BAYESIAN AND FUZZY KRIGING INTERPOLATION TECHNIQUES FOR SPATIAL ESTIMATION IN MINING FIELD

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This dissertation is dedicated to my family for their endless support and encouragement.

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ABSTRACT

The focus of this research is in the area of spatial estimation. Such a study is very important in order to improve the spatial prediction performance. Many techniques of prediction that are based on the regionalized variables, and the surface trend change from linear to quadratic or cubic that produces inaccurate results in the prediction process. In this thesis, Bayesian and fuzzy kriging methods are suggested to solve the problem of uncertainty, which requires obtaining a minimum error in the prediction process. This study aims to improve the mixed approaches among methods of spatial prediction that are used for evaluation of prediction. The study also finds the performance of variation interpolation methods of minerals needed to develop the relationship between Bayesian techniques and fuzzy kriging and apply the results for further modeling a spatial relationship. This spatial prediction assumes stationary property. The findings of this study are mathematical models of covariance functions. The variogram and cross variogram functions are computed for all compass directions for the phenomena under the study and its parameters are estimated. Another aspect is to obtain Bayesian predictor, kriging predictor, and Bayesian kriging variance which represent the minimum variance of prediction. In addition, the constraints weights of linear prediction were computed. The practical side of this study includes the applications of the Bayesian and fuzzy kriging techniques on real spatial data with their locations in the mining fields of Australia, Canada, and Colombia. All the computations were carried out by using Matlab software. In conclusion, this study uses two different methods (Bayesian and fuzzy kriging techniques) for incorporating the spatial autocorrelation in order to improve the accuracy of uncertainty and estimation with minimum error. The approach combines more than one prediction methods to determine a model which is based on a cross validation that satisfies the best optimal prediction.

ABSTRAK

Tumpuan kajian ini adalah dalam bidang penganggaran ruang. Kajian ini sangat penting untuk mempertingkatkan prestasi ramalan ruang. Pelbagai teknik ramalan berdasarkan pembolehubah regionalisasi dan perubahan jalan permukaan daripada linear kepada kuadratik atau kubik menghasilkan keputusan yang tidak tepat dalam proses ramalan. Kaedah Bayesian dan kriging kabur dicadangkan untuk menyelesaikan masalah ketidaktentuan yang memerlukan supaya ralat minimum dalam proses ramalan dapat diperoleh matlamat kajian ini adalah untuk meningkatkan pendekatan gabungan antara kaedah bagi ramalan ruang yang digunakan dalam penilaian ramalan. Prestasi bagi kaedah interpolasi variasi juga ditemukan bagi mineral yang diperlukan untuk membangunkan hubungan antara teknik Bayesian dan kriging kabur dan melanjutkan dapatan bagi pemodelan atau hubungan ruang. Ramalan ruang ini mengandaikan ciri pegun. Dapatan kajian ini adalah model bermatematik bagi fungsi kovarians. Fungsi variogram dan variogram bersilang dikira untuk semua arah kompas bagi fenomena dalam kajian dan parameternya dianggarkan. Aspek lain adalah untuk mendapatkan peramal Bayesian, peramal kriging dan varian kriging Bayesian yang mewakili varians minimum bagi ramalan. Tambahan lagi, kekangan pemberat bagi ramalan linear dikira. Bahagian praktikal kajian ini termasuk penggunaan teknik Bayesian dan kriging kabur bagi data ruang sebenar dengan lokasi di kawasan perlombongan di Australia, Canada dan Colombia. Semua pengiraan dilakukan menggunakan perisian Matlab. Kesimpulannya, kajian ini menggunakan dua kaedah yang berbeza (Teknik Bayesian dan kriging kabur) untuk menggabungkan auto korelasi ruang bagi meningkatkan ketepatan bagi ketidaktentuan dan mendapatkan penganggaran dengan ralat yang minimum. Pendekatan yang digunakan menggabungkan lebih daripada satu kaedah ramalan untuk menentukan suatu model berasaskan pengesahan bersilang bertujuan memenuhi ramalan optima terbaik.

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LIST OF SYMBOLS

(x)	-	Locations x of Spatial Variable
(x') or $(x+h)$	-	Locations $(x+h)$ of Spatial Variable
$Z(x)$	-	Regionalized Variable of location (x)
$Z(x_o)$	-	Regionalized Variable of location (x_o)
$Z(x+h)$	-	Regionalized Variable of location $(x+h)$
Z , or Z_β	-	Vector of Real Data
$\bar{Z}$, $\bar{d}$	-	Sample Mean
h	-	Euclidian Distance. or Lag
(x,y)	-	Points of location
$N(\Delta x, \Delta y)$, or $N(h)$	-	Number of pairs of observations
λ_i	-	The Weights in Estimation of Ordinary Kriging
λ'	-	The Weights Transpose
λ_β	-	The Weights Vector
ω_i	-	The Weight in Estimation of Universal Kriging
$\gamma(\mathrm{x}, h)$	-	Semivariogram Function
$2\gamma(\mathrm{x}, h)$	-	Variogram Function
$\hat{\gamma}(h)$	-	Estimator of Semivariogram Function
γ^*_{ij}	-	Cross Semivariogram Function
$C(h)$, ϕ	-	Covariance Function
$C_{12}(h)$	-	Cross Covariance Function
C^{-1}, K^{-1}	-	Inverse Matrix
ϕ^{-1}	-	Covariance Inverse

C_o	-	Nugget Effect
$C + C_o$	-	Sill
a	-	Range
$\hat{z}(x)$	-	Predictor Values of Variable
z^b	-	Variable of Bayes
μ	-	Mean of Stationary Spatial Process
$\mu(x)$, $\mu_m(x)$	-	Mean of Non-Stationary Spatial Process
σ^2 , σ_E^2	-	Variance Parameter
$\sigma^2{}_{OK}$	-	Ordinary Kriging Variance
$\sigma^2{}_{uK}$	-	Universal Kriging Variance
$\sigma^2{}_{COK}$	-	COKriging Variance
LP	-	Lagrange Multiple
P	-	Dimensions
P(A)	-	Probability of Event A
P(x)	-	Probability of Event x
P(m)	-	Probability of Event m
P(D)	-	Probability of Event D
P(x|A)	-	Probability x Condition A
P(A|x)	-	Probability A Condition x
P(x, A)	-	Joint Probability of x and A
P(m|D)	-	Probability m Condition D
P(D|m)	-	Marginal Likelihood
P(A $\cup$ B)	-	Probability of Event Union
P(A $\cap$ B)	-	Probability of Event Intersection
$\rho(\psi, d)$	-	The Matérn Covariance
ρ	-	Correlation Coefficient
$\rho_{12}(h)$	-	Cross Correlation Coefficient

$\varepsilon(\mathrm{x})$	-	Random Error Vector
β	-	Vector Unknown Parameter
$\widehat{\beta}_{BK}$	-	Posterior of Bayesian Kriging
$\forall$	-	For all variables
D	-	Domain (or Region)
D_2	-	Special Distance
R	-	Real Numbers
R^p	-	Real Numbers R in P Dimension Space
∞	-	Infinity
$\sum$	-	Summation
$\sum\sum$	-	Double Summation
$\int$	-	Integral
F	-	Information Function
F^T	-	Matrix Transpose
θ	-	Angle (or Trend)
$K_u(\bullet)$	-	Bessel Function
$\tilde{A}$	-	Fuzzy Set
$\mu_A(x)$	-	Membership Function
$\mu_{\tilde{A}}(x)$	-	Membership Function of Fuzzy Set $\tilde{A}$
M^-	-	Left Spreads
M^m	-	Model Value
M^+	-	Right Spreads
$\mu_{\tilde{A}}(x)$	-	Membership Function of Fuzzy Set $\tilde{A}$
$\tilde{O}^\beta$	-	Trend Parameter
$\tilde{C}^\beta$	-	Covariance Matrices
Δ	-	Change in or Difference

$\vert\ \vert$	-	Absolute Value
$\Vert\ \Vert$	-	Euclidian Norm on R^p
$\sim$	-	Approximate
$\approx$	-	Approximate to Equal to
$\triangleq$	-	Equal by Definition
$\rightarrow$	-	Go to or Lead
$\begin{pmatrix} & \\ & \end{pmatrix}$	-	Matrix
$\{\ ,\ \}$	-	The Set of
$(\ ),[\ ]$	-	Brackets
(a,b)	-	Open Interval
$[a,b]$	-	Closed Interval
$\oplus$	-	Direct Sum
$\mathrm{E}(Z\vert\bullet)$	-	Conditional Expectation of Z
$\mathrm{E}(\bullet)$	-	Expectation Value
$\mathrm{Var}(\bullet)$	-	Variance
I	-	Identity Matrix
$f(\bullet)$	-	Non Linear Function

LIST OF ABBREVIATIONS

SK	-	Simple Kriging
OK	-	Ordinary Kriging
UK	-	Universal Kriging
COK	-	Cokriging
COV	-	Covariance
KED	-	Kriging with the External Drift
RK	-	Regression Kriging
RF	-	Random Function
BLUE	-	Best Linear Unbiased Estimator
ME	-	Mean Error
MAE	-	Average Absolute Error
MSE	-	Mean Square Error
KRMSE	-	Kriged Random Mean Square Error
CORMSE	-	Cokriged Mean Square Error

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CHAPTER 1

INTRODUCTION

1.1 Overview

Geostatistics is centrally focused areas of applied statistics. This branch of statistics is concerned with the spatial or temporal outlook of the data studied and the corresponding contour distribution of that data. Using geology, which is at the root of geostatistics, was pioneered by two researchers Krige (1951) and Matheron (1963). It was originally meant to project changes in the quality of ore from particular mines.

In the mid-nineties, Krige, a seasoned engineer working in South African mines, developed a model for interpolating true random spatial variables of a sample space. Krige's spatial statistic model was later modified by Matheron, a French mathematician create the best kriging. The trend of kriging was evident in the research conducted by Journel and Huijbregts (1978), Ripley (1981), Lam (1983), Davis (1986), and Cressie (1990) in their geostatistical investigation of various data sets.

1.2 Research Background

The spatial statistics theory has its roots from the theory of regionalized random spatial variables. This spatial statistics theory is concerned with spatial data such as deposits of mineral ore, oil reserves and data on rainfall distribution, epidemics, or data on various types of pollution. These spatial variables may be generated from geographic locations on the earth's surface, underground or from the atmosphere.

Spatial interpolation has close connections to geology and it entails a series of mathematical computational methods. It is also; concerned with the spatial phenomena. It uses methods that emphasis regionalized random variables so as to generate spatial distribution. Studies of spatial statistics based on regionalized variables theory have successfully formulated mathematical models for determining the nature of the spatial distribution through use of functions such as a variogram. A variogram is one of the most commonly employed functions used for modeling in spatial statistics.

A variogram functions is employed to determine existing variations in observed phenomenon. The function was employed in studies by Journel (1992), Cressie (1993), and Chiles and Defflar (1999). The variogram function uses spatial prediction. Spatial prediction uses the kriging method to estimate the Best Linear Unbiased Estimator (BLUE) for a set of spatial real data.

There are various types of kriging including simple kriging, ordinary kriging (which is the most commonly used), anisotropic kriging (uses accounting geometric variance), universal kriging (uses local accounting trends), cokriging (where there is more than one variables). Each of these kriging techniques types employs either a semivariogram or a variogram function.

The kriging school of thought uses geostastistical interpolation to estimate the value of unknown parameters using available location data (Cressie, 1985; Burrough, 1986). Some of the key studies that employed spatial interpolation models are Burrough and Mcdonnel (2000), and Cressie (2003). Other works that employed universal kriging, include Martinez and Zinck (2004) and Gooavaerts (1997a) who estimated parameters using approaches such as maximum likelihood estimation, least squares quadratic method, and Bayesian. Bayesian kriging is; where model parameters are generated with the aid of regionalized random variables.

1.3 Significance of the study

The significance of the current study lies in the fact that it will further enhance understanding of contour maps using regionalized random variables; and outline situations where the kriging technique for estimating the parameters of experimental functions and knowledge properties can be used.

1.4 Research Questions

The spatial estimation of interpolation methods was based on the following research questions:

i. Does mineral ore data satisfy the stationary assumptions?
ii. Are isotropic variations accounted for?
iii. Is the robust estimator trend required?
iv. In which method do different variogram estimation techniques, contribute to variations in the estimated covariance parameters?
v. How can a reasonable estimate of the nugget variance be achieved?
vi. How can we obtain a mathematical model that fits with covariance functions?
vii. Which model provides the best spatial estimation performance?

1.5 Problem Statement

Spatial interpolation techniques are an essential input for the development of spatial prediction models that use a stochastic process. In practice, location estimation using geostatistics or spatial statistics is associated with some level of precision error and uncertainty. This can be attributed to the fact that the surface trend changes from linear to quadratic or cubic thereby resulting in the inaccuracy. As such, the current research seeks to solve this problem by developing prediction models for enhancing the spatial prediction performance with minimal prediction error. Therefore, Bayesian kriging with fuzzy is proposed as a mixed approach to solve this prediction problem. The main motivation is to jointly handle different types of uncertain information such as uncertainty in the variogram parameters and uncertainty property in the variogram model. Kriging possesses advantages over the interpolation method as it has the ability to determine an uncertainty estimate for the value of the regionalized variable.

1.6 Objectives of the research

The objectives of the current research are as follows:

i. To analyze the spatial data using regionalized variables in the mining industry.
ii. To make a comparison between various types of kriging models (ordinary, universal, universal cokriging, Bayesian) to know the best performance.
iii. To establish the ability and accuracy of fuzzy method for enhancing the spatial prediction.
iv. To combine kriging techniques with the Bayesian fuzzy kriging so as to observe their output.
v. To identify the uncertainty features present in the mining industry and to analyze the effect of these features on the prediction process.
vi. To establish a mathematical model and to compare it with the covariance functions; and study the performance of the variation interpolation model.
vii. To enhance the performance of the spatial prediction models and to ascertain the environment effect of mineral ores in the study area.

1.7 Scope of the study

The current study was concerned with the analysis of data generated from metal ore mines and how the contour maps of the data are distributed in the study area. The idea was to allow for predictions that use spatial data and the theory of random spatial process to explore for minerals such as gold, silver, nickel, lead, zinc and copper.

1.8 Thesis Organization

The thesis is organized into five chapters. Chapter 1 includes an introduction of spatial statistics and gives an idea of research background, significance of the study problem statement, objectives of the research and the scope of the study. Chapter 2 provides a review of the literature on kriging which includes the origins of kriging, applications of the kriging, universal kriging, and cokriging techniques, applications of the Bayesian and fuzzy approach, and other applications of interpolation methods. Chapter 3 contains the research methodology that starts with introduction of geostatistics and goes on defining regionalized variables, the experimental variogram function, and spatial predictions. Chapter 4 illustrates the data analysis and the results based on the real spatial data in different areas of Colombia, Australia, and Britain, by using interpolation methods for predictions. Finally, Chapter 5 is reserved for conclusions and recommendations for future work.

REFERENCES

Bandemer, H. and Gebhardt, A. (2000). Bayesian fuzzy kriging. Freiberg (Sachs), Germany. *Fuzzy Sets and Systems*. 112, 405-418.

Beliaeff, B. and Cochard, M. L. (1995). Applying geostatistics to identification of spatial patterns of fecal contamination in a mussel farming area. (Havre de la Vanlee, France), *Water Research*. 29, 1541-1548.

Bivand, R. S. Pebesma, E. and Virgilio, R. (2008). Applied spatial data analysis with R, R series. New York. *Springer.*

Brochu, Y. Marcotte, D. and Chapuis, R. (2002). *Kriging of Hydraulic Head Field for a Confined Aquifer.* GEOENV2002.

Brus, D. J. and Heuvelink, G. (2007). Optimization of sample patterns for universal kriging of environmental variables. *Geoderman.* 138(1): 86-95.

Bourennane, H. and King, D. (2003) Using multiple external drifts to estimate a soil variable, *Geoderma*. 114(1): 1-8.

Budiman, M. Jasper A. and Alex, B. (2011) Confronting uncertainty in model based geostatistics using Markov Chain Monte Carlo simulation. *Asturalia, Elsevier Geoderma.* 163, 150-162.

Buland, A. Kolbjørnsen, O. and Omre, H. (2003). Rapid spatially coupled AVO inversion in the Fourier domain. *Geophysics*. 68, 824-836.

Burrough, P. (1986) *Principals of geographical information systems for land resources assessment*. Oxford, Clarendon Press.

Burrough, P. and McDonnell, R. (1998). *Principles of Geographical Information Systems.* NewYork: Oxford University Press Inc, page 333.

Burrough, P. and McDonnell, R. (2000). *Principles of Geographical Information Systems*. New York, Oxford University Press.

Burgess, T. M. and Webster, R. (1980). Optimal interpolation and is arithmic mapping of soil properties: I. The semivariogram and punctual kriging. *Journal of Soil Science.* 31, 15–331.

Carlin, B. P. and Louis, T. A. (1997) Bayes and empirical Bayes methods for data analysis. *Journal of Statistics and Computing*. 7, 153–154.

Chalkiadakis, G. and Boutilier, C. (2004). Bayesian reinforcement learning for coalition formation under uncertainty, Proceedings of the Third Joint *Conference on Autonomous Agents and Multiage Systems, IEEE Computer Society*.

Chalk, G. and Bouti, C. (2004). *Bayesian Reinforcement Learning for Coalition Formation under Uncertainty*. New York, USA.

Chang, C. L. Lo, S. L., and Yu, S.L. (2005). Applying fuzzy theory and genetic algorithm to interpolate precipitation. *Journal of Hydrology*. 314, 92–104.

Chilés, J. and Delfiner, P. (1999). *Conditioning by Kriging. In: Geostatistics: Modeling Spatial Uncertainty*, edited by V. Barnett et al., 465-472, John

Christensen, R. (2001). *Linear* Wiley and Sons, New York. *models for multivariate time series and spatialy data. Spring verlag* New York 393.

Cressie, N. A. and Hawkins, D. (1980). Robust estimation of the variogram. *Mathematical Geology*. 12, 115-125.

Cressie, N. A. (1985). Fitting Variogram Models by Weighted Least Squares. *Journal of the international association for Math. Geol*. 17, 563–586.

Cressie, N. A. (1986). .Kriging non stationary data, *Journal of the international association for Mathematical Geology*. 81, 625-634.

Cressie, N. A. (1990). The Origins of Kriging. *Journal of the international association for Mathematical Geology*. 22(3): 239-252.

Cressie, N. A. (1991). Modeling Growth with random sets in spatial statistics and imaging. Institute of mathematical statistics. Hayward. 20, 31-45.

Cressie, N. A. (1993). *Statistics for Spatial Data*. (2nd Edition), John Wiley and Sons, New York.

Cressie, N. A. (2003). *Statistics for Spatial Data*.Revised Edition, New York, John Wiley and Sons.

Cressie, N. A. and Johansson, G. (2008). Fixed rank kriging for very large statistical Society: Series B (Statistical spatial data sets), *Journal of the Royal Methodology*. 70, 209-226.

Curran, P. (1988). *The semivariogram in remote sensing: an introduction.* Remote Sensing of Environment.

Dagdelen, K. and Nieto, A. (1997). Geostatistics applied to mine waste characterization at Leadville, Colorado, USA, *International Journal of Surface Mining Reclamation and Environment*. XI (4): 175-188.

Daly, C. Neilson, R. and Phillips, D. L. (1994). A statistical-topographical model for mapping climatological precipitation over mountainous terrain. *Journal of Application*, Meteor. l33, 140–158.

David, M. (1977). Geostatistics Ore resource estimation, *Elsevier*, Amsterdam.

Davis, J. C. (1986). *Statistics and data analysis in geology*. (2nd Edition). Wiley, New York.

Delhomme, J. P. (1978). Kriging in the hydrosciences. *Advances in Water Resources,* 1, 251-266.

Diggle, P. J. (1983). *Statistical analysis of spatial point paterns*. academic press, London, England.

Diggle, P. J. and Ribeiro, J. (2007). Model- based Geostatistics. Springer Science, Business Media, LLC, 288.

Diggle, P. J. Ribeiro, P. J., and Christensen, O. (2003). *An introduction to model based geostatistics. Book chapter in spatial statistics and computational methods.* Springer, New York. 173, 43–86.

Diggle, P. J. Menezes, R. and Su T. l. (2010). Geostatistical inference under preferential sampling Journal of the Royal Statistical Society: Series C (Applied Statistics) Article first published online: 9 FEB 2010.

Dirks, K. Hay, J. Stow, C. and Harris, D. (1998). High-resolution studies of rainfall on Norfolk Island. Part II: Interpolation of rainfall data. *Journal of Hydrology*. 208(3): 187–193.

Dingman, S. Seely, D. and Reynolds, J. (1988). Application of kriging to estimating mean annual precipitation in a region of orographic influence. *Water Resource Bull.* 24(2): 329–339.

Dowd, P. (1984). The variogram and kriging: robust and resistant estimators. In: Geostatistics for Natural Resources, edited by V.G., 91-106.

Dubois, G. and Galmarini, S. (2004). Introduction to the spatial interpolation and comparison. *Applied GIS*. 1(2): 9-11.

Dutina, I. (2013). Atico updates activities in preparation for operating the El Roble Mine**.** Technical report. Colombia. Vancouver, BC V6C 1X8.

Ecker, M. and Gelfand, A. (1999). Bayesian modeling and inference for anisotropic spatial data. *Mathematical Geology*. 31(1): 67-83.

Eldeiry, A. and Garcia, L. (2009). *Comparison of Regression Kriging and Cokriging Techniques to Estimate Soil Salinity Using Landsat Image*. Department of Civil and Environmental Engineering, ColoradoState University Fort Collins, CO 80523-1372. Hydrology Days 27.

Fewell, M. P. (1995). The atomic nuclide with the highest mean binding energy. *American Journal of Physics*. 63(7): 653–58.

Fisher, R. A. (1935). The logic of inductive inference, *Journal of the Royal Statistical Society*. 98, 39-54.

Fisher, R. A. and Yates, F. (1938). Statistical Tables for Biological, Agricultural and Medical Research, (5th edition, 1957). Edinburgh: Oliver and Boyd.

Fleming, K. L. (2000). Evaluating farmer defined management zone maps for variable rate fertilizer application. *Precis. Agric*. 2, 201–215.

Genton, M. G. (1998). Highly robust variogram estimation.Mathematical. Geology, *Geoderma*. 106, 173-190.

Grauso, S. Diodato, N. and Verrubbi, V. (2010). Calibrating a rainfall erosivity assessment model atregional scale in Mediterranean area. *Environ Earth Sci.* 60, 1597-1606.

Goovaerts, P. (1997a). *Geostatistics for natural resources evaluation*. Oxford University Press, New York.

Goovaerts, P. (1997b). Kriging with a trend model In: Geostatistics for natural resources evaluation. Edited by *A.G. Journal,* Oxford University Press, New York. 139-152.

Goovaerts, P. (1997c). Visualization of spatial uncertainty. In: Geostatistics for natural resources evaluation, edited by *A.G. Journal*, 431-434, Oxford University Press, New York.

Goovaerts, P. (1997d). The practice of modeling. In: Geostatistics for natural resources evaluation, edited by *A.G. Journal*. 97-107, Oxford University Press, New York.

Goovaerts, P. (2000). Geostatistical approaches for incorporating elevation into the spatial interpolation of rainfall. *Journal of Hydrology* 228, 113–129.

Gorsich, D. J. and Genton, M. G. (2000). Variogram Model Selection via Nonparametric Derivative Estimation. *Cambridge, Massachusetts Mathematical Geology*. 32 (3): 249-270.

Hambidge, K. M. and Krebs, N. F. (2007). Zinc deficiency: a special challenge. *J. Nutr*. 137 (4): 1101–5.

Handcock, M. S. and Stein, M. L. (1993). *A* Bayesian analysis of kriging. *Technometrics.* 35(4): 403-410

Hengl, T. Heuvelink, Q. B. M. and Stein, A. (2003b). Comparison of kriging with external drift and regression kriging technical note. ITC, <http//www.itc.nl/library/Academic-output/ (5 Dec 2011)>.

Hengl, T. (2007). A practical guide to geospatial mapping of environmental variables. JRC, Scientific and Technical Research series, office for official Publications of the European Communities, Luxembourg, page 143.

Hevesi, J. Istok, J. and Flint, A. (1992). Precipitation estimation in mountainous terrain using Multivariate geostatistics: P1, *J. application of Meteorology*. 31, 661-688.

Hofstra, N. Haylock, M. New, M. Jones, P. and Frei, C. (2008). Comparison of six methods for the interpolation of daily European climate dat. *Journal of Geophysical Research* 113, D21110, doi10.1029/2008JD010100.

Huijbregts, C. J. and Matheron, G. (1970). Universal kriging an optimal an approach to trend surface analysis.special volume CIMM Montreal. 12, 159-169.

Isaaks, E. H. and Srivastava, R. M. (1989). *An introduction to applied geostatistics*. Oxford University Press, 561 P. N.Y.

Johansson, B. (2000). Precipitation and temperature in the HBV model: A comparison of interpolation methods, SMHI Reports. Hydrology Gostatistics: Models and tools for the earth sciences, *Mathematical Geology*. 18, 119-140.

Johansson, B. and Chen, D. (2005). The influence of wind and topography on precipitation distribution in Sweden: Statistical analysis and modeling, International Journal of Climatology. 23, 1523-1535.

John, A. and William, A. (2010). *Assessment report: Soil Geochemical survey and Core Dilling on Krof property* .British, Columbia,Canada NAD83Zone10N Map: 092H052, MinFile: 092HNW070.

John, A. (2012). Breakaway Resources Limited**,** Annual Report for the Year Ended 30 June 2012, ABN 16 061 595 051

Johnston, K. VerHoef, J. M. Krivoruchko, K. and Lucus, N. (2001). *Using Arc GIS Geostatistical Analyst.* Redlands, ESRI, 300.

Journel, A. (1992). Geostatistics models and tools for the earth science. *Mathematical Geology*. 18. 119-140.

Journel, A. and Huijbregts, C. J. (1978). *Mining Geostatistics*. Academic press, London.

Kim, D. Park, J. and Joo, Y. (2005). Fuzzy Classifier with Bayes Consequent. 3809, 1130-1133.

Khorsandi, N. Mahdian, M. Pazira, E. Nikkami, D. and Chamheridar, (2012). Spatial variability of imprecise values of rainfall Erosivity index, *world applied science journal*, ISSN. 18(2): 243-250.

Krebs, R. E. (2006). *The history and use of our earth's chemical elements: A reference guide* (2 ed.). *Greenwood Publishing Group*. Page 107.

Krige, D. G. (1951). A statistical approach to some basic mine valuation problems on the Witwatersrand. *Journal of the Chemical*, Metallurgical and Mining Society of South Africa. 52, 119-139.

Kolmogorov, A. N. (1941). *The local structure of tutbulence in an incompressible fluid at every large Reynolds numbers*. Doklady Academii Nauk SSSR. 30, 301-05.

Kotas, J. and Stasicka, Z (2000). Chromium occurrence in the environment and methods of its speciation. *Environmental Pollution*. 107(3): 263–283.

Kuhlman, K. and Pardo, I. (2010). Universal cokriging of hydraulic accounting for boundary conditions, *Journal of hydrology*. 384, 14-25.

Kumar, V. and Ramadevi, *(2006*). Kriging of groundwater levels a case study; *Journal of Spatial Hydrology*. 6(1): 81–94.

Lacher, R. C. (1993). Expert networks paradigmatic conflict, technological rapprochement. *Minds and Machines.*

Lark, R. M. (2000a). Designing sampling grids from imprecise information on soil variability, an approach based on the fuzzy kriging variance. Bedford, Great Britain. *Geoderma*. 98, 35-59.

Lark, R. M. (2000b). Ageostatistical extension of the sectioning procedure for handling differences of scale between functional models of soil processes and information on one or more input variables. *Geoderma.* 95, 89–112.

Lark, R. M. (2000c). A comparison of some robust estimators of the variogram for use in soil survey. *European Journal of Soil Science*. 51, 137–157.

Lark, R. M. (2008). Some results on the spatial breakdown point of robust point estimates of the variogram. *Mathematical Geosciences*. 40, 729–751.

Lark, R. M. Culis, B. R. and Welham, S. J. (2006). On spatial prediction of soil properties in the presence of a spatial rend: the empirical best linear unbiased predictor. *European Journal of Soil Science*. 57, 787-799.

Lam, N. (1983). Spatial interpolation methods. The American Cartographer. 10, 129-149.

Liao, H. Chen, W. and Seinfeld, J. H. (2006). Role of climate change in global predictions of future tropospheric ozone and aerosols. *Journal of Geophysical Research*. 111(1): 1-4.

Lioyd, C. D. (2005). Assessing the effect of integrating elevation data into the estimation of monthly precipitation in Great Britain. *Journal of Hydrology*. 308, 128–150.

Lopez, F. Jurado, M., Pena, J. and Garcia, L. (2005). Using geostatistical and remote sensing approaches for mapping soil properties. *European Journal of Agronomy*. 23(3): 279-289.

Mabit, L. Bernard, C. Makhlouf, M. and Laverdiere, M. R. (2008). Spatial variability of erosion and soil organic matter content estimated from measurements and geostatistics. *Geoderma.* 145(3): 245-251.

Marchant, B. P. Saby, N. Jolivet, C. Arrouays, D. and Lark, R. (2011). Spatial prediction of soil properties with copulas. *Geoderma.* 162, 327–334.

Matérn, B. (1986). *Spatial Variation*. (2nd Edition). *Springer-Verlag*, Berlin, Heidelberg, New York, London, Paris, Tokyo.

Matheron, G. (1963). Principles of Geostatistics, *Journal of Economic Geology*. 58, 1246–1266.

Matheron, G. (1971). *The Theory of Regionalised Variables and its Applications*: Les Cahiers du Centre de Morphologie Math´ematique de Fontainebleau, No. 5, Paris, France.

Matheron, G. (1973). The intrinsic random functions and their application. *Journal of Advance Application Probability*. 5, 439-68.

Mark, F. and Peter K.(1997). A geostatistical approach to contaminant source identification Department of Civil Engineering, Stanford, California, *Water Resource Management*. 33(4): 537-546.

Martínez, A. (1995). Estimation of mean annual precipitation as affected by elevation using multivariate geostatistics, *Water Resource Management*. 9, 139–159.

Martinez, L. J. and Zinck, J. A. (2004). Temporal variation of soil compaction and deterioration of soil quality in pasture areas of Colombian Amazonia. *Soil and Tillage Research*. 75, 3–17.

McBratney, A. B., Webster, R. Burgess, T. M. (1981). The design of optimal sampling schemes for local estimation and mapping of regionalised variables. *Computers and Geosciences*. 7, 331–334.

Meyers, D. (1994). Spatial interpolation: an overview. *Geoderma*. 62, 17-28.

Miller, J. Franklin, J. and Aspinall, R. (2007). *Incorporating spatial dependence in predictive vegetation models. Ecol. Modelling*. 202(3):225-242.

Mingoti and Rosa, G. (2008). A note on robust and non-robust variogram estimators. *Universidad Federal de Minas Gerais*. 61(1): 370-4467.

Mueller, T. G. and Pierce, F. (2003). Soil carbon maps:Enhancing spatial estimates with simple terrain attributes at multiple scales soil. *Am. Journal*. 67, 258-267.

Muller, W. G. and Zimmerman, D. L. (1999). Optimal designs for variogram estimation. *Environ metrics*. 10, 23–37.

Myers, D. E. (1982). Matrix formulation of co-kriging. *Mathematical geology*. 14, 249-257.

Myers, D. E. (1991). Interpolation and estimation with spatially located data. *Lab. Syst.* 11: 209-228.

Nielsen, A. A. (2004). *Kriging*, Builduy 321, Dk-2.800, Denmark, page 2.

Oman, H. (1992). Not invented here? Check your history. *Aerospace and Electronic Systems Magazine*. **7** (1): 51–53.

Omre, and Halvorsen, K. B. (1989) The Bayesian bridge between simple and universal kriging, *Mathematical geology*. 21, 767-786.

Pardo, E. (1997). A computer program for the inference of spatial covariance parameters by maximum likelihood and restricted maximum likelihood. Computers and Geosciences. 23, 153–162.

Pardo, E. and Chica, M. (2004). Estimation of gradients from sparse data byuniversal kriging. University of Granada, Granada,Water Resources, 40: 17.

Pardo, E. Dowd, P. A. (1998). Maximum likelihood inference of spatial covariance parameters of soil properties. Soil Science. 163, 212–219.

Philip, R. and Kitanidis, P.K. (1989). Geostatistical Estimation of Hydraulic Head Gradients Ground Water. 27(6): 855–865.

Prudhomme, C. and Reed, D. W. (1999). Mapping extreme rainfall in a mountainous regionusing geostatistical technique: A case study in Scotland. *International Journal of Climatology*. 19, 1337-1356.

Ripley, B. D. (1981). *Spatial statistics*, New York, John Wiley.

Robertson, G. P. (1987). Geostatistics in ecology: Interpolating with known variance. Ecology. 68(3): 744-748.

Rudolf, D. (2003). *Introduction to geostatistics and variogram analysis.* Vienna University of Technology.

Schabenberger, O. and Gotway, C. A. (2005). *Statistical Methods for Spatial Data Analysis Texts in Statistical Science.* Chapman and Hall.

Shilton, S. Trow, J. Hii, V. and Archer, N. (2005). *Accuracy implications of using the WG-AEN Good practice* Guide Toolkits. Forum Acusticum.

Simbahan, G. Dobermann, A. Goovaerts, J. P. and Haddix, M. L. (2006). Fine-resolution mapping of soil organic carbon based on multivariate secondary data. *Geoderma*. 132, 471-489.

Smith, William F. and Hashemi, Javad (2003). *Foundations of Materials Science and Engineering*. McGraw-Hill Professional. Page 223.

Stein, M. and Handcock, M. (1989). Some Asymptotic Properties of kriging with the covariance is Misspecified, *Math. Geology*. 21(2): 171-189.

Stein, M. L. (1999). Interpolation of Spatial Data: Some Theory for kriging. *Springer*. New York.

Student, Mercer, E. B. and Hall, A.D. (1911). Appendix to the experimental error of field trials. *Journal of Agricultural science*. 4, 107-32.

Sumfleth, K. and Duttmann, R. (2008). Prediction of soil property distribution in panddy soillandscapes using terrain data and satellite information as indecators. *Ecological indicators*. 8(5): 485-502.

Thurmer, K.; Williams, E; Reutt-Robey, J. (2002). Autocatalytic Oxidation of Lead Crystallite Surfaces. *Science*. 297 (5589): 2033–5.

Tveito, O. E. and Schöner, W. (2002). *Application of spatial interpolation of climatological and meteorological elements by the use of geographical information systems (GIS*), No. 28.

Wackernagel, H. (2003). *Multivariate Geostatistcs: An introduction with application. Springer*, Varlg Berlin, Germany, page 256.

Webster, R. and Oliver, M. A. (2007). *Geostatistics for Environmental Scientists,* Statistics in Practice Series, John Wiley and Son, page 315.

Webster, R. and Oliver, M.A. (2001a). *Reliability of the experimental variogram. In:Geostatistics for Environmental Scientists*, John Wiley and Sons, Chichester. edited by V. Barnett, 78-92.

Webster, R. and Oliver, M. A. (2001b). *Authorized models. In: Geostatistics for Environmental Scientists,* John Wiley and Sons, Chichester. edited by V. Barnett.

Whittle, P. (1956). On the variation of yield variance with plot size. *Biometrika* 43, 337-343.

Willbold, M.; Elliott, T.; Moorbath, S. (2011). The tungsten isotopic composition of the Earth's mantle before the terminal bombardment. *Nature*. 477 (7363): 195–198.

Wim, C. M. and Jack, P.C. Kleijnen (2002). *Kriging for Interpolation in Random Simulation* Department of Information Management. Tilburg University (KUB), Netherlands, Center for Economic Research.

Yates, F. and Cochran, W. G. (1938). The analysis of groups of experiments. *Journal of Agricultural Science*. 28, 556-580.

Yeh, T. C. Gutjahr, A. L. and Jin, M. (1995). An iterative cokriging-like technique for groundwater flow modelling. *Ground Water*. 33(1): 33–41.

Zadeh, L. (1975). *The Calculus of Fuzzy Restrictions in Fuzzy Sets Applications to Cognitive and Decision Making Processes*, edited by Academic Press, New York, 1-39.

Zhang, J. and Goodchild, M. (2002). *Uncertainty in Geographical Information*. Taylor and Francis, London.

Zhang, R. Myers, D. and Warrick, A. (1992). Estimation of the spatial distribution of the soil chemicals using pseudo cross-variograms. Soil Sci. Soc. *Am. J*ournal. 56, 1444–1452.

Zhu, Q. and Lin, H.S. (2010). C*omparing ordinary kriging and regression kriging for soil proportion in contrasting land scapes.* Pedospheve. 20: 594-606.

Zidek, J. V. and Lee, N. D. (1999). Interpolation with Uncertain spatial covariance: A Bayesian alterative to kriging. *Journal of Multivariate Analysis.* 43(2): 351-374.

Zimmerman, D. and Cressie, N. (1992). Mean square prediction error in the spatial linear model with estimated covariance parameters. 44, 27-43.

Zimmerman, D. Pavlik, C., Ruggles, A. and Armstrong, M. (1999). An experimental comparison of ordinary and universal kriging and inverse distance weighting. *Journal of Mathematical Geology*. 31(4): 376-389.