IRREDUCIBLE REPRESENTATIONS OF SOME FINITE GROUPS AND GALOIS STABILITY OF INTEGRAL REPRESENTATIONS

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To my beloved parents,

Yahya bin Mat Isa & Pahlawi binti Hassim

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ABSTRACT

Representation theory is a branch of mathematics that studies properties of abstract groups via their representations as linear transformations of vector spaces. A representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in terms of matrix addition and matrix multiplication. In this research, representations of certain finite groups are presented. The aims of this research are to find the matrix representations of dihedral groups, irreducible representations of dihedral groups and to relate the representations obtained with their isomorphic point groups. For these purposes, some theorems presented by previous researches on the representations of groups have been used. The fact that isomorphic groups have the same properties has also been applied in this research. Another part of this research is to explore on the representations of finite groups over algebraic number fields and their orders under field extension. Thus, this research also aims to prove the existence of abelian Galois stable subgroups under certain restriction of field extensions. The concept of generalized permutation modules has been used to determine the structure of the groups and their realization fields. Matrix representations of dihedral groups of degree six and the irreducible representations of all point groups which are isomorphic to the dihedral groups have been constructed. The existence of abelian Galois stable subgroups under certain restriction of field extensions have also been proven in this research.

ABSTRAK

Teori perwakilan adalah satu cabang matematik mengenai ciri-ciri kumpulan abstrak melalui perwakilan mereka sebagai transformasi linear dalam ruang vektor. Satu perwakilan menjadikan sesuatu objek aljabar yang abstrak lebih konkrit dengan menerangkan unsur-unsurnya dalam bentuk matriks dan operasi aljabarnya dalam bentuk penambahan matriks dan pendaraban matriks. Dalam kajian ini, perwakilan untuk beberapa kumpulan terhingga dibentangkan. Tujuan kajian ini dijalankan adalah untuk mencari perwakilan matrik kumpulan dwihedron, perwakilan tereduksi kumpulan dwihedron dan untuk mengaitkan perwakilan yang diperolehi dengan kumpulan titik yang isomorfik dengan kumpulan dwihedron. Untuk tujuan ini, beberapa teori daripada kajian-kajian sebelum ini mengenai perwakilan kumpulan telah digunakan dalam kajian ini. Fakta yang mengatakan kumpulan isomorfik mempunyai ciri-ciri yang sama juga telah diaplikasikan. Sebahagian lain kajian ini ialah untuk meneroka perwakilan kumpulan terhingga dalam medan nombor aljabar dan peringkat mereka di bawah bidang perluasannya. Justeru, kajian ini juga bertujuan untuk membuktikan kewujudan subkumpulan abelan stabil Galois di bawah bidang perluasannya dengan syarat tertentu. Konsep pilih atur modul teritlak telah digunakan untuk menentukan struktur kumpulan dan bidang realisasinya. Perwakilan matrik untuk kumpulan dwihedron dengan darjah enam dan perwakilan tereduksi untuk semua kumpulan titik yang isomorfik kepada kumpulan dwihedron telah dibina. Kewujudan subkumpulan abelan stabil Galois di bawah bidang perluasannya dengan syarat tertentu juga telah dibuktikan dalam kajian ini.

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LIST OF SYMBOLS

$A(n)$	-	Alternating group of degree n
S	-	Dedekind rings
$[E:F]$	-	Degree of the field extension E/F
$\det M$	-	Determinant of a matrix M
D_n	-	Dihedral group
$dim_K A$	-	Dimension of K-algebra A over the field K.
$(V:K)$	-	Dimension of V over K
$\mathrm{discr}_{E/F}$	-	Discriminant of E/F
$\in$	-	Element of
$\sigma, \lambda \in \Gamma$	-	Elements of Γ
E/F	-	Field extension $F \subseteq E$
$F(G)$	-	Field obtained by adjoining to F all matrix coefficients of all matrices $g \in G$ for G is a finite linear group.
E, F, K, L	-	Fields
$\mathbb{C}$	-	Fields of complex numbers
$\mathbb{Q}$	-	Fields of rational numbers
$\mathbb{R}$	-	Fields of real numbers
G, H	-	Finite groups
$M_n(S)$	-	Full matrix algebra over a ring S
$Gal(E/F)$	-	Galois group of E/F
Γ	-	Galois groups
Γ-stable	-	Galois stable

$\phi_K(t) = [K(\zeta_t) : K]$	-	Generalized Euler function for a field K
$GL_n(K)$	-	General linear group of degree n over the field K
$GL_n(S)$	-	General linear group over a ring S
$>$	-	Greater than
$\geq$	-	Greater than or equal to
$\langle a, b \ldots \rangle$	-	Group generated by $a, b, \ldots$
$GL(V)$	-	Group of all invertible transformations of a vector space V onto itself
$\mathbb{Z}_n$	-	Groups under addition modulo n
$\mathbb{Z}_{2n}$	-	Groups under addition modulo $2n$
ε_i	-	Idempotents
1, I	-	Identity element in a group
id	-	Identity functions
1_V	-	Identity mapping on V
$1_\wedge$	-	Identity of exterior square
g^σ	-	Image of g under σ-operation, where σ element of Galois group and g element of any group
$\cap$	-	Intersection
$\cong$	-	Isomorphic
ϕ	-	Isomorphism mapping
$<$	-	Less than
$\leq$	-	Less than or equal to
O_K	-	Maximal order of a number field K
$\notin$	-	Not element of
$\neq$	-	Not equal to
$\|G\|$	-	Order of a group G
ζ_t	-	Primitive t-root of 1
$\subset$	-	Proper subset
T	-	Representation of a group G

$f\|_s$	-	Restriction of f to S
$\mathbb{Z}$	-	Rings of rational integers
$\subseteq$	-	Subset
$S(n)$, Σ_n	-	Symmetric group of degree n
$Tr_{E/F}$	-	Trace of E/F
I_m	-	Unit $m \times m$-matrix
V	-	Vector space
$\wedge$	-	Wedge product
$\{0\}$	-	Zero subspace

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Representation theory is a mathematical topic that studies properties of abstract groups via their representations as linear transformations of vector spaces. Representation theory is important because it enables many group-theoretic problems to be reduced to problems in linear algebra which is a very well-understood theory. The term representation of a group is also used in a more general sense to mean any description of a group as a group of transformations of some mathematical object.

In the past century, this theory has been developed in many interesting directions. Besides being a natural tool in the study of the structure of finite groups, it has turned up in many branches of mathematics such as probability and number theory and also extensive applications in chemistry such as atoms and molecules and physics such as elementary particles have been found.

A matrix representation of degree n of a finite group G over a field F is a homomorphism mapping from the group G into the general linear group, $GL_n(F)$. The representation is said to be faithful if the homomorphism is injective. Thus a faithful representation identifies G with some group of $n \times n$ matrices. The matrix representation is a type of representation that is easy to understand but requires tedious calculations in obtaining the possible matrix representations. Dihedral groups are among the simplest example of finite groups and they play an important role in group theory, geometry and chemistry. Thus, at the beginning of this research, matrix representations of dihedral groups of certain order are given.

A subrepresentation of a representation T is a subspace $U \subset T$ which is invariant under all operators in a vector space V. A nonzero representation T of V is said to be irreducible if its only subsepresentations are 0 and T itself. The irreducible representations are important and useful because many properties of a group can be obtained from its irreducible representations. The irreducible representations of dihedral groups and their isomorphic point groups are provided in this research.

Another interesting part of representations of finite groups is by considering Galois action on these representations. The structure of the representations and their realization fields are studied in this research. It is actually motivated from the work of Ritter and Weiss [1] who have done the research on Galois action on integral representations.

1.2 Research Background

Representations of dihedral groups are of great interest to many researchers since many years before until now. Various studies on representations of dihedral groups have been done. In 1942, Miller [2] studied on the automorphism of the dihedral groups. Meanwhile, Lee [3] in 1964 discovered the integral representations of dihedral groups. Charlap and Vasquez [4] studied multiplication of integral representations of some dihedral groups in 1979. Three years later, Wang [5] developed an effective method to transform group matrices for generalized dihedral group to a useful block-diagonal form.

Aguade $et\ al.$ [6] have also done the research on the integral representations of the infinite dihedral group in 2002. In 2010, Ishguro [7] studied the invariant rings and dual representations of dihedral groups whereas the invariant of the dihedral group, D_{2p} in characteristic two where p is an odd prime was presented by Kohls and Sezer [8] in 2012. The representations of quantum double of dihedral groups had been done by Dong and Chen [9] in 2013. From these studies, current researchers are motivated to find matrix representations of certain dihedral groups.

The irreducible representations of dihedral groups have been obtained from the previous study by Galin [10] in 2007. In his research, some theorems on irreducible representations of dihedral groups are presented. The theorems have been used in this research to find irreducible representations of their isomorphic point groups.

1n 1992, Ritter and Weiss [1] studied Galois action on integral representations. From this study, the relationship between the representations over fields K and some related representations over Dedekind rings S in K is interesting to be studied. As well as establishing extra properties of these representations, in this research, it is interesting to look at the property of stability of the representations under the natural action of Galois group. This condition was considered earlier for Galois stability of groups and orders in [1] and some other papers.

1.3 Motivation of the Study

The irreducible representations of groups are important to be studied. There are few methods to find irreducible representations of a group such as Burnside method and Orthogonality Theorem method. Finding irreducible representations by using these two methods require tedious calculations. Thus, in this research, different approach has been used to obtain the irreducible representations of groups. In this research, by using the concept of isomorphism, the irreducible representations of some point groups are obtained from the theorem on irreducible representations of dihedral group by Galin [10].

The property of stability of representations was considered earlier for Galois stability of groups and orders of the groups in [1] and some other papers. In previous studies, many results are related to representations of abelian groups. It would be interesting to consider some other classes of groups. Thus, in this research, some arithmetic problems for representations of finite groups over algebraic number fields and arithmetic rings under the ground field extensions are studied.

1.4 Problem Statement

The irreducible representations of dihedral groups have been found by previous researchers. As a continuation, this research aims to find the matrix representations of dihedral groups and the irreducible representations of the point groups which are isomorphic to them. The Galois action on integral representations has been explored by previous researchers for Galois stability of groups and orders. Thus, in this research the existence of abelian Galois stable subgroups under certain restrictions are proven. The interplay between the Galois stable groups G over algebraic number fields and over rings of integers is also explored in this research.

1.5 Research Objectives

The objectives of this research are:

(i) to determine matrix representations of certain dihedral groups,

(ii) to find the irreducible representations of point groups which are isomorphic to dihedral groups,

(iii) to prove the existence of abelian Γ-stable subgroups G under certain restrictions of field extensions,

(iv) to explore the interplay between the Γ-stable groups G over algebraic number fields and over rings of integers.

1.6 Scope of the Study

This research focuses on matrix representations of dihedral groups, irreducible representations of dihedral groups and point groups, representations of finite groups over algebraic fields and their orders under ground field extensions.

1.7 Significance of Findings

This research gives new results in the field of representation theory. The benefit of the results are far reaching. The result obtained can be used for further research in this field. In this research, detailed calculations to determine the matrix representations of a finite group are given, which will be very useful as a practice for new researchers in the field of representation theory. For example, one can use the same method to find matrix representations of dihedral groups of order 14 and above.

This research also presents an example on how representations can become a very useful applications. For instance, the irreducible representations of certain groups can be used to find irreducible representations of their isomorphic groups. Furthermore, this research studies arithmetic problems for representations of finite groups in different context from previous researches. Thus, the results will enhance more ideas and researches in representations of finite groups.

1.8 Research Methodology

In determining the matrix representations of dihedral groups, similar concept shown in Curtis [11] has been used in this research. The irreducible representations of dihedral groups has been obtained by Galin in [10]. The results of his research are used to find irreducible representations of certain point groups which are isomorphic to the dihedral groups. This research also used some definitions and theorems from previous results by Bartels and Malinin [12], Roggenkamp [13] and Weiss [14] in determining the structure of groups G and their realization fields where G is a subset of the general linear group, $GL_n(E)$. In this research, G is assumed to be a subgroup stable under the natural operation of the Galois group of field extension E/F. The concept of generalized permutation modules has also been used in this research. The research framework are given in Figure 1.1 on page 8.

1.9 Thesis Organization

This thesis is divided into five chapters, starting with the introduction chapter. It then continues with literature review, representations of dihedral groups and certain point groups, representations of some finite groups and Galois stability and lastly, conclusion (refer Figure 1.2).

The first chapter serves as an introduction to the whole thesis including the research background, problem statement, research objectives, scope of the study, significance of findings and research methodology. This chapter also gives some concepts on finite groups, some useful representations, the structure of representations and their realization fields.

The literature review of this research is presented in Chapter 2. Various works from previous researchers are listed in this chapter. The definitions and theorems which have been used in obtaining the results of this research are also stated in Chapter 2.

Chapter 3 presents representations of dihedral groups and certain point groups. The chapter gives the matrix representations of dihedral group of order 12 in the first section. In the next section of this chapter, irreducible representations of point groups which are isomorphic to dihedral groups are given.

Chapter 4 discusses on representations of some finite groups and Galois stability. Firstly, this chapter provides the structure of groups G and their realization fields where G, a subset of the general linear group of a field E, $GL_n(E)$, is assumed to be a subgroup stable under the natural operation of the Galois group with field extension E/F. It then compares the possible realization fields of G in the cases if G is a subset of the general linear group, $GL_n(E)$ and if all coefficients of matrices in G are algebraic integers.

Finally, the summary and conclusion of this research are stated in Chapter 5. Some useful suggestions and ideas for future research on representations of finite groups are also given in this chapter.

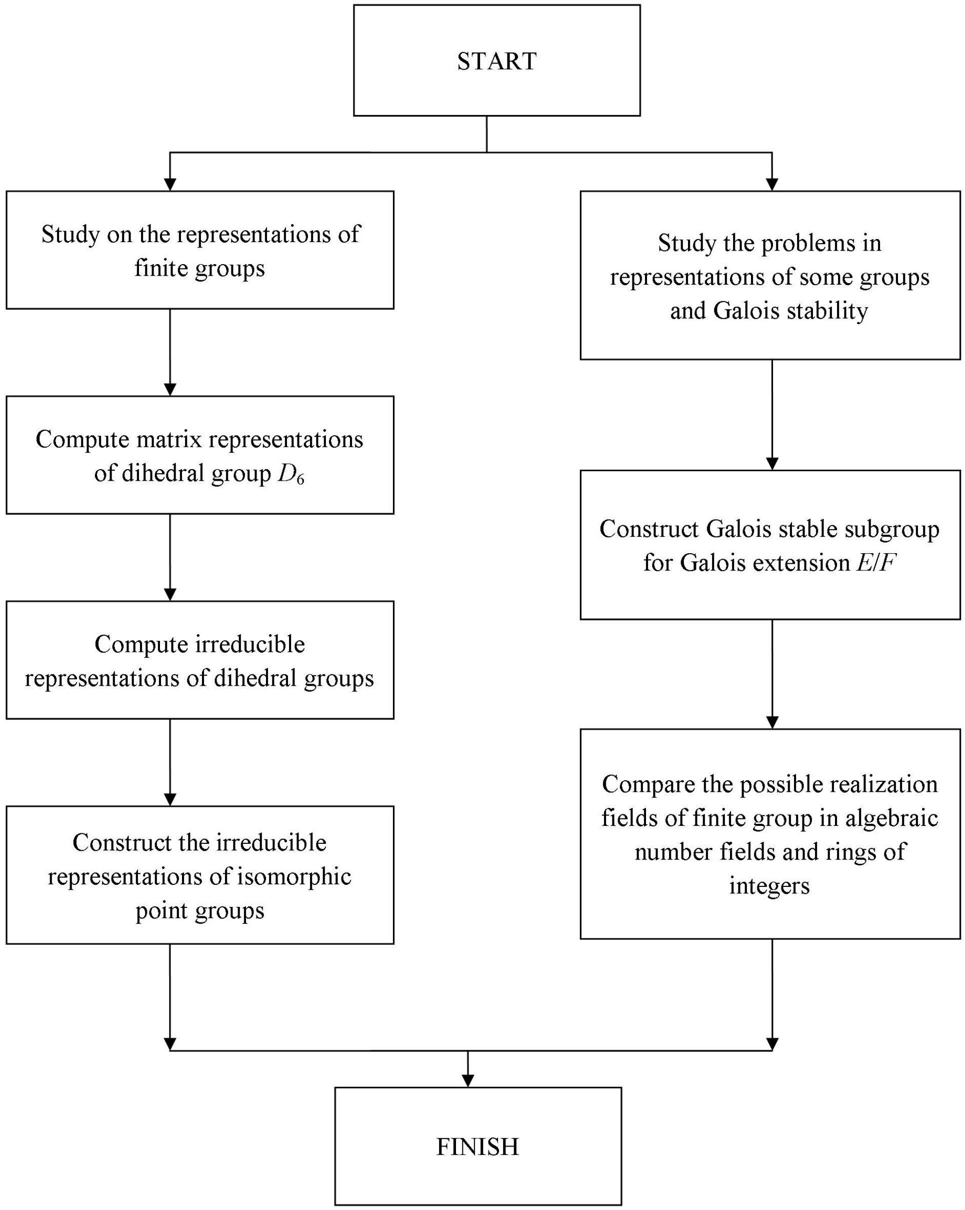

Figure 1.1 Research framework

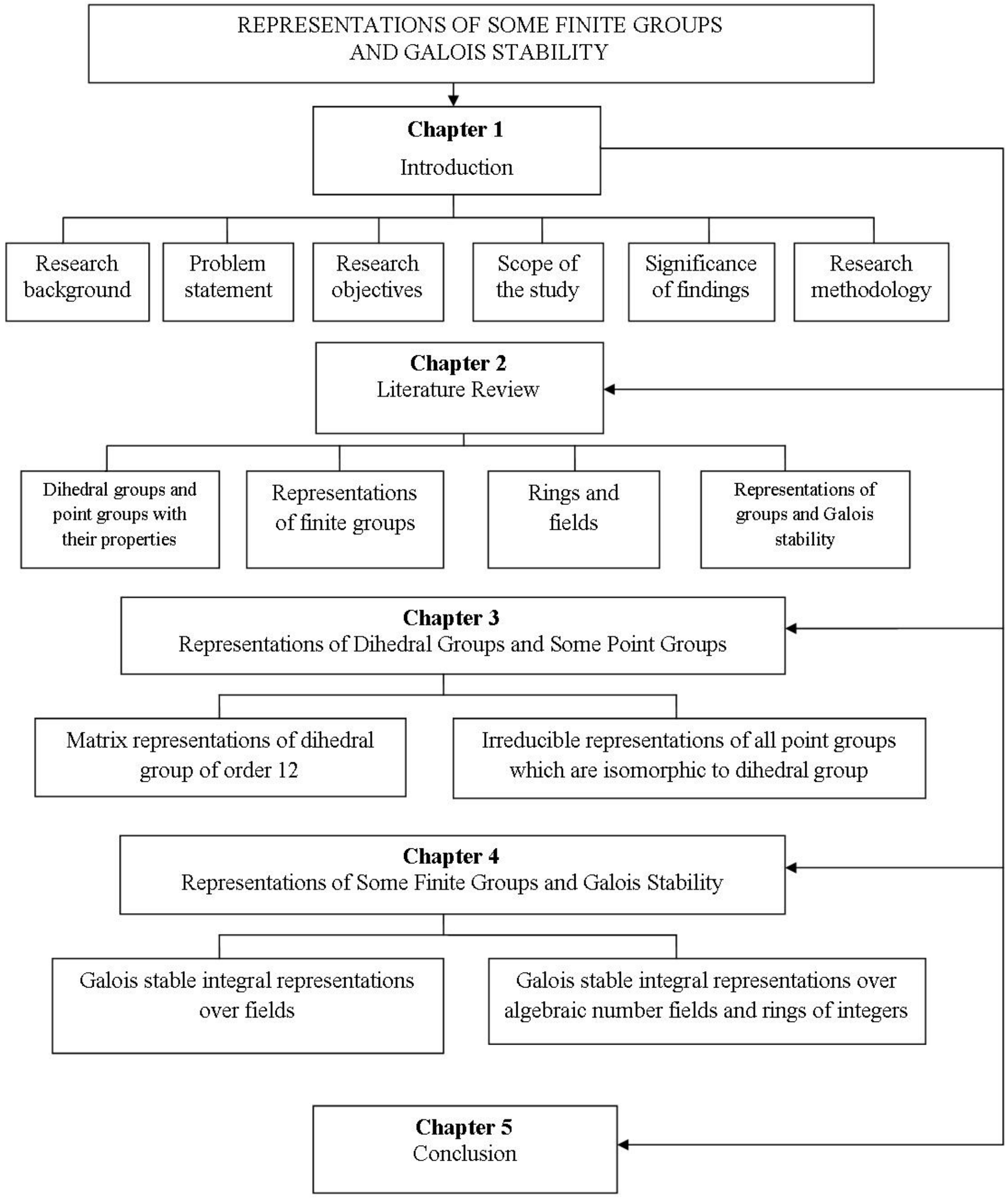

Figure 1.2 Thesis organization

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