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A simulation-based product diffusion forecasting method using geometric Brownian motion and spline interpolation

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Abstract: This study addresses the problem of stochasticity in forecasting diffusion of a new product with scarce historical data. Demand uncertainties are calibrated using a geometric Brownian motion (GBM) process. The spline interpolation (SI) method and curve fitting process have been utilized to obtain parameters of the constructed GBM-based differential equation over the product's life cycle (PLC). The constructed stochastic differential equation is coded as the forecast model and is simulated using MATLAB. The results are several sample demand paths generated from simulation of the forecast model. To evaluate the forecasting performance of the proposed method it is compared with Holt's model, using actual data from the semiconductor industry. The comparison results confirm the applicability of the proposed method in the semiconductor industry. The method can be helpful for policy-makers who require the prediction of uncertain demand over a time horizon, such as decisions associated with aggregate production planning, capacity planning, and

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PUBLIC INTEREST STATEMENT

Nowadays, the success of innovative industries relies on how they encounter certain challenges and arrange activities in order to meet customer requirements. In such situations, accurate forecasting of product demand and its penetration to market are very crucial in providing sufficient amounts of required resources at the right time and the right place. Since for new and innovative products, planners are faced with the rare or very short history of data, forecasting is a very difficult task. Particularly, the available forecasting methods' performance highly relies on the availability of historical sales data. Therefore, developing a forecast method for a new product in its pre-launch phase which is not exclusively based on historical sales data is very crucial. This study applies mathematical concepts together with engineering approaches to propose a forecast method which is able to predict penetration of a new product to an uncertain market without exclusively relying on historical sales data.

supply chain network design. Especially for the semiconductor industry with intensive capital investment the proposed approach can be useful for making decisions associated with capacity allocation and expansion.

Subjects: Operations Research; Mathematics & Statistics for Engineers; Engineering Management; Customer Relationship Management (CRM); Supply Chain Management

Keywords: demand forecast; stochastic differential equation; simulation; uncertainties; GBM; interpolation

1. Introduction

Nowadays, the success of innovative industries relies on how they encounter certain challenges and arrange activities in order to meet customer requirements. In such situations, accurate forecasting of product demand and its diffusion into the market are very crucial in providing sufficient amounts of required resources at the right time and the right place. As pointed out by Chien, Chen, and Peng (2010) demand forecasting provides a basis for strategic decisions, such as technology selection, tool purchasing, outsourcing, and capacity expansion.

For new products, demand forecasting acts as a foundation for all supply chain planning activities (Chopra & Meindl, 2007). Companies are interested in higher accuracy levels of demand forecasting due to its direct effect on future profit (Cheng, Chen, & Wu, 2009). Sales managers of many innovative products attempt to understand new product diffusion by moving toward using certain analysis tools and acquiring professional knowledge. Based on Scitovski and Meler (2002), diffusion is defined as the process through which an innovation is accepted by the market. The theory of diffusion has shaped a scientific basis for studying the Product Life Cycle (PLC) phenomenon. The focus of many developed diffusion models is around the development and evolution of the associated PLC curve (Mahajan, Muller, & Bass, 1990). Knowing the product diffusion characteristics and PLC features helps firms develop appropriate contingency plans in order to respond to possible future changes in market demand. Therefore, the dynamic and uncertain nature of demand through the PLC, particularly for newly innovated products, deserves more attention.

The shape of the PLC as a model of product diffusion is, to a great extent, affected by a multitude of variables (Scitovski & Meler, 2002). Qin and Nembhard (2012) suggested there are two general modeling approaches for formulating product diffusion through the PLC, namely the Bass model and the geometric Brownian motion (GBM) model. The standard Bass model operates entirely on a macro-variable level in the total number of adopters and non-adopters (Chen & Chen, 2007). On the other hand, several stochastic diffusion models have been suggested that utilize the GBM process to incorporate influential factors that may not always be transparent. Although the developed GBM-based models are meant to capture the uncertainty of demand, they may fail to address the inherent dynamism and possible fundamental changes in the PLC. The reason can be the fact that a constant drift rate of demand (considered in most relevant studies conducted) is not able to reflect the complexity associated with the growth of demand through the PLC. In a new study, Qin and Nembhard (2012) tried to deal with this matter by considering a linear model for demand drift. However, for products with different growth paces on the PLC, a linear growth pattern may not address the complex behavior of demand. This concern intensifies when dealing with demand forecasting of a new or pre-launched product, which has no data or a short history of sales (demand). Regarding these types of products, Lee, Kim, Park, and Kang (2014) suggested a machine learning and statistical-based approach utilizing the Bass model. However, their study mostly focused on estimating the deterministic parameters of the Bass model and failed to reflect on the possible stochastic nature of demand. To be summarized, there are three main shortcomings in the extant literature which are to be addressed by this study. First, the available forecasting models do not address new products in pre-launch phases which are suffering from short history of sales data. Second, the existent forecasting methods need involvement of uncertainty causing the drift from the standard bell-shaped PLC pattern which is mostly considered in the extant literature. Finally, the

third shortcoming is referred to the lack of an analytical approach to see how moving from linear growth to upper degree polynomials can affect the accuracy of forecast results? To fill the aforementioned gaps, the aims of this study are to:

- Suggest a pre-launch forecasting method for a new product with scarce or no historical data.
- Address possible uncertainties in demand and generate possible trajectories of future demand.
- Show the effect of considering different growth patterns on the accuracy of the proposed forecasting method.

The scope of this study is limited to the semiconductor industry. Nonetheless, the proposed approach may benefit all managers, practitioners, and researchers who strive to predict demand in uncertain and dynamic environments for new or pre-launched products. This study is organized over different sections. The following section provides a review of relevant literature. The applied methodology is presented in Section 3. The results and performance evaluation of the method are discussed in Section 4. Section 5 expresses the sensitivity analysis performed. Section 6 provides information about managerial implications of the proposed method, and Section 7 is dedicated to the conclusions of this study.

2. Literature review

This section comprises a review of the most related literature. The Bass model and its limitations is discussed in the first part and in the second part, the review covers some previous literature on the area of demand forecasting in uncertain environments.

2.1. The Bass model and its limitations

Since the introduction of the Bass model in 1969, it has been used for diffusion forecasting in many areas including industrial technologies, retail services, agriculture, education, pharmaceuticals, and consumer durable goods markets (Mahajan et al., 1990). The main concept of the Bass model is founded on the premise that the probability of a buyer purchasing a product is influenced by the number of previous consumers. The model is generated from the probability that a purchase will occur at time t , assuming that it has not happened before. Thus,

$$\frac{f(t)}{[1 - F(t)]} = p + qF(t) \quad (1)$$

The above is the basic proposition that forms the foundation of the Bass model, where $f(t)$ is the density function representing the probability of purchase by a potential consumer, $F(t)$ is the fraction corresponding to the cumulative consumers at time t (Mahajan et al., 1990) and p and q are the Bass model parameters. Therefore, the cumulative number of purchasers (adopters) over time can be obtained by solving the resulting nonlinear differential equation, where the values of the Bass model parameters, i.e. p and q , are available. Initially, the observational data of adoption rates for several consumer durable goods were fitted to the Bass model to obtain the cumulative S-shaped curve associated with the number of adopters up to a desired time (Laciana & Oteiza-Aguirre, 2014). However, since 1980, several estimation procedures have been proposed for estimating the Bass model parameters (Peres, Muller, & Mahajan, 2010). For instance, Schmittlein and Mahajan (1982) employed the maximum likelihood estimation method to estimate the parameters of the Bass model. Scitovski and Meler (2002) suggested a method for Bass model parameter estimation using the finite-difference method and the moving least squares method, in which the analytical solution of the differential equation associated with the cumulative number of adopters, is not required.

Although the Bass model is simple to use, its application is limited to similar known products (Laciana & Oteiza-Aguirre, 2014). In general, there are some matters that limit the application of the Bass model, such as the possibility of multiple purchases by an adopter (purchaser) being ignored or the lack of reflective marketing strategies or competitive structures in the model (Chen & Chen,

2007). The latter deficiency has led to the proposal of various expansions of the Bass model. The expanded models are intended to reflect the increasing complexity of new products' diffusion on the market caused by various influential factors. For instance, in 1990 Jain and Rao incorporated the price factor and proposed an expansion of the Bass model assuming the population of eventual purchasers as a function of price (Jain & Rao, 1990). In 1994, a generalized version of the Bass model called the generalized Bass model was developed to incorporate two decision variables, i.e. price and advertisement costs, into the diffusion model (Bass, Krishnan, & Jain, 1994). Cheng et al. (2009) used a trend-weight fuzzy time-series model to model growth diffusion. In a different study, Chien et al. (2010) considered product life cycle and applied technology diffusion theories in the proposed model. Laciana and Oteiza-Aguirre (2014) proposed a model for the diffusion of several products competing on a common market (Potts model). They showed how the topological changes in a social network influence the adoption process.

The Bass diffusion model has been expanded to be applied for different scopes. For instance, Chen and Chen (2007) applied the system dynamics method to the Bass model to be utilized for multi-period forecasting purposes. They also incorporated factors such as advertising, pricing and market competitors, assuming they have impact on the probability of purchase and consequently, on the coefficients of the Bass model. Another expansion of the Bass model was presented in a study performed by Peres et al. (2010). The model incorporates generic cross-country influence and was constructed based on the fact that the success of an innovation in one country is perceived as a signal by consumers in other countries who perceive lower levels of risk associated with consuming the new product.

Given the above descriptions along with Table 1, it can be concluded that the framework proposed by Bass in 1969 constitutes the main string of proposed diffusion models in the literature. However, most of the suggested analytical diffusion models are not able to address the entire complexity, dynamism, feedback loops and impact of factors that are not always transparent (Qin & Nembhard, 2012). Generally, as mentioned by Chen and Chen (2007), demand management requires complete

Table 1. Examples of models constructed for predicting product diffusion

Author(s) and year	Forecasting methodology	Stochastic or deterministic	Influential factors considered	Base of the model
Schmittlein and Mahajan (1982)	Maximum likelihood estimation	Deterministic	No additional factors	Bass
Scitovski and Meler (2002)	Finite difference method and the moving least squares method for data smoothing	Deterministic	No additional factors	Bass
Chen and Chen (2007)	System dynamics	Deterministic	Pricing, advertising, market competitors	Bass
Cheng et al. (2009)	Fuzzy logic relevant approach	Deterministic	No factor	A trend-weighted fuzzy time-series
Chien et al. (2010)	Nonlinear least squares method	Deterministic	Seasonal factors, market growth rate, price, repeat purchases and technological substitution	Bass
Laciana and Oteiza-Aguirre (2014)	Small world networks approach and simulation	Stochastic	The introduction of an additional parameter (temperature) for the quantification of the uncertainty in the decision process	Ising model of statistical mechanics (Potts model)

understanding of all factors that impact product diffusion rate. Ignoring the uncertain behavior of demand that originates from known or unknown influential factors has important managerial implications and may result in imprecise and erroneous capital investment decisions (Mahajan et al., 1990). This study is an attempt to address this gap and propose a demand forecasting method in which the stochastic behavior of demand and its drift through the PLC is taken into consideration. In 2004, Bass emphasized this drift should be considered as important as the classical PLC pattern (Bass, 2004).

2.2. Dealing with uncertainties in demand forecasting

Although some literature exists in response to the issue of demand forecasting in uncertain environment, the studies conducted in this area are quite limited. Scitovski and Meler (2002) tried to model uncertainty using an error factor with normal distribution with zero mean and constant variance over the entire time horizon. However, the constant variance assumed in the constructed model does not reflect the fact that as time elapses from the point of prediction, uncertainty would increase (Qin & Nembhard, 2012). Furthermore, in the majority of studies conducted, the uncertainty in demand is calibrated by generating random data as representative of the uncertain demand in the corresponding time period. The disadvantage of this approach is in ignoring the dynamism and pattern of demand growth through the PLC. To consider both dynamic growth and possible stochasticity in future demand, this study suggests a methodology for generating sufficient demand paths as representatives of possible demand trajectories through the PLC. To do so, the GBM model is applied. GBM is a renowned model that has been frequently employed to represent the movement and variability of stock prices. It is a mathematical model with applications in different areas, such as supply chain management, biology, physics, economy, financial engineering and stochastic calculus (Hsu & Wu, 2011). For instance, Marathe and Ryan (2005) analyzed four data-sets in the energy, transportation, and telecommunication sectors and found that using electric power exhibits a good fit to the GBM model. In the supply chain management area, GBM has found a place as well. Table 2 represents some applications of GBM in supply chain management. The application of GBM in dealing with demand uncertainties has been justified in some previous literature. Based on Yao, Jiang, Young, and Talluri (2010), GBM is a good first approximation for uncertainties. Chou, Cheng, Yang, and Liang (2007) described GBM as a mathematical tool with the capability of calibrating demand volatility very reasonably and accurately. In a study by Qin and Nembhard (2012) the stochastic nature of the diffusion process was considered as a GBM process with a linear expected growth rate. The GBM model has also been applied in a recent study by Chou, Sung, Lin, and Jahn (2014) to generate sample paths of demand in the semiconductor manufacturing industry.

Table 2. Some applications of GBM in the supply chain field

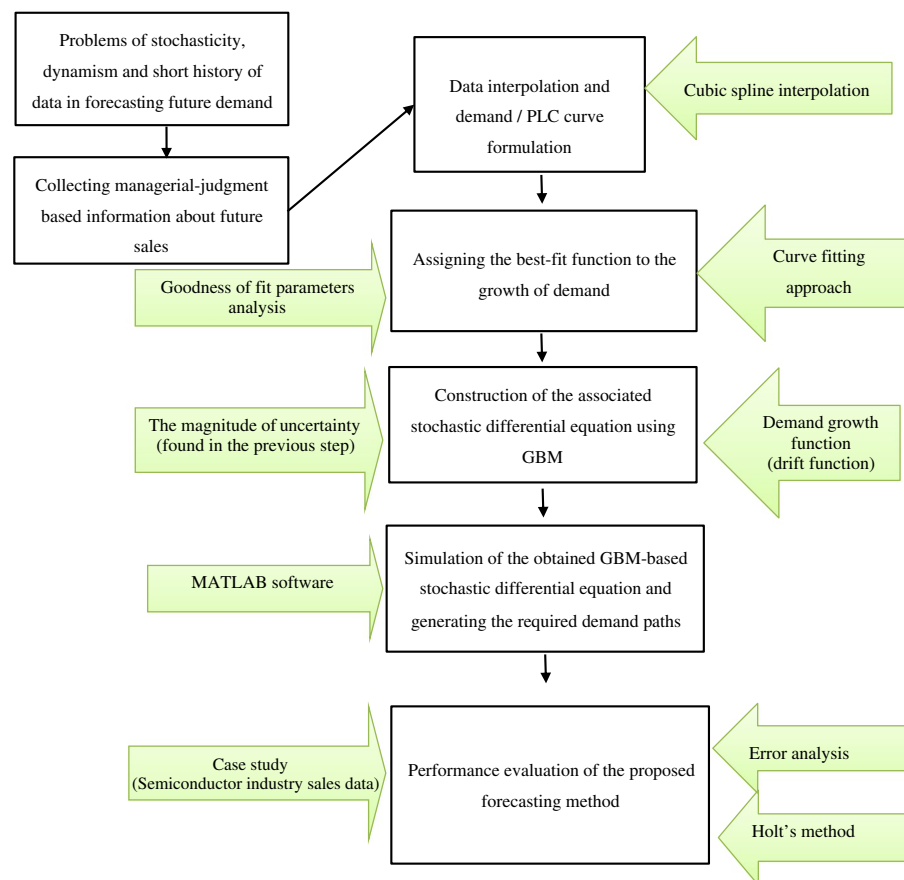
Author(s) and year	PLC consideration (Dynamic demand growth)	Demand growth	Area of application	Status of historical/ analogous data
Chou et al. (2014)	No	Different scenarios and constant drift parameter levels	Capacity planning	Available
Chou et al. (2007)	No	Constant growth rate for different scenarios	Capacity planning	Available
Benavides et al. (1999)	No	—	Capacity planning	Available
Qin and Nembhard (2012)	Yes	Linear growth	Forecasting diffusion	Available
This study	Yes	Nonlinear growth	Forecasting diffusion	Unavailable

The review of the applications of GBM in dealing with uncertainty reveals that most studies have been conducted under the condition of historical or analogous data availability. In other words, in those studies, the role of available historical data for measuring the drift of stochastic demand over the PLC is highly significant (e.g. Qin & Nembhard, 2012). Consequently, a number of analogical approaches have followed, based on the assumption that a new product will behave as an analogous product or will represent attributes of several current products on the market (Seol, Park, Lee, & Yoon, 2012). However, as pointed out by Lee et al. (2014), due to the lack of a clear standard related to benchmark selection purposes, analogous products are usually selected with shallow procedures, causing impractical prediction results with some challenges in real-world situations. Lee et al. (2014) responded to this problem themselves using managerial-based information and by proposing a statistical and machine learning algorithm. The study focused on estimating the deterministic parameters of the proposed Bass-based model and ignored the demand behavior complexity caused by demand uncertainties. In this study, an attempt is made to involve the unavailability of historical or analogous data to fill the gap of previously mentioned literature. Therefore, unlike some similar studies, model construction is not based on the deterministic Bass model. In addition, more emphasis is placed on estimating the dynamic growth of demand over the PLC as well as reflecting the uncertain nature of demand. Therefore, we can claim that this study addresses both subjects of dynamism and stochastic nature of demand for products with rare historical or analogous sales (demand) data.

3. Research methodology

The steps taken in performing the proposed forecasting method are represented in Figure 1. A brief explanation about the procedure is given in the next section. Additional explanations and details of each step in the procedure as applied on data from the semiconductor industry are provided in Section 3.2.

Figure 1. Overall procedure.



3.1. Overall procedure

The aim of this study is to propose a statistical and simulation-based method for forecasting the demand of a product with rare historical sales data in an uncertain and dynamic environment. An instance of this method's application is in demand forecasting of an immature product with short or no history of sales data. Following sections provide detailed information about the procedure taken in performing this research.

3.1.1. Collecting managerial-based qualitative data

As demonstrated in Figure 1, the procedure begins with collecting qualitative managerial information about the future sales. The information is translated into at least four data points on the PLC curve. Each data point represents a coordinate (x, y) with x representing time period and y representing predicted future sales by the managerial board.

3.1.2. Interpolation of the future sales data predicted by the managerial board

The next step is to interpolate the data points translated from qualitative managerial information about the future sales. The aim is to generate a general formulation of demand by spline interpolation (SI) of the data points. Once the general function of demand is formulated, the growth function is generated based on the data obtained from the relative increment of demand. Since a product may experience different growth during different time intervals, considering a constant growth rate throughout the entire PLC will not explain the complex behavior of stochastic and uncertain demand. Although utilizing piecewise functions can address this problem to some extent, preserving smoothness at the knots is a concern. One way to address this matter is to connect sequential points in such a way that the overall curve passing through all the points remains continuous and appears smooth. The SI method helps obtain such a curve that preserves smoothness at the knots (Neill, 2002).

In mathematics, a spline is a numeric function that is defined by a number of piecewise polynomial functions. This function has a sufficiently high degree of smoothness at the points where two polynomial pieces are connected. The SI method has been applied in many fields and industries so far. It originated from a tool applied by engineers to make smooth curves that pass through certain points. The spline includes some weights bound to a flat surface at connection points. A smooth curve is then generated by bending a flexible strip upon each of the weights (Neill, 2002). Here, the same concept is considered when referring to a mathematical spline. The difference is that this study deals with numerical data instead of the points and also coefficients of a polynomial instead of the weights. These polynomial coefficients interpolate the numerical data and bend the curve to pass through each data, preserving continuity without having any drifting and irregular behavior.

3.1.3. Formulation of demand growth during the PLC

The output of the interpolation process provides key insight for understanding the trend of the PLC curve and consequently the growth rate of future demand. In this step, using statistical approaches, a mathematical function that best represents the growth rate of demand is formulated. In order to obtain more accurate results, there should be no limitation on the degree of the growth rate function. In constructing the prediction model, different polynomials with various degrees and complexities will be tested in order to find the best-fit growth rate function. In the next section, it will be shown that utilizing a growth rate function with better goodness-of-fit parameter values results in higher accuracy of the proposed prediction model. In this regard, sensitivity analysis is performed to demonstrate the effect of different growth rate functions on the accuracy of results.

3.1.4. Construction of stochastic differential equation as the forecast model

According to a study conducted by Qin and Nembhard (Qin & Nembhard, 2012), a GBM process enables the formulation of uncertainty in demand over the PLC as shown in Equation (2).

$$dD_t = D_t \mu_t dt + D_t \sigma dW_t \quad (2)$$

where $dW_t = \varepsilon \sqrt{dt}$ leads to Equation (3).

$$dD_t = D_t \mu_t dt + D_t \sigma \varepsilon \sqrt{dt} \tag{3}$$

where $\varepsilon = N(0, 1)$. The term $D_t \sigma \varepsilon \sqrt{dt}$ demonstrates the magnitude of uncertainty and μ_t is the growth rate function through which the complexity in demand behavior is reflected. The magnitude of uncertainty is calibrated by parameter σ as the root square mean error (RSME) of the best-fit function found for μ_t .

With the values obtained from interpolating the managerial judgment-based data, the growth rate in each time period t can be attained using Equation (4).

$$\mu(t_i) = \frac{D(t_{i+1}) - D(t_i)}{D(t_i)}, \quad i = 1, \dots, n. \tag{4}$$

in which n is the number of time periods that constitutes the PLC duration. Then the best-fit function of demand growth μ_t is found and utilized in Equation (4). Substituting the RSME value in parameter σ is the final step in formulating the targeted stochastic differential equation presented in Equation (3).

3.1.5. Simulation of the constructed forecast model

MATLAB (R2015b) software is used to simulate the resulted stochastic differential equation model and generate various possible demand paths. The following section gives additional information about the different steps taken in constructing and simulating the model.

3.2. Illustrative example

In the relevant literature, it is assumed that in the semiconductor industry, the stochastic movement of demand over time is similar to the GBM process (Benavides, Duley, & Johnson, 1999; Qin & Nembhard, 2010). Global semiconductor industry’s sales data between the years 1997 and 2014 is also applied in this study to better elucidate the proposed method. Furthermore, the same data are applied for evaluating the performance of the proposed methodology and are described in the following sections. It is worth mentioning that sales data are considered in this study as a close estimation of demand in different time periods.

3.2.1. Spline interpolation

Six records of semiconductor industry’s sales data were selected to be utilized as input data to the SI process (see Table 3). Note that for cubic spline interpolation at least four points must be considered (Neill, 2002).

The SI method provides the general trend of the PLC curve and in order to obtain the targeted demand function, the following equation is applied.

Table 3. Management-judgment-based information for future demand

Period (t)	Expected demand
1	45
6	77
12	150
18	227
24	300
27	336

Table 4. Coefficients corresponding to the piecewise spline interpolated polynomials

	a_i	b_i	c_i	d_i
$i = 1$	-0.0331	1.0534	1.95967	45
$i = 2$	-0.0331	0.5573	10.0135	77
$i = 3$	-0.0019	-0.0380	13.1292	150
$i = 4$	0.0036	-0.0719	12.4698	227
$i = 5$	0.0036	-0.0078	11.9915	300

$$D(t) = \begin{cases} D_1(t) & 1 \leq t < 6 \\ D_2(t) & 6 \leq t < 12 \\ D_3(t) & 12 \leq t < 18 \\ D_4(t) & 18 \leq t < 24 \\ D_5(t) & 24 \leq t \leq 27 \end{cases} \quad (5)$$

where $D_i(t)$ for $i = 1$ to 5 is the third-degree curve fitted to each two sequential points produced based on the following formula:

$$D_i(t) = a_i(t - t_i)^3 + b_i(t - t_i)^2 + c_i(t - t_i) + d_i \quad \text{for } i = 1 \text{ to } 5 \quad (6)$$

MATLAB (R2015b) was used to obtain the coefficients corresponding to each fitted polynomial $D_i(t)$ for $i = 1, \dots, 5$ as shown in Table 4. More explanations on the algebraic procedure of cubic SI are given in a study conducted by Neill in 2002 (Neill, 2002).

Finally, Equation (7) suggests a unique Lagrangian-based polynomial that best fits the PLC curve.

$$D(t) = \sum_{i=1}^5 l_i(t)D_i(t), \quad (7)$$

where

$$l_i(t) = \prod_{j=1, i \neq j}^n \frac{t - t_j}{t_i - t_j} \quad \text{for } i = 1 \text{ to } 5 \quad (8)$$

Figure 2 shows polynomial $D(t)$ obtained from Equation (7) vs. the actual sales data. As can be observed from this figure, the interpolated plot gives clear insight into the trend of sales (demand) during the considered time horizon.

Figure 2. Interpolated curve vs. actual sales data.

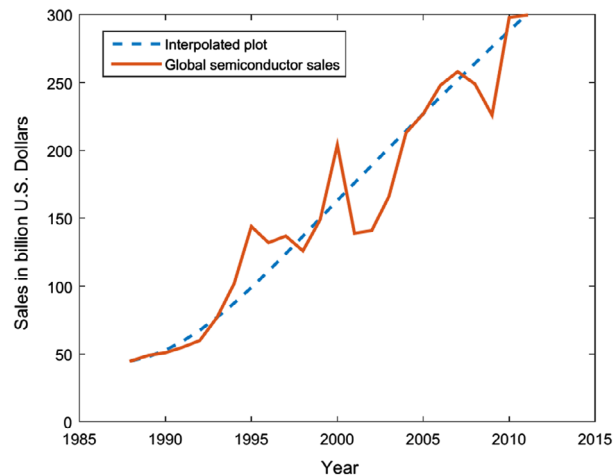
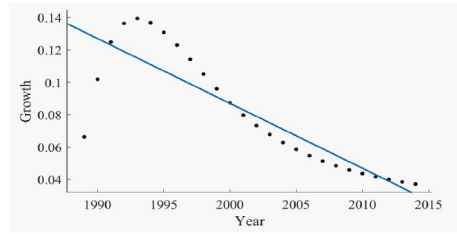


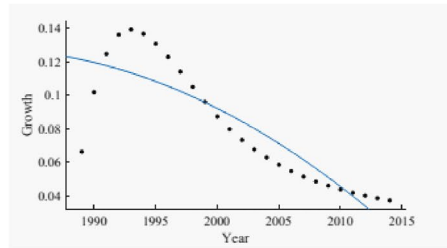
Figure 3. Fitting different polynomials to the growth rate data.



3.1. Linear

Fitted polynomial:
 $f(x) = p1 * x + p2$
 Coefficients (with 95% confidence bounds):
 $p1 = -0.004005 (-0.005004, -0.003006)$
 $p2 = 0.135 (0.1196, 0.1504)$

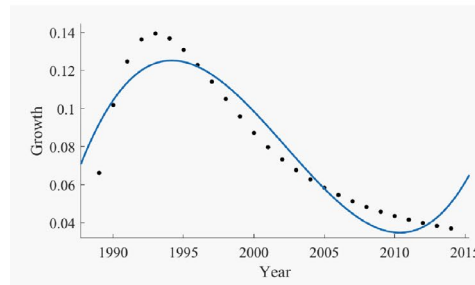
Goodness of fit:
 SSE: 0.00822
 R-square: 0.7405
 Adjusted R-square: 0.7297
 RMSE: 0.01851



3.2. 2nd degree polynomial

Fitted polynomial:
 $f(x) = p1 * x^2 + p2 * x + p3$
 Coefficients (with 95% confidence bounds):
 $p1 = -0.00009627 (-0.0002433, 5.076e-05)$
 $p2 = -0.001406 (-0.005496, 0.002684)$
 $p3 = 0.1229 (0.09893, 0.1469)$

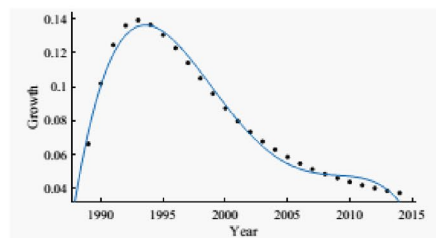
Goodness of fit:
 SSE: 0.007613
 R-square: 0.7597
 Adjusted R-square: 0.7388
 RMSE: 0.01819



3.3. 3rd degree polynomial

Fitted polynomial:
 $f(x) = p1 * x^3 + p2 * x^2 + p3 * x + p4$
 Coefficients (with 95% confidence bounds):
 $p1 = 0.00004247 (2.917e-05, 5.578e-05)$
 $p2 = -0.001816 (-0.002362, -0.00127)$
 $p3 = 0.01752 (0.01112, 0.02393)$
 $p4 = 0.07633 (0.05598, 0.09669)$

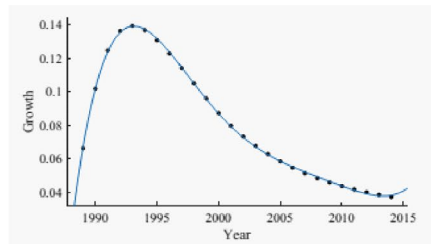
Goodness of fit:
 SSE: 0.002545
 R-square: 0.9197
 Adjusted R-square: 0.9087
 RMSE: 0.01076



3.4. 4th degree polynomial

Fitted polynomial:
 $f(x) = p1 * x^4 + p2 * x^3 + p3 * x^2 + p4 * x + p5$
 Coefficients (with 95% confidence bounds):
 $p1 = -0.000004349 (-5.095e-06, -3.604e-06)$
 $p2 = 0.0002773 (0.0002368, 0.0003179)$
 $p3 = -0.005946 (-0.006682, -0.005211)$
 $p4 = 0.04343 (0.03841, 0.04844)$
 $p5 = 0.03547 (0.02529, 0.04565)$

Goodness of fit:
 SSE: 0.0003178
 R-square: 0.99
 Adjusted R-square: 0.9881
 RMSE: 0.00389



3.5. 5th degree polynomial

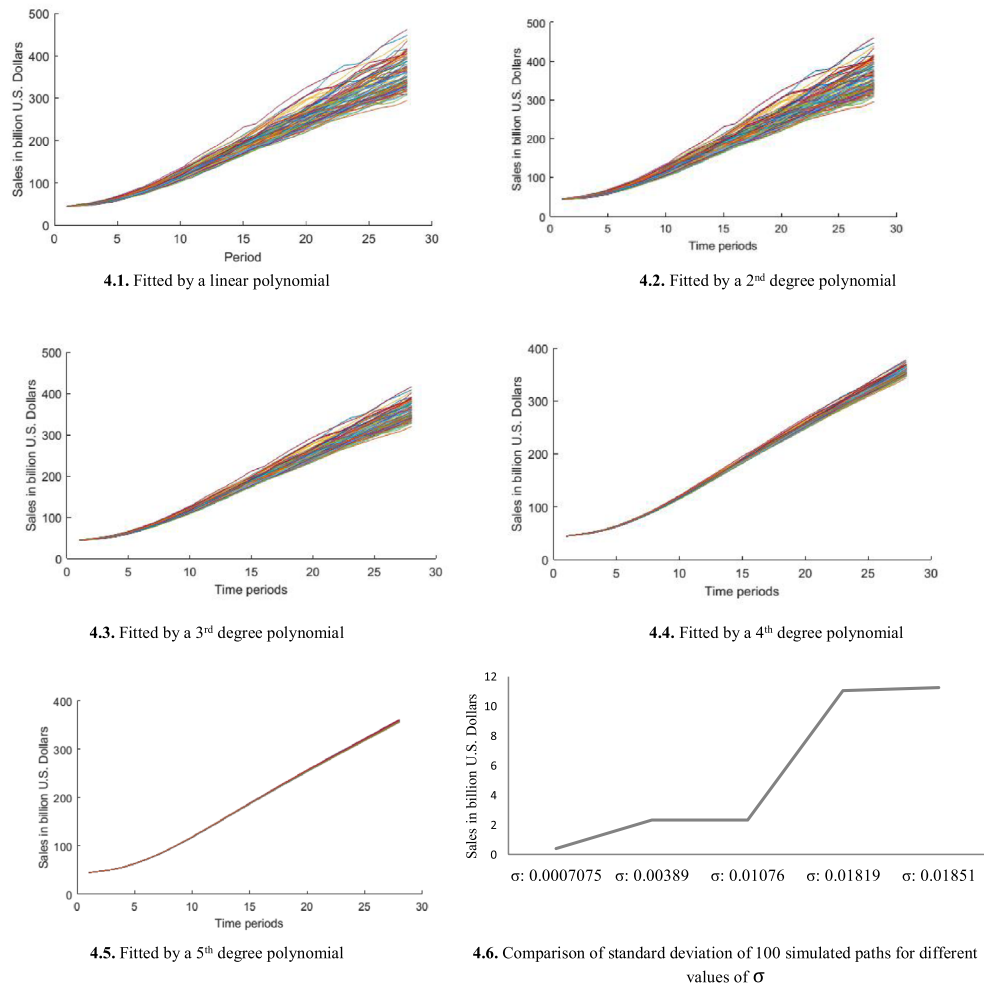
Fitted polynomial:
 $f(x) = p1 * x^5 + p2 * x^4 + p3 * x^3 + p4 * x^2 + p5 * x + p6$
 Coefficients (with 95% confidence bounds):
 $p1 = 0.0000002522 (2.31e-07, 2.734e-07)$
 $p2 = -0.00002137 (-2.281e-05, -1.994e-05)$
 $p3 = 0.0006901 (0.0006546, 0.0007256)$
 $p4 = -0.01025 (-0.01064, -0.009867)$
 $p5 = 0.06136 (0.0596, 0.06313)$
 $p6 = 0.01506 (0.01253, 0.01759)$

Goodness of fit:
 SSE: 1.001e-05
 R-square: 0.9997
 Adjusted R-square: 0.9996
 RMSE: 0.0007075

3.2.2. Finding the best fitted curve to the demand growth

Parts 1–5 of Figure 3 show the growth rate function fitted to the growth data obtained from Equation (4). Although greater complexity would be involved for approximation purposes when

Figure 4. The effect of considering different degrees of growth rate polynomials on the volatility of simulated paths.



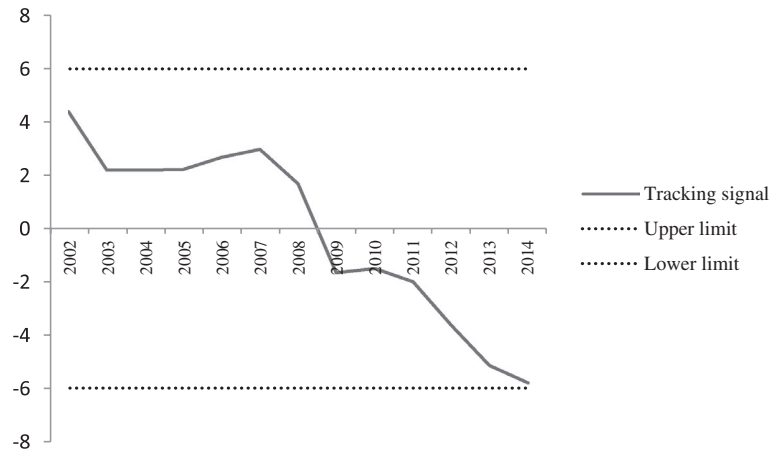
using higher order polynomials (Wang, Yang, Ding, & Wang, 2010), an attempt should be made to find the best-fit growth function. Figure 3 shows different approximations for the growth function (see Equation (4)) with the corresponding root mean square error (RMSE) values (see Equation (9)). A comparison of the values obtained for the RMSE parameter justifies the selection of the 5th degree polynomial as it exhibits the best goodness-of-fit values. It is worth noting that in the GBM-based prediction model (Equation (3)), the magnitude of uncertainty is calibrated by parameter σ , whose value is set based on the RMSE parameter value obtained from the best fitted function of growth. For the case under study and as demonstrated in Figure 3, the 5th degree polynomial indicates better fit with a value of 0.0007075 for the RMSE parameter.

3.2.3. Simulation of the possible demand paths (as possible scenarios)

To generate possible events of demand through the PLC, a simulation-based approach is followed in this section. Given the best fitted function to the demand growth as well as the value obtained for parameter σ , the targeted stochastic differential equation is made based on Equation (3). The simulation algorithm was coded in MATLAB R2015b to generate 100 demand paths.

Figure 4 is an illustration of the 100 demand paths generated with five different values of parameter σ set based on values obtained for RMSE (see Equation (9)) in curve fitting process. The figure clearly demonstrates an increase in volatility of the demand paths as the value of parameter σ increases. An analysis of the standard deviation of 100 simulated paths for each value of σ shows that, an increase in the value of σ leads to higher volatility in the 100 simulated paths (see Figure 4.6).

Figure 5. Tracking signals.



$$RMSE = \sqrt{1/n \sum_{t=1}^n E_t^2} \tag{9}$$

4. Performance evaluation of the proposed method

The expected path of demand obtained from average of 100 simulated demand paths is considered as the predicted diffusion through the PLC. According to Chopra and Meindl (2007), the tracking signal (TS), as the ratio of bias and mean absolute deviation (MAD), can reveal any bias in the values predicted by a forecasting method. If TS is under -6, it is a signal that the forecasting method underestimates the demand and if it is found to be over 6, it is a sign that the forecasting method overestimates the demand. To evaluate the performance of the proposed method, the value of TS for time period t, as given in Equation (10), is calculated and represented in Figure 5. The results demonstrate the unbiased prediction of the forecasting method between the years 2001 and 2014, as there is no value outside the above-mentioned range.

$$TS_t = \frac{bias_t}{MAD_t} \tag{10}$$

in which

$$MAD_n = 1/n \sum_{t=1}^n |E_t|, \tag{11}$$

Figure 6. Performance comparison of the proposed method with Holt’s model.

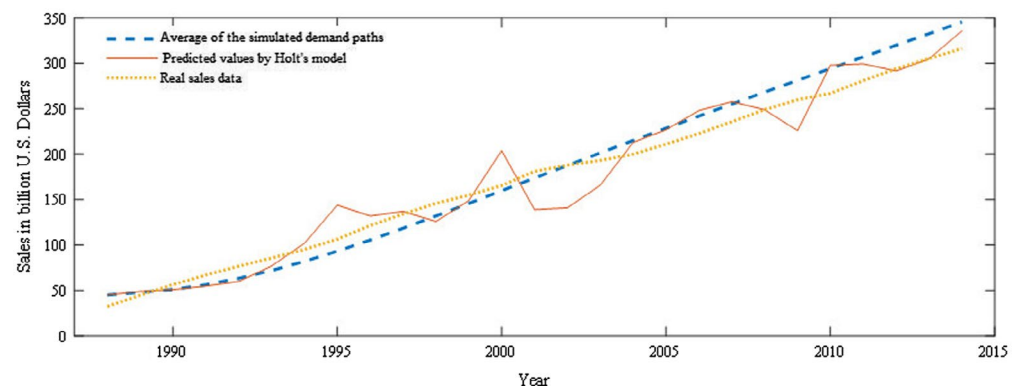
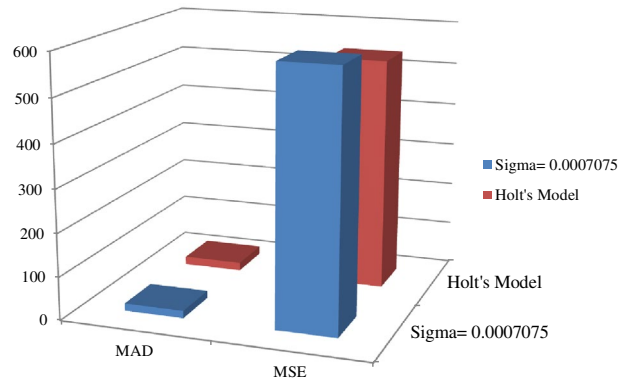


Figure 7. Comparisons of forecast error measures obtained from the proposed method and Holt's model.



And

$$E_t = \text{forecasted demand} - \text{actual demand} \tag{12}$$

To obtain further insight into the performance of the proposed method, a conventional prediction method, Holt's method, was also applied on the same data from the semiconductor industry. A comparison was made between the results from the average of 100 simulated paths and the values predicted by Holt's method (Figure 6). The figure also provides a comparison of the outputs obtained from utilizing these two methods and the real sale values within the time horizon comprising 27 years from 1988 to 2014. Evidently, both models predicted the sale values relatively close to each other, which confirm the applicability of the proposed method with respect to Holt's conventional forecasting method. In addition, a comparison was done for two significant measures of forecast error, namely MAD and RMSE. To do so, the same error measures were calculated based on the average path obtained from the outputs of the proposed method (simulated paths). Figure 7 demonstrates this comparison.

It should be noted that to perform demand forecasting using Holt's method, some historical data are required. Furthermore, the data should be dynamically updated after observing the demand of each time period. As a result, Holt's method does not provide the sequences of demand values assigned to the desired time horizon consisting of several time periods. Therefore, the proposed method outperforms Holt's model when the aim is to predict a sequence of demand values for a time horizon comprising several periods. Figure 6 also reveals the fact that Holt's model cannot capture the complexity of demand behavior stemming from the volatility in the semiconductor field. The same problem arises when using the average of 100 simulated paths. Therefore, generating several different demand paths based on the GBM process as described earlier, can address the problem caused by demand uncertainty. This is because it is a valuable input in proposing a feasible and optimal plan for all generated demand paths as representatives of scenarios of possible demand trends during the considered time horizon.

Figure 8. Comparison of MAD measures for different values of σ .

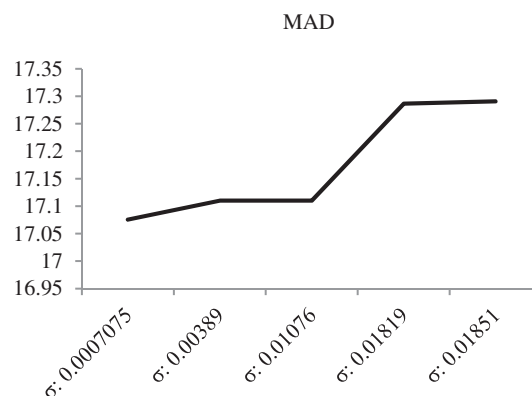
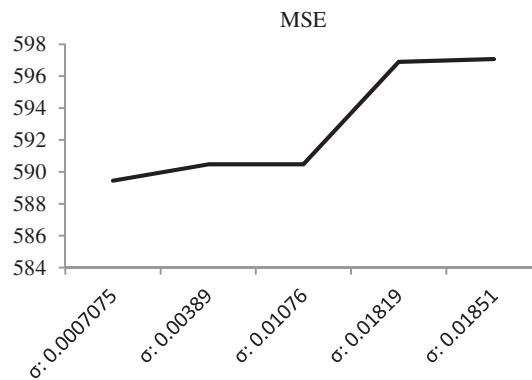


Figure 9. Comparison of MSE measures for different values of σ .



5. Sensitivity analysis

In this section, sensitivity analysis is performed on parameter σ to thoroughly understand its effect on the accuracy of the proposed forecasting method. To quantify this effect, some measures of forecast error including MAD and mean squared error (MSE) are analyzed, where the following equation holds:

$$MSE = 1/n \sum_{t=1}^n E_t^2 \tag{13}$$

The results of the performed error analysis are demonstrated in Figures 8 and 9, respectively. The figures indicate an increase in the value of both error measures as the value of σ is increased. Note that the value of σ changes based on different RMSE values found from curve fitting to the growth rate data.

6. Managerial implications

The suggested procedure is helpful for sales managers to gain insight into possible challenges that may arise when dealing with an uncertain future market, particularly when the source of uncertainty is unknown and there is insufficient past information available. The proposed method also gives valuable input for developing more accurate and practical contingency plans in areas with more vulnerability caused by uncertainties in demand, as the method proposes a scenario aggregation approach by utilizing path-based scenarios (Kall & Wallace, 1994). The method can be helpful for policy-makers who require the prediction of uncertain demand over a time horizon, such as decisions associated with aggregate production planning, capacity planning, and supply chain network design. Especially for the semiconductor industry with intensive capital investment (Chien et al., 2010), the proposed approach can be useful for making decisions associated with capacity allocation and expansion.

7. Conclusion

Through this study, a forecast method was suggested for capturing both the dynamism and stochasticity of future demand in the semiconductor industry. The proposed method aims to address the problem stemming from unavailability of historical data in predicting future demand trends, by utilizing the SI approach. In addition, uncertainties in future demand were calibrated using the GBM process. It was shown that utilizing non-linear polynomials for demand growth function improves forecasting accuracy, since they can reflect the actual demand growth rate over different PLC intervals. The proposed GBM-based forecast model permits involving possible uncertainty in predicting future demand. The outputs of the proposed forecast model are several demand paths whose volatility represents future demand uncertainty. The performance of the proposed method was tested against Holt's model by performing forecast error analysis. Comparisons of values obtained for two significant measures of forecast error, namely MSE and MAD, confirmed the capability of the proposed method in demand forecasting in the semiconductor manufacturing field. The method contributes well to developing strategic plans in dynamic and uncertain markets when a robust scenario analysis is required. The method facilitates the generation of high numbers of demand scenarios, leading to more practical and accurate plans, particularly in the case of stochastic programming.

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References

- Bass, F. M. (2004). Comments on "a new product growth for model consumer durables the bass model!". *Management Science*, 50, 1833–1840. doi:10.1287/mnsc.1040.0300
- Bass, F. M., Krishnan, T. V., & Jain, D. C. (1994). Why the bass model fits without decision variables. *Marketing Science*, 13, 203–223. doi:10.1287/mksc.13.3.203
- Benavides, D. L., Duley, J. R., & Johnson, B. E. (1999). As good as it gets: optimal fab design and deployment. *IEEE Transactions on Semiconductor Manufacturing*, 12, 281–287. doi:10.1109/66.778191
- Chen, W.-S., & Chen, K.-F. (2007). Modeling product diffusion by system dynamics approach. *Journal of the Chinese Institute of Industrial Engineers*, 24, 397–413. doi:10.1080/10170660709509055
- Cheng, C.-H., Chen, Y.-S., & Wu, Y.-L. (2009). Forecasting innovation diffusion of products using trend-weighted fuzzy time-series model. *Expert Systems with Applications*, 36, 1826–1832. doi:10.1016/j.eswa.2007.12.041
- Chien, C.-F., Chen, Y.-J., & Peng, J.-T. (2010). Manufacturing intelligence for semiconductor demand forecast based on technology diffusion and product life cycle. *International Journal of Production Economics*, 128, 496–509. doi:10.1016/j.ijpe.2010.07.022
- Chopra, S., & Meindl, P. (2007). Supply chain management. Strategy, planning & operation. *Das Summa Summarum des Management*, 265–275.
- Chou, Y.-C., Cheng, C.-T., Yang, F.-C., & Liang, Y.-Y. (2007). Evaluating alternative capacity strategies in semiconductor manufacturing under uncertain demand and price scenarios. *International Journal of Production Economics*, 105, 591–606. doi:10.1016/j.ijpe.2006.05.006
- Chou, Y.-C., Sung, W.-C., Lin, G., & Jahn, J. (2014). A comparative study on the performance of timing and sizing models of capacity expansion under volatile demand growth and finite equipment lifetime. *Computers & Industrial Engineering*, 76, 98–108. doi:10.1016/j.cie.2014.07.027
- Hsu, Y.-S., & Wu, C.-H. (2011). A generalization of geometric brownian motion with applications. *Communications in Statistics - Theory and Methods*, 40, 2081–2103. doi:10.1080/03610921003764167.
- Jain, D. C., & Rao, R. C. (1990). Effect of price on the demand for durables: modeling, estimation, and findings. *Journal of Business & Economic Statistics*, 8, 163–170. doi:10.2307/1391978
- Kall, P., & Wallace, S. W. (1994). *Stochastic programming* (p. 307). Chichester: Wiley.
- Laciana, C. E., & Oteiza-Aguirre, N. (2014). An agent based multi-optional model for the diffusion of innovations. *Physica A: Statistical Mechanics and its Applications*, 394, 254–265. doi:10.1016/j.physa.2013.09.046
- Lee, H., Kim, S. G., Park, H., & Kang, P. (2014). Pre-launch new product demand forecasting using the Bass model: A statistical and machine learning-based approach. *Technological Forecasting and Social Change*, 86, 49–64. doi:10.1016/j.techfore.2013.08.020
- Mahajan, V., Muller, E., & Bass, F. M. (1990). New product diffusion models in marketing: A review and directions for research. *Journal of Marketing*, 54(1), 1–26. doi:10.2307/1252170
- Marathe, R. R., & Ryan, S. M. (2005). On the validity of the geometric brownian motion assumption. *The Engineering Economist*, 50, 159–192. <http://dx.doi.org/10.1080/00137910590949904>
- Neill, C. O. (2002). Cubic spline interpolation. *Acta mathematica hungarica*, 107, 493–507. doi:10.1007/s10474-005-0180-4
- Peres, R., Muller, E., & Mahajan, V. (2010). Innovation diffusion and new product growth models: A critical review and research directions. *International Journal of Research in Marketing*, 27, 91–106. doi:10.1016/j.ijresmar.2009.12.012
- Qin, R., & Nembhard, D. A. (2010). Workforce agility for stochastically diffused conditions—A real options perspective. *International Journal of Production Economics*, 125, 324–334. doi:10.1016/j.ijpe.2010.01.006
- Qin, R., & Nembhard, D. A. (2012). Demand modeling of stochastic product diffusion over the life cycle. *International Journal of Production Economics*, 137, 201–210. doi:10.1016/j.ijpe.2012.01.027
- Schmittlein, D. C., & Mahajan, V. (1982). Maximum likelihood estimation for an innovation diffusion model of new product acceptance. *Marketing Science*, 1(1), 57–78. doi:10.1287/mksc.1.1.57
- Scitovski, R., & Meler, M. (2002). Solving parameter estimation problem in new product diffusion models. *Applied Mathematics and Computation*, 127, 45–63. doi:10.1016/S0096-3003(00)00164-8
- Seol, H., Park, G., Lee, H., & Yoon, B. (2012). Demand forecasting for new media services with consideration of competitive relationships using the competitive Bass model and the theory of the niche. *Technological Forecasting and Social Change*, 79, 1217–1228. doi:10.1016/j.techfore.2012.03.002
- Wang, X. J., Yang, S. L., Ding, J., & Wang, H. J. (2010). Dynamic GM(1,1) model based on cubic spline for electricity consumption prediction in smart grid. *China Communications*, 7, 83–88.
- Yao, T., Jiang, B., Young, S. T., & Talluri, S. (2010). Outsourcing timing, contract selection, and negotiation. *International Journal of Production Research*, 48, 305–326. doi:10.1080/00207540903174858



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