investigate the use of rate-compatible low-density parity-check (RC-LDPC) codes as part of a soft decode-and-forward (SDF) strategy over the half-duplex relay channel. We introduce the concept of a *K-layer doping matrix* that enables one to design the rate-compatible LDPC code with a lower triangular parity-check matrix, subsequently allowing the additional parity bits to be linearly and systematically encoded at the relay. As a result of our concept, the asymptotic performance of RC-LDPC codes can be analyzed and predicted using the multi-edge-type density evolution. Then, we developed the soft-decoding and soft-re-encoding algorithms for the designed RC-LDPC codes, which allows the relay to forward soft messages to the destination when the relay fails to decode the source's message. A special attention is given to the problem

of modeling the soft-errors, which can lead to poor performance if it is not done properly. We propose a novel method, called *soft fading*, to deal with this problem.

Chapter 7 - Here we conclude the thesis and provide some key points for future research.

Note that the chapters with original contributions are Chapters 3, 4, 5 and 6.

1.4 Research contributions

To a certain extent, the chapters in this thesis are self contained and can be read independently. The main contributions of this thesis are:

- The achievable rate analysis of DF strategy in the half-duplex relay channels under simplified and pragmatic conditions (equal power or equal time allocation).
- A new ensemble of codes, called multi-edge-type bilayer-expurgated LDPC codes and its corresponding design methodology.
- The design of distributed LDPC codes in time-division multiple-access relay channel and time-division two-way relay channel.
- The design of LDPC codes with systematic and linear encoding complexity.
- Detection of transmitted data under the soft decode-and-forward scheme in the half-duplex relay channels.

In the following, a detailed list of the research contributions in each chapter is presented.

Chapter 3

The main results of this chapter deal with the problem of finding the closed-form expression for the achievable rate of DF strategy in the half-duplex relay channel under simplified and pragmatic conditions. These results have been accepted for publication in a conference paper.

- M.H. Azmi, J. Li, R. Malaney, and J. Yuan, “Optimization for Pragmatic Half-Duplex Relay Network,” accepted for publication in Proc. IEEE Global Telecommunications Conference (GLOBECOM), Houston, Texas, USA, December 2011.

Chapter 4

The main results of this chapter deal with the design of bilayer-expurgated LDPC codes for DF strategy in the relay channels. These results have been published in one conference paper and have been accepted for publication in a journal.

- M. H. Azmi, and J. Yuan, “Design of multi-edge type bilayer-expurgated LDPC codes,” in Proc. IEEE International Symposium on Information Theory (ISIT), Seoul, Korea, July 2009.
- M.H. Azmi, J. Yuan, G. Lechner, and L.K. Rasmussen, “Design of Multi-Edge-Type Bilayer-Expurgated LDPC Codes for Decode-and-Forward in Relay Channels,” accepted for publication in IEEE Transactions on Communications, August 2011.

Chapter 5

The main results of this chapter deal with the design of distributed LDPC codes in time-division multiple-access relay channel and time-division two-way relay channels. These results have been published in two conference papers and a journal is in the stage of preparation for submission.

- M.H. Azmi, J. Li, J. Yuan, and R. Malaney, “Design of Distributed Multi-Edge Type LDPC Codes for Multiple-Access Relay Channels,” in Proc. Australian Communications Theory Workshop (AusCTW), Melbourne, Australia, February 2011.
- M.H. Azmi, J. Li, J. Yuan, and R. Malaney, “Design of Distributed Multi-Edge Type LDPC Codes for Two-Way Relay Channels,” in Proc. IEEE International Conference on Communications (ICC), Kyoto, Japan, June 2011.
- M.H. Azmi, J. Li, J. Yuan, and R. Malaney, “Design of Distributed Multi-Edge Type LDPC Codes for Relay Networks,” in preparation.

Chapter 6

The main results of this chapter deal with the design of rate-compatible LDPC codes for soft decode-and-forward in half-duplex relay channels. These results have been published in one conference paper and have been submitted for publication in a journal.

- M.H. Azmi, J. Li, J. Yuan, and R. Malaney, “Soft Decode-and-Forward using LDPC coding in Half-Duplex Relay Channels,” in Proc. IEEE International Symposium on Information Theory (ISIT), Saint Petersburg, Russia, August 2011.
- M.H. Azmi, J. Li, J. Yuan, and R. Malaney, “LDPC codes for Soft Decode-and-Forward in Half-Duplex Relay Channels,” submitted to IEEE Journal on Selected Areas in Communications, August 2011.

Other contributions not presented in this thesis

- M. H. Azmi, and J. Yuan, “Improved bilayer LDPC codes using irregular check node degree distribution,” in Proc. IEEE International Symposium on Information Theory (ISIT), Toronto, Canada, July 2008.
- M. H. Azmi, and J. Yuan, “Performance of bilayer-lengthened LDPC codes under joint decoding,” in Proc. IEEE Information Theory Workshop (ITW), Taormina, Sicily, Italy, October 2009.
- D. Duyck, M. Moeneclaey, M. H. Azmi, J. Yuan, and J. Boutros, “Universal LDPC codes for Cooperative Communications,” in Proceedings of the 6th International Symposium on Turbo codes and iterative processing, France, September 2010.

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