GROUP-LIKE ALGEBRAIC STRUCTURES OF FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING FOR SOLVING NEUROMAGNETIC INVERSE PROBLEM

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To **my beloved father, mother, Lai Kin, Li Meng and Li Jing**

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ABSTRACT

Fuzzy Topographic Topological Mapping (FTTM) is a novel mathematical model for solving neuromagnetic inverse problem. It is given as a set of mathematical operations, namely topological transformations with four components and connected by three different algorithms. At this moment, Fuzzy Topographic Topological Mapping 1 (FTTM 1) and Fuzzy Topographic Topological Mapping 2 (FTTM 2), which are used to solve the inverse problem for determining single current source and multiple current sources respectively, have been developed. The purpose of this research is to establish the topological and the algebraic structures of the components of FTTM 1 and FTTM 2. Firstly, the topological structures of the components of FTTM 2 were established and the homeomorphisms between the components of FTTM 2 were shown by using the proving techniques of the topological structures of the components of FTTM 1 and the homeomorphisms between the components of FTTM 1, then followed by the establishment of the algebraic structures of the components of FTTM 1 and FTTM 2. In the process, several definitions and theorems of group theory were adopted and the proving technique by construction was highlighted. In addition, FTTM was then generalized as a set which led to the proving the existence of infinitely many forms of FTTM. Finally, these structures were interpreted physically in order to study the information content of the inverse problem for determining single and multiple current sources.

ABSTRAK

Pemetaan Topologi Topografi Kabur (FTTM) merupakan model matematik baru untuk menyelesaikan masalah songsangan neuromagnetik. Ia terdiri daripada satu set operasi matematik, iaitu transformasi topologi dengan empat komponen dan dihubungkan oleh tiga algoritma yang berlainan. Kini, Pemetaan Topologi Topografi Kabur 1 (FTTM 1) dan Pemetaan Topologi Topografi Kabur 2 (FTTM 2) yang masing-masing digunakan untuk menyelesaikan masalah songsangan untuk menentukan sumber arus tunggal dan sumber arus berbilang telah dibangunkan. Tujuan penyelidikan ini adalah untuk membina struktur-struktur topologi dan aljabar bagi komponen-komponen FTTM 1 dan FTTM 2. Pada mulanya struktur-struktur topologi bagi komponen-komponen FTTM 2 telah dibina dan homeomorfisma-homeomorfisma antara komponen-komponen FTTM 2 telah dibuktikan dengan menggunakan teknik-teknik pembuktian struktur-struktur topologi bagi komponen-komponen FTTM 1 dan homeomorfisma-homeomorfisma antara komponen-komponen FTTM 1. Struktur-struktur aljabar bagi komponen-komponen FTTM 1 dan FTTM 2 telah dibina. Dalam pembinaan struktur-struktur aljabar ini, beberapa takrif dan teorem dari teori kumpulan digunakan dan pembuktian secara pembinaan diketengahkan. Selain daripada itu, FTTM telah diungkapkan sabagai satu set mengakibatkan pembuktian kewujudan bentuk-bentuk FTTM yang lain yang tak terhingga banyaknya. Akhirnya, struktur yang terbina diinterpretasikan secara fizikal untuk mengkaji kandungan maklumat bagi masalah sonsangan untuk menentukan sumber arus tunggal dan berbilang.

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LIST OF ABREVIATIONS

BM	-	Base Magnetic Plane
BMI	-	Base Magnetic Image Plane
FM	-	Fuzzy Magnetic Field
FMI	-	Fuzzy Magnetic Image Field
FTTM	-	Fuzzy Topographic Topological Mapping
FTTM 1	-	Fuzzy Topographic Topological Mapping 1
FTTM 1	-	Fuzzy Topographic Topological Mapping 2
MC	-	Magnetic Contour Plane
MI	-	Magnetic Image Plane
TM	-	Topographic Magnetic Field
TMI	-	Topographic Magnetic Image Field

LIST OF SYMBOLS

$\mathbb{C}$	-	field of complex numbers
$\mathbb{N}$	-	set of natural numbers
$\mathbb{Q}$	-	field of rational numbers
$\mathbb{R}$	-	field of real numbers
$\mathbb{Z}$	-	set of integers
$\mathbb{R}^n$	-	*n*-dimensional Euclidean space
max	-	maximum
min	-	minimum
$\in$	-	member of
$\notin$	-	not member of
$\subset$	-	subset of
$\exists$	-	exists
$\forall$	-	for all
$\ni$	-	such that
$\Rightarrow$	-	if …… then
$>$	-	more than
$<$	-	less than
$\geq$	-	more than or equal to
$\leq$	-	less than or equal to
$\lvert\ \rvert$	-	absolute value
$\lVert\cdot\cdot\rVert$	-	norm
$\{\ldots\}$	-	set consisting of …
$f : X \rightarrow Y$	-	*f* is a mapping from *X* to *Y*

$g \circ f$	-	composition of mappings f and g
f^{-1}	-	inverse mapping of f
$N(a)$	-	neighborhood of a
$\cong$		homeomorphic / isomorphic

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Generally speaking sets have no intrinsic structure, they are just collections of things. Much like a generic collection of boards, they do not have any structure (Hrabovsky, 2003). In mathematics, a structure on a set consists of additional mathematical objects that in some manner attach to the set, making it easier to visualize or work with, or endowing the collection with meaning or significance. A partial list of possible structures are measures, algebraic structures, topological structures, metric structures, orders, and equivalent relations. Sometimes, a set is endowed with more than one structure simultaneously; this enables mathematicians to study it more richly. For example, if a set has a topology and is a group, and the two structures are related in a certain way, the set becomes a topological group.

In this thesis, we start with the introduction of topological and algebraic structures. We start with the traditional joke that a topologist does not know the difference between a coffee cup (with a handle) and a doughnut (with a hole), since a sufficiently pliable doughnut could be smoothly manipulated into the shape of a coffee cup, by creating a dimple and progressively enlarging it, while shrinking the hole into a handle, which does not require the discontinuous action of a tear or a punching of holes (Levin, 2000). In other words, the coffee cup and the doughnut are two objects endowed with respective topological structures, which are topologically equivalent. However, a topologist can tell the difference between a ball and a doughnut since they are two objects endowed with respective topological structures, which are not topologically equivalent. Intuitively, a topological structure on an object (a set) is a collection of subsets with certain properties (Anthony, 2003), which concerns itself with how the object is connected, but not how it look.

Formally, a topological structure (or, more briefly, a topology) on a set X is a structure given by a set τ of subsets of X, having the following properties (called axioms of topological structures) (Bourbaki, 1989):

i. Every union of sets of τ is a set of τ.

ii. Every finite intersection of sets of τ is a set of τ.

The sets of τ are called open sets of the topological structure defined by τ on X. A topological space is a set endowed with a topological structure (Bourbaki, 1989). Two topological spaces are topologically equivalent if there is a homeomorphism between them. Formally, a homeomorphism is defined as an open continuous bijection (Christie, 1976). However, a more informal criterion gives a better visual sense: two spaces are topologically equivalent if one can be deformed into the other without cutting it apart or gluing pieces of it together. In other words, a homeomorphism maps points in the first object that are “close together” to points in the second object that are close together, and points in the first object that are not close together to points in the second object that are not close together. For example, a sphere and an ellipsoid are topologically equivalent. We can show that a sphere and an ellipsoid are topologically equivalent by stretching a sphere into an ellipsoid or by pressing an ellipsoid into a sphere. Besides, we can show that a sphere and an ellipsoid are topologically equivalent analytically by defining a homeomorphism between a sphere and an ellipsoid (Liau and Tahir, 2003).

The great importance and wide application of topological structures: mathematicians merely have to show that a given set is endowed with a topological structure, and then the topological properties of the set remain unchanged under a homeomorphism. In other words, if two topological spaces are topologically equivalent, then they have the same topological properties. For example, the impossibility of arranging a walking route through the town of Königsberg (now Kaliningrad) that would cross each of the seven bridges formed over four lands (and areas) exactly once, which was published in Leonhard Euler's 1736 paper on Seven Bridges of Königsberg (Morikawa and Newbold, 2003), can be applied to any arrangement of bridges topologically equivalent to those in Königsberg. The great importance of topological structures presents in almost all areas of today's mathematics and also other fields of study.

What is algebraic structure? Out of numerous possible approaches to answer this question, we should pay attention to Weyl's conception in the following sentence. Weyl mentioned:

> …now we are coming back to old Greek viewpoint, according to which every sphere of things requires its own numeric system defined on its own basis. And this happens not only in geometry but in new quantum physics: physical quantities, belonging to a certain given physical structure, permit themselves (but not those numeric values which they may assume due to its different states), in accordance with quantum physics, perform addition and non-commutative multiplication, forming by this some world of algebraic quantities, corresponding to this structure, the world, which cannot be regarded as fragment of the system of real numbers.
>
> (Rososhek, 1999)

According to Rososhek (1999), the ideas of Weyl mentioned in the preceding paragraph can be summarized in such a way by using the ideas of Shafarevich (1986):

i. Every phenomenon, every process of real world (also in mathematics itself) may be "coordinatized" in the frame of some system of coordinatizing quantities.
ii. Subject of Algebra is a study of various systems of coordinatizing quantities as concrete (for example numbers, polynomials, permutations, matrices, functions and so on) as well as abstract (groups, rings, fields, vector spaces and so on).

iii. If some phenomenon is not yet coordinatized by any familiar system of coordinatizing quantities, the problem of coordinatization arises to develop a system of coordinatizing quantities for the phenomenon.

From the ideas (i) and (ii) of Shafarevich (1986), every phenomenon may be represented with some systems of coordinatizing quantities such as numbers, polynomials, permutations, matrices, functions, groups, rings, fields, vector spaces and so on. We recognized that groups, rings, fields, vector spaces are algebraic structures. Therefore algebraic structures are systems of coordinatizing quantities. In other words, a phenomenon may be represented with an algebraic structure or a set may has an algebraic structure. For example, most sets dealt with in mathematics are sets which have an algebraic structure (Burton, 1965). An algebraic structure comes out when we impose certain suitably restricted rules on how elements of a set can combine (Hrabovsky, 2003). These rules enable the mathematicians to combine the elements of the sets in useful ways. Formally, an algebraic structure is a nonempty set together with one or more binary operations which obey certain rules known as axioms or postulates (Burton, 1965)

From the idea (ii) of Shafarevich (1986), algebraic structures are systems of coordinatizing quantities as abstract or regarded in an abstract way. In other words, an algebraic structure captures common abstract notion and properties of different sets, which satisfy the basic laws of that algebraic structure. Any particular example we encounter which satisfies the basic laws of a given algebraic structure will also satisfy all the theorems, which are true for that algebraic structure (Burton, 1965). For example, if a set has a group structure, then the whole range of proved results or properties for group in general will be valid for the phenomenon.

From the preceding paragraph, we recognized the great importance and wide application of algebraic structures: mathematicians merely have to show that a given set satisfies the basic laws of an algebraic structure (usually a simple matter) and then the whole range of results is ready to apply where necessary. Although the basic laws of algebraic structures are few and simple, mathematicians can built a surprisingly large amount of algebraic structures of sets with them and of course any result proved in the general theory are true in such sets. Therefore algebraic structures have embraced a wide variety of other fields of study (Sheth, 2002). For example, in science and engineering, scientists and engineers routinely use physical quantities to represent the measured properties of physical objects and some mathematicians have studied the algebraic structures of the physical quantities in order to

i. study the physical quantities from a more abstract standpoint, with the aim of better understanding the nature and use of those quantities,
ii. derive a number of meaningful results from the algebraic structures of the physical quantities (Sheth, 2002).

Therefore, algebraic structures, which are tools for exploring, for inquiring, and for understanding, interact with other fields of study to illuminate and advance them. Now, let us switch our attention to the idea (iii) of Shafarevich (1986). If some phenomena are not yet represented with any familiar system of coordinatizing quantities, mathematicians can carry out researches to represent these phenomena with some systems of coordinatizing quantities.

Finally, we summarize that topological and algebraic structures are mathematical structures that are important and widely used in many fields of study. Furthermore, in this research, we will study the topological and algebraic structures that exist in a novel mathematical model known as Fuzzy Topographic Topological Mapping, shortly FTTM. In the next section, we will have a brief discussion on FTTM and the role of this research in the development of FTTM.

1.2 Influential Observation

Fuzzy Modelling Research Group, shortly FMRG, which is led by Associate Professor Dr. Tahir Ahmad, has been developing a software for determining the location of epileptic foci in epilepsy disorder patients since 1999. At the present time, FMRG has developed FTTM for solving neuromagnetic inverse problem to determine the cerebral current sources, namely epileptic foci (Tahir *et al.*, 2000).

FTTM is given as a set of mathematical operations, namely topological transformations with four components and connected by three different algorithms, which are three different sets of mathematical instructions that must be followed in a fix order, and that, especially if given to a computer via a computer program, will help to calculate an answer to a neuromagnetic inverse problem (Tahir *et al*., 2003) (see Figure 1.1).

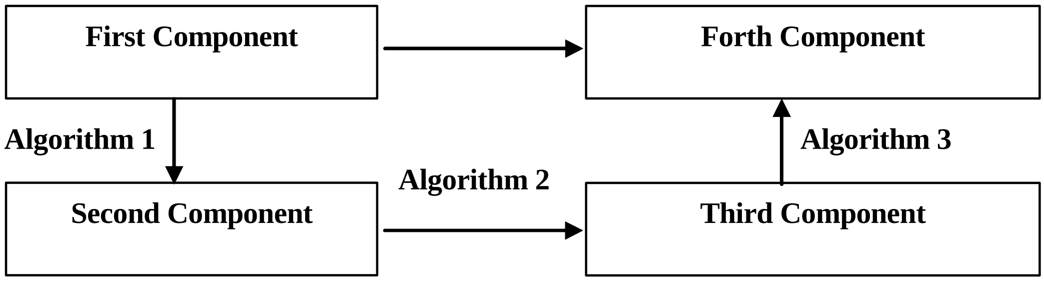

Figure 1.1 FTTM

There are FTTM 1 and FTTM 2 up to now. FTTM 1 consists of three different algorithms that link between the four components of the model: Magnetic Contour Plane (*MC*), Base Magnetic Plane (*BM*), Fuzzy Magnetic Field (*FM*) and Topographic Magnetic Field (*TM*). The three different algorithms that link between the four components of FTTM 1 are three different sets of mathematical instructions that must be followed in a fix order, and that, especially if given to a computer via a computer program, will help to solve the inverse problem for determining single current source (Fauziah *et al*, 2000; 2002; Tahir, 2000; Tahir *et al*., 2000; 2001; 2003; 2004a; 2005). On the other hand, FTTM 2 consists of three different algorithms that link between the four components of the model: Magnetic Image Plane (*MI*), Base Magnetic Image Plane (*BMI*), Fuzzy Magnetic Image Field (*FMI*) and Topographic Magnetic Image Field (*TMI*). The three different algorithms that link between the four components of FTTM 2 are three different sets of mathematical instructions that must be followed in a fix order, and that, especially if given to a computer via a computer program, will help to solve the inverse problem for determining multiple current sources (Tahir *et al*., 2003; 2004a; 2004b; Wan Eny Zarina *et al*., 2001; 2002; 2003a; 2003b; 2004).

The appearance of mathematical structures, especially topological structures in FTTM was preconceived by Tahir (2000) and was shown by Tahir *et al*. (2005). Tahir *et al*. (2005) established the topological structures of all components of FTTM 1 and showed that they are topologically equivalent. Therefore, all components of FTTM 1 have the same topological properties. However, in this work, we will establish the topological structures of all components of FTTM 2. We will show that all components of FTTM 2 are topologically equivalent too. Besides, we will show that every component of FTTM 1 and its corresponding component of FTTM 2 are topologically equivalent. According to Tahir *et al*. (2003), there exists the duality for the topological structures of the components of FTTM 1 and FTTM 2. Therefore, this research will probe into one of the preconceived dualities mentioned in Tahir *et al*. (2003), which are the existence of algebraic structures of the components of FTTM 1 and FTTM 2 in detail.

After studying the topological and the algebraic structures of the components of FTTM 1 and FTTM 2, we will interpret the physical meanings of some of the results in order to study the information content of the inverse problem of single and multiple current sources. In other words, we are going to find out which internal parameters of magnetic field data inaccessible to measurement can be determined in a stable and unique manner.

1.3 Problem Statement

The components of FTTM 2 are topologically equivalent (see Figure 1.2).

$$\begin{array}{ccc} \boldsymbol{MI} & \cong & \boldsymbol{TMI} \\ \wr\Vert & & \wr\Vert \\ \boldsymbol{BMI} & \cong & \boldsymbol{FMI} \end{array}$$

Figure 1.2 The components Of FTTM 2 are topologically equivalent

Besides, corresponding components of FTTM 1 and FTTM 2 are topologically equivalent (see Figure 1.3).

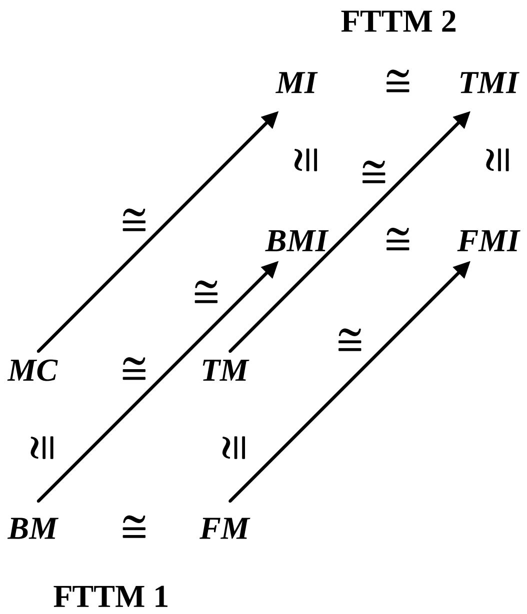

Figure 1.3 Corresponding components of FTTM 1 and FTTM 2 are topologically equivalent

Furthermore, the study of the topological and the algebraic structures of the components of FTTM 1 and FTTM 2 contributes the information of which internal parameters of magnetic field data inaccessible to measurement can be determined in a stable and unique manner.

1.4 Objectives of Research

The objectives of this research are given as follows:

i. To show the homeomorphisms between the components of FTTM 2.
ii. To show a homeomorphism between every component of FTTM 1 and its corresponding component of FTTM 2.
iii. To show the algebraic structures of the components of FTTM 1 and FTTM 2.
iv. To interpret the physical meanings of the topological and algebraic structures of the components of FTTM 1 and FTTM 2.

1.5 Scope of Research

In this research, we will study the topological and the algebraic structures of the components of FTTM 1 and FTTM 2. In studying the topological structures of the components of FTTM 1 and FTTM 2, we will focus on showing the topological structures of the components of FTTM 2. Besides, we will show that all components of FTTM 2 are topologically equivalent. In addition, we will show that every components of FTTM 1 and its corresponding components of FTTM 2 are topologically equivalent. Finally, we will derive other additional results from the topological structures of the components of FTTM 1 and FTTM 2.

On the other hand, in studying the algebraic structures of the components of FTTM 1 and FTTM 2, we will focus on showing the algebraic structures of the components of both FTTM 1 and FTTM 2. In other words, we will establish the algebraic structures of the components of FTTM 1 and FTTM 2. Furthermore, we will focus on interpreting some results to show

i. which internal parameters of magnetic field data inaccessible to measurement can be determined in a stable and unique manner,
ii. the behaviour of neuromagnetic fields, and
iii. some features of FTTM.

In the next section, we will discuss the importance of this research.

1.6 Importance of Research

Most mathematical problems in science, technology and medicine are inverse problems. For example, determination of the current sources underlying a measured distribution of the magnetic field is an inverse problem.

According to Anger and Moritz (2003), one of the important points to solve an inverse problem is development of algorithms for the numerical solution of an inverse problem. Therefore, the development of algorithms for determining single and multiple current sources from the detected magnetic field distributions is of great importance and contained in the development of FTTM. The homeomorphisms between the components of FTTM 1 make up algorithms for determining single current source (Tahir *et al.*, 2005). However, in this research, we will show the homeomorphisms between the components of FTTM 2, which will make up algorithms for determining multiple current sources.

On the other hand, another important point is studying the information content of the inverse problem, i.e., to find out which inner parameters of a system inaccessible to measurement can be determined in a stable and unique manner (Anger and Moritz, 2003). Therefore, in this research, the study of the topological and the algebraic structures of the components of FTTM 1 and FTTM 2 will be carried out in order to study the information content of the inverse problem in determining single and multiple current sources. We will show which internal parameters of magnetic field data inaccessible to measurement can be determined in a stable and unique manner.

1.7 Framework of Thesis

In general, this thesis contains six chapters. Chapter 1 deals with the introduction to the research. It discusses the background and motivation, influential motivation, problem statement, objectives, scope and importance of the research. It is then followed by Chapter 2, which deals with literature review of the research. It discusses the human brain, FTTM, mathematical background and formulation, and the concept of inverse problem. Chapter 3 presents the proof of the existence of the homeomorphisms between the components of FTTM 2. Besides, it presents the proof of the existence of a homeomorphism between every component of FTTM 1 and its corresponding components of FTTM 2. It also presents the generalization of FTTM and other additional results. Next, Chapter 4 presents the establishment of the algebraic structures of the components of FTTM 1 and FTTM 2. The physical interpretations of the topological and the algebraic structures of the components of FTTM 1 and FTTM 2 are presented in Chapter 5. Finally, this thesis will be ended with a conclusion and some future works presented in Chapter 6.

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