

Investigation on the Impact of Tropospheric Delay on GPS Height Variation near the Equator

Abstract—One of the major problems currently facing satellite-based positioning is the atmospheric refraction of the GPS signal caused by the troposphere. The tropospheric effect is much more pronounced at the equatorial region due to its hot and wet conditions. This significantly affects the GPS signal due to the variability of the refractive index, which in turn affects the accuracy of GPS positioning, especially in the height components. This paper presents a study conducted in Malaysia located at the equatorial region, to investigate the impact of tropospheric delay on GPS height variation. Five GPS reference stations forming part of the Malaysian real-time kinematic GPS network (MyRTKnet) in Johor were used. RINEX data from these stations were integrated with GPS and ground meteorological data observed from a GPS station located at the Universiti Teknologi Malaysia (UTM), at varying antenna heights for each session of observation in four campaigns with each campaign lasting for three days. A computer program called TROPO.exe was developed based on Saastamoinen tropospheric model. The result shows variations in the height component of GPS measurement with a maximum value of 119.100 cm and a minimum value of 37.990 cm. Similarly, the results show that, the tropospheric delay is a distance-dependent error, which varies with changes in meteorological condition. Furthermore, result of simulated data shows decrease in tropospheric delay with increase in antenna height.

Keywords— Ground meteorological data, height component, Saastamoinen model, tropospheric delay.

I. INTRODUCTION

THE troposphere is the lower part of the atmosphere close to the Earth's surface; it is 9 km over the poles and 16 km over the equator [7], which extends from the sea to about 50 km [2]. It is considered as a neutral atmosphere, with an index of refraction that varies with altitude. The variability of refractive index causes an excess group delay of the GPS signal usually referred to as *tropospheric delay*. This delay induces variation in GPS positioning and is a matter of great concern to the geodetic community in terms of high accuracy applications. The positioning error due to improper estimation of the tropospheric delay can be over 10 m because; the tropospheric delay can range from 2 m at the zenith to over 20 m at lower elevation angle [1].

There are two classes of tropospheric biases that affect GPS measurement; there are those that influence the height component and others affecting the scale having significance

in terms of positional accuracy [4].

The tropospheric delay consists of the hydrostatic component, also known as the *dry part* and the non-hydrostatic component, also known as the *wet part*. Several researchers have made attempts to model the tropospheric delay. The most widely use expression for tropospheric refractivity N is [3] and given by the expression:

$$N = 77.6 \frac{P}{T} + 3.73 \times 10^5 \left(\frac{e}{T^2} \right) \quad (1)$$

where:

P , the total atmospheric pressure in mbar; T , temperature in Kelvin; e , partial pressure of water vapour in mbar. [9] asserts that, the hydrostatic contributes approximately 90% of the total tropospheric delay. Nevertheless, the hydrostatic part can be computed from pressure measured at the receiver antenna. It is given by the expression:

$$D_{dz}^{trop} = (77.62) \frac{P}{T} \quad (2)$$

where D_{dz}^{trop} is the hydrostatic tropospheric delay at given angle from the zenith. The wet component only accounts for 10% of the total tropospheric delay. However, it is more difficult to model due to the diversity of the water vapour distribution. As a result of this, error in the wet component contributes the most significant factor of the signal refraction. It is given by the expression:

$$D_{wz}^{trop} = -(12.96) \frac{e}{T} + (3.718 \times 10^5) \frac{e}{T^2} \quad (3)$$

where D_{wz}^{trop} is the wet tropospheric delay at given angle from zenith.

There are two basic types of models for estimating the tropospheric delay. The first relates the meteorological parameters in (1) to surface meteorological measurements. These surface meteorological models are based on radiosonde profiles measurements taken at the ground surface. Examples include the Hopfield tropospheric delay model [5] and the Saastamoinen tropospheric delay model [6]. The second relates to global standard atmosphere.

The refined Saastamoinen tropospheric model is used in this study. It is expressed in the form [10]:

$$D_z^{trop} = \frac{0.002277}{\cos z} \left[P + \left(\frac{1255}{T} + 0.05 \right) e - B \tan^2 z \right] + \delta R \quad (4)$$

where:

D_z^{trop} : propagation delay in terms of range (m)

z : zenith angle of the satellite

P : atmospheric pressure at the site in milibar (mbar)

T : temperature at the station in Kelvin (K)

e : partial pressure of water vapour in milibar (mbar)

$B, \delta R$ are the correction terms for height and zenith angle

Based on equation (4), e is calculated as a fractional of 1 from the relative degree of moisture. It is expressed as [8]:

$$e = 6.108RH \times \exp \left[\frac{17.15T - 4684}{T - 38.45} \right] \quad (5)$$

where:

RH is the relative humidity. The pressure P at height above sea level h (in kilometres) is given in terms of the surface pressure P_s and temperature T . Pressure P can be defined as:

$$P = P_s \left[\left(\frac{T - 4.5h}{T} \right) \right]^{7.58} \quad (6)$$

II. FIELD DATA COLLECTION

Static GPS observations using LeicaTM System 500 dual frequency receivers and a ground meteorological sensor called Davis GroWeatherTM System were set up next to one another at GPS station G11 in UTM. Fig. 1 shows the observation set up.

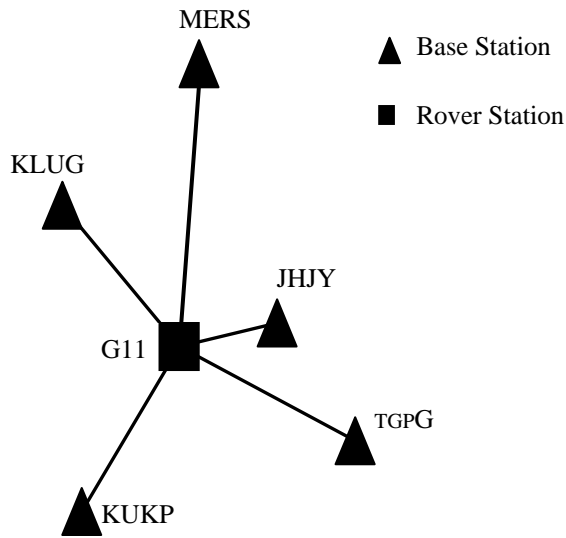


Fig. 1. An overview of the field Setup

Four GPS campaigns were conducted as shown in Table 1. Series of field observations were carried out for a total of nine hours per day and divided into three sessions of 3 hours each.

For each session, the antenna height was increased systematically. Ten minutes interval of ground meteorological data of temperature, pressure, and relative humidity were measured in each session. The procedures were repeated in all the campaigns forming four sets of observation where each set consists of three consecutive days of data collection. Table 1 shows the scheduling of the field observation.

TABLE 1
TIME SCHEDULLING OF FIELD OBSERVATION

GPS Campaigns		1	2	3	4
Observation Period		29–31 Aug 06	01-03 Dec 06	06–08 Jan 07	09–11 Jan 07
9 hours	1st Session (9 am – 12 pm)	Antenna Height : 0.5 m			
	2nd Session (12 pm – 3 pm)	Antenna Height : 1.0 m			
	3rd Session (3 pm – 6 pm)	Antenna Height : 1.5 m			

Five GPS reference stations forming MyRTKnet stations in Johor were used as the base stations, thus producing the baselines for processing and analysis. Table 2 shows the description of the selected MyRTKnet stations relative to the rover station G11 located in UTM.

TABLE 2
DESCRIPTION OF MYRTKnet STATIONS IN JOHOR

ID	JHJY	KUKP	TGP	KLUG	MERS
Station	Johor Bahru	Pontian	Pengerang	Mersing	Mersing
Location	SMK Taman JohorJaya(1)	JPS Bandar Permas	SK Tanjung Pengelih	Pejabat Daerah Kluang	SMK Mersing
Latitude	01° 32' 12.517586"	01°19' 59.79030 3"	01° 22' 2.678994"	02° 01' 31.361182"	02° 27' 12.482131"
Longitude	103° 47' 47.510364"	103° 27' 12.35534 2"	104° 06' 29.730485"	103° 19' 0.520982"	103° 49' 43.505376"
Ellipsoidal Height (m)	39.1959	15.4282	18.0874	73.5879	18.0812
Distance Relative to G11 (km)	17.9051	32.1902	56.5244	62.7530	101.2633

A. Multi-station Analysis

In order to establish the availability of the GPS satellites during the observation sessions, a Multi-station Analysis is carried out. This allow for checking simultaneous observation of same satellite, satellite elevation and the Dilution of Precision (DOP). Low Geometry Dilution of Precision (GDOP) indicates strong satellite geometry with a higher possibility of accuracy. Tables 3-5 present the GDOP of satellites for the 4th campaign between 9th and 11th January 2007. Good GDOP were obtained between 1500 hours and 1800 hours in all cases. However, best GDOP of 1.67 is obtained on 11th January 2007.

TABLE 3
MULTISTATION ANALYSIS OF GEOMETRY DILUTION OF
PRECISION FOR 9TH JAN. 2007

	Time	No. GPS Satellite	GDOP
09/01/07	09:00	6	4.79
	10:00	8	6.76
	11:00	10	2.38
	12:00	9	2.34
	13:00	10	2.26
	14:00	9	2.87
	15:00	9	2.34
	16:00	11	2.41
	17:00	11	2.10
	18:00	12	1.83

TABLE 4
MULTISTATION ANALYSIS OF GEOMETRY DILUTION OF
PRECISION FOR 10TH JAN. 2007

	Time	No. GPS Satellite	GDOP
10/01/07	09:00	6	5.01
	10:00	8	6.29
	11:00	10	2.35
	12:00	9	2.35
	13:00	10	2.34
	14:00	9	2.79
	15:00	9	2.36
	16:00	11	2.45
	17:00	11	2.08
	18:00	12	1.82

TABLE 5
MULTISTATION ANALYSIS OF GEOMETRY DILUTION OF
PRECISION FOR 11TH JAN. 2007

	Time	No. GPS Satellite	GDOP
11/01/07	09:00	6	5.22
	10:00	8	5.72
	11:00	10	2.32
	12:00	9	2.37
	13:00	10	2.42
	14:00	9	2.71
	15:00	9	2.38
	16:00	11	2.48
	17:00	11	2.06
	18:00	13	1.67

The elevations of the satellites during the observation periods were determined. Satellites at low elevation angle (in this case below 10^0) contribute to errors in propagating signals through the atmosphere. Figures 2-4 show satellite elevation plots for the 4th campaign. None of the satellite was found below 10^0 cut-off angle.

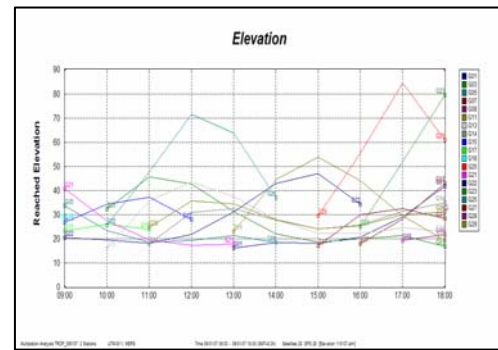


Fig. 2. Satellite Elevation for UTM and MERS on 9th Jan. 2007

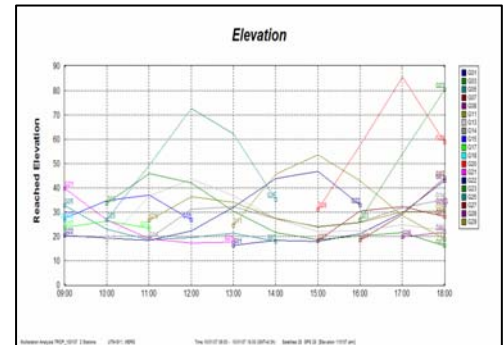


Fig. 3. Satellite Elevation for UTM and MERS 10th Jan. 2007

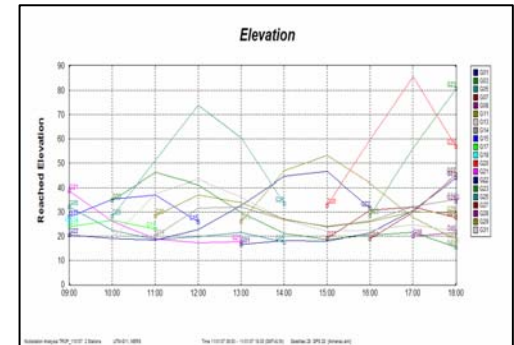


Fig. 4. Satellite Elevation for UTM and MERS on 11th Jan. 2007

III. DATA PROCESSING

In order to study the impact of troposphere on height determination, the tropospheric effect has been left uncompensated as no standard tropospheric model was applied during processing. To eliminate the effect of ionosphere, satellite and receiver clock bias, the ionospheric free double difference solution was applied. Multipath effects were assumed to be eliminated entirely by the long hours of observations. Each observation session was 3 hours long. The GPS receivers were calibration and in excellent condition, antenna phase centre variation in this study has also been neglected.

The processing is done at 1 hour interval using the broadcast and precise ephemerides to gauge at what baseline lengths the use of the precise ephemerides becomes worthwhile. The horizontal and vertical components residual for each baseline in each case (i.e. broadcast and precise

ephemerides) as a function of the baseline length is presented in Table 6. The 3D error in each case is computed as follows:

$$3D \text{ Error} = \left[(\Delta E)^2 + (\Delta N)^2 + (\Delta U)^2 \right]^{\frac{1}{2}}$$

where ΔE , and ΔN are errors in the horizontal component and ΔU is the error in the height component. The result is presented in Table 7

TABLE 6
HORIZONTAL AND VERTICAL COMPONENTS RESIDUAL FOR
BROADCAST AND PRECISE EPHEMERIDES

BASELINE E	LENGTH (KM)	PRECISE EPHEMERIS			BROADCAST EPHEMERIS		
		N E	E E	U E	N E	E E	U E
UTM - JHJY	17.9051	1.3560	0.3050	0.5910	1.3560	0.3050	0.5910
UTM - KUKP	32.1902	1.3600	0.3140	0.5830	1.3600	0.3140	0.5830
UTM - TGPG	56.5244	1.3540	0.2960	0.5680	1.3540	0.2960	0.5680
UTM - KLUG	62.7530	1.3580	0.3270	0.5920	1.3580	0.3270	0.5920
UTM - MERS	101.2633	1.3710	0.2970	0.6130	1.5680	0.2470	0.8840

TABLE 7
OBSERVED 3D ERROR AS A FUNCTION OF BASELINE LENGTH
USING 1 HOUR OF DATA

Baseline	Length (km)	3D Error (cm)	
		Precise Ephemeris	Broadcast Ephemeris
UTM - JHJY	17.9051	1.5103	1.5103
UTM - KUKP	32.1902	1.5126	1.5126
UTM - TGPG	56.5244	1.4979	1.4979
UTM - KLUG	62.7530	1.5171	1.5171
UTM - MERS	101.2633	1.5309	1.8169

From Tables 6 and 7, the precise and broadcast ephemerides 3D error values are virtually identical. The largest difference of 0.286 cm is seen at baseline UTM-MERS. It is evident that, with the current improvement on the broadcast ephemeris, there is no clear benefit to using the precise ephemeris for baselines of less than 100 km. Therefore, as baselines range from only 17 to 100 km in this research, the broadcast ephemeris has been used. Table 8 shows a summary of the processing parameters.

TABLE 8
SUMMARY OF PROCESSING PARAMETERS USED

Cut-off angle	10^0
Orbit Type	IGS Broadcast
Solutions	Ionosphere-free double difference fixed
Tropospheric Models	None

IV. ANALYSIS OF RESULTS

A. Tropospheric Effect on the Ellipsoidal heights

Residuals in the computed ellipsoidal height at G11 of four sets of field observation compared to the known value were calculated first. As mentioned earlier, in this process, tropospheric effects have been left uncompensated. To visualize the variation on the height component of GPS measurement due to the tropospheric delay, discrepancies of ellipsoidal height between computed and known value for each baseline in the four campaigns have been plotted against each hour of observation as shown in Figures 5-16

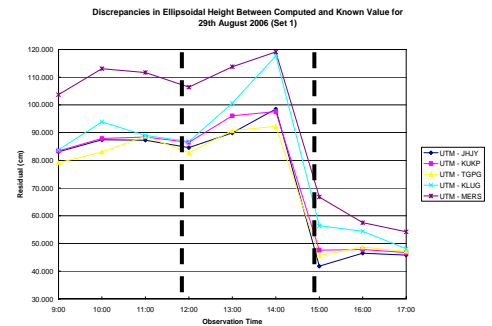


Fig. 5. Discrepancies of Ellipsoidal Height Between Computed and Known Value of 1st Campaign of 29th August 2006

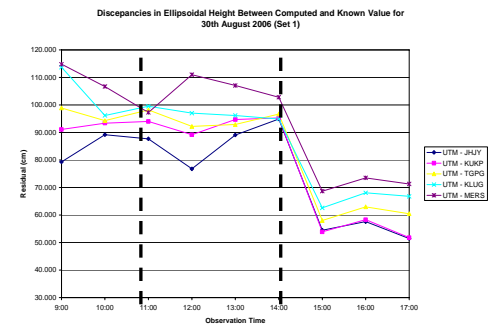


Fig. 6. Discrepancies of Ellipsoidal Height Between Computed and Known Value of 1st Campaign of 30th August 2006

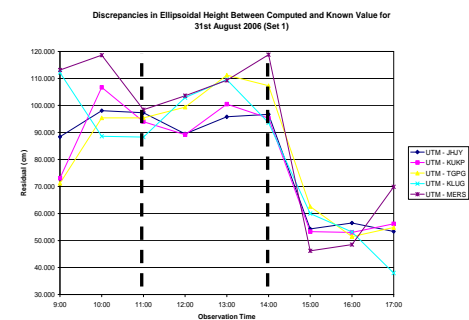


Fig. 7. Discrepancies of Ellipsoidal Height Between Computed and Known Value of 1st Campaign of 31st August 2006

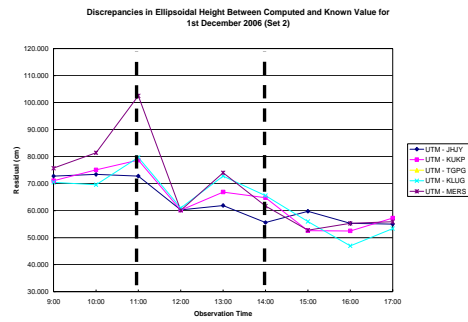


Fig. 8. Discrepancies of Ellipsoidal Height Between Computed and Known Value of 2nd Campaign of 1st December 2006

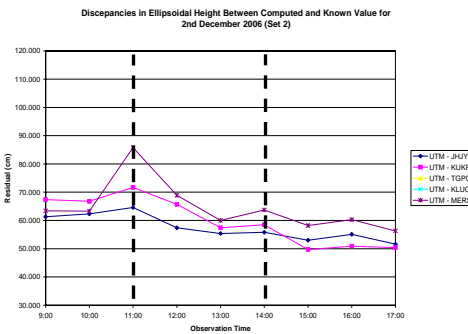


Fig. 9. Discrepancies of Ellipsoidal Height Between Computed and Known Value of 2nd Campaign of 2nd December 2006

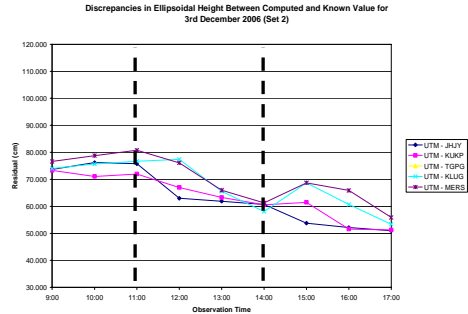


Fig. 10. Discrepancies of Ellipsoidal Height Between Computed and Known Value of 2nd Campaign of 3rd December 2006

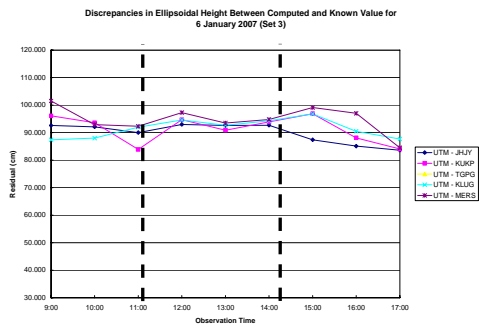


Fig. 11. Discrepancies of Ellipsoidal Height Between Computed and Known Value of Set 3rd Campaign of 6th January 2007

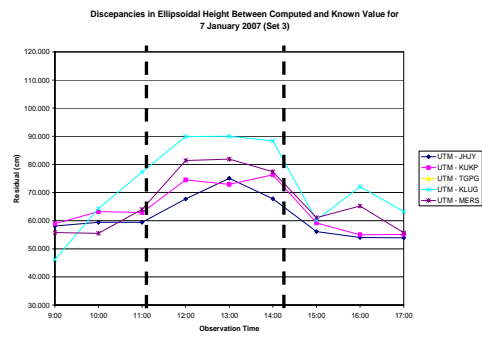


Fig. 12. Discrepancies of Ellipsoidal Height Between Computed and Known Value of Set 3rd Campaign of 7th January 2007

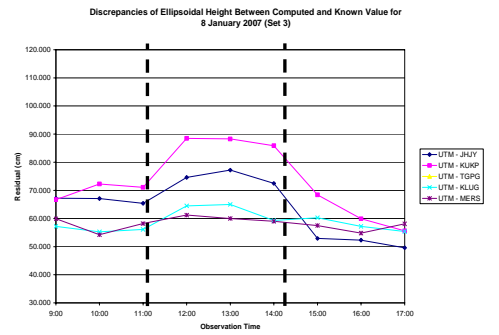


Fig. 13. Discrepancies of Ellipsoidal Height Between Computed and Known Value of Set 3rd Campaign of 8th January 2007

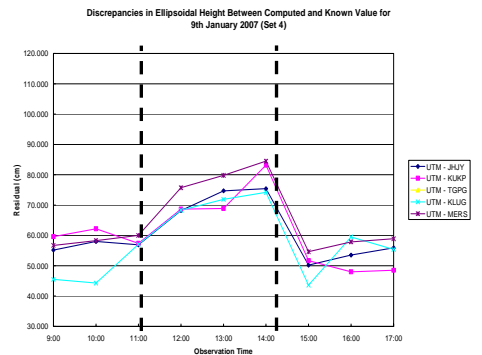


Fig. 14. Discrepancies of Ellipsoidal Height Between Computed and Known Value of Set 4th Campaign of 9th January 2007

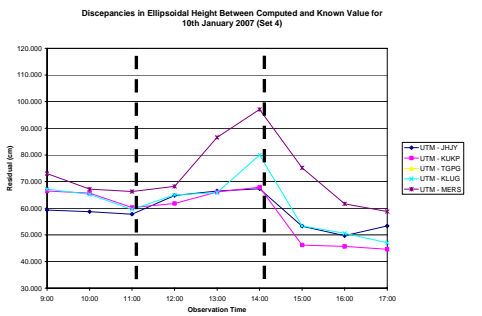


Fig. 15. Discrepancies of Ellipsoidal Height Between Computed and Known Value of Set 4th Campaign of 10th January 2007

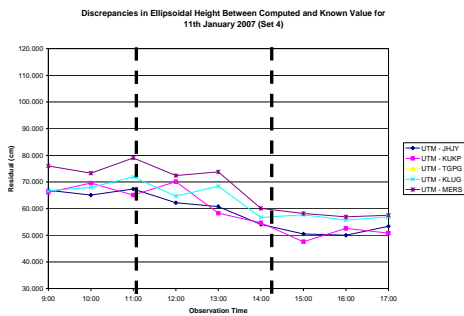


Fig. 16. Discrepancies of Ellipsoidal Height Between Computed and Known Value of Set 4th Campaign of 11th January 2007

From the results obtained, neglecting the use of a standard tropospheric model leads to variations in the height components of the GPS measurement. A maximum difference of 119.100 cm and minimum of 37.990 cm in the height component were obtained between computed and known value. This value increases between 10 am and 12 noon followed by another occurrence period at 2 pm to 3 pm. On the other hand, better results in computed height were generally confined around 5 pm to 6 pm.

The result of the computed baseline residual at maximum and minimum between UTM-MERS during the 4th campaign were analyzed and compared with the meteorological value at maximum and minimum. The result, as shown in Table 9 indicates differences in terms of meteorological condition at occurrence time of maximum and minimum residual.

It is clear that slight changes in meteorological condition can affect the amount of computed discrepancies. This is attributed to satellite geometry as shown in Tables 3-5 and the satellite signal refraction through the atmosphere. Similarly, the location of Malaysia in the equatorial and tropical region makes it susceptible to strong atmospheric effect.

Differences up to 29.9 cm between maximum and minimum residuals (9/1/2007) were detected when changes in temperature and pressure were at 0.9 C and 0.4 Hpa respectively. However for observation on 10/1/2007, differences up to 39 cm between maximum and minimum residuals were detected when changes in temperature, pressure and relative humidity were at 2.9 C, 2.4 Hpa and 3% respectively. For observation on 11/1/2007, differences up to 22.1 cm between maximum and minimum residuals were detected when changes in temperature, pressure and relative humidity were at -0.3 C, 2.9 Hpa and 2% respectively. Based on these results, conclusion can be made that there is a direct correlation between the meteorological condition and the amount of discrepancies due to tropospheric delay.

TABLE 9
METEOROLOGICAL DATA CONDITION AT MAXIMUM AND MINIMUM RESIDUAL VALUE FOR UTM-MERS BASELINE OF 4TH CAMPAIGN

UTM-MERS Baseline				
4 th Campaign	9/1/2007	Max Residual(cm)		84.500
		Min Residual(cm)		54.600
		Met Value@ Max Residual	Temperature(C)	24.6
			Pressure(Hpa)	1009.4
			R.Humidity(%)	37
		Met Value@ Min Residual	Temperature(C)	23.7
			Pressure(Hpa)	1009.0
			R.Humidity(%)	37
	10/1/2007	Max Residual(cm)		97.090
		Min Residual(cm)		58.000
		Met Value@ Max Residual	Temperature(C)	31.9
			Pressure(Hpa)	1010.4
			R.Humidity(%)	38
		Met Value@ Min Residual	Temperature(C)	29.0
			Pressure(Hpa)	1008.0
			R.Humidity(%)	35
	11/1/2007	Max Residual(cm)		79.000
		Min Residual(cm)		56.900
		Met Value@ Max Residual	Temperature(C)	23.8
			Pressure(Hpa)	1012.9
			R.Humidity(%)	43
		Met Value@ Min Residual	Temperature(C)	24.1
			Pressure(Hpa)	1010.0
			R.Humidity(%)	41

B. Tropospheric Delay on differences in Baseline lengths

In order to investigate whether tropospheric delay is also a distance-dependent error, comparisons have been made on the residuals between short (UTM-JHJY) and long (UTM-MERS) baselines from each of the campaigns. Figures 17 –20 show the differences of height value derived from both baselines of a set of observation taken from the four campaigns each.

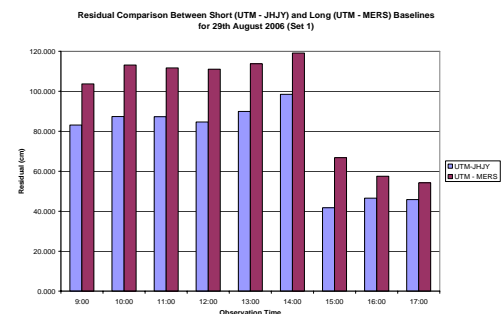


Fig. 17. Residual Comparison Between Short (UTM - JHJY) and Long (UTM - MERS) Baselines of 1st campaign of 29th August 2006

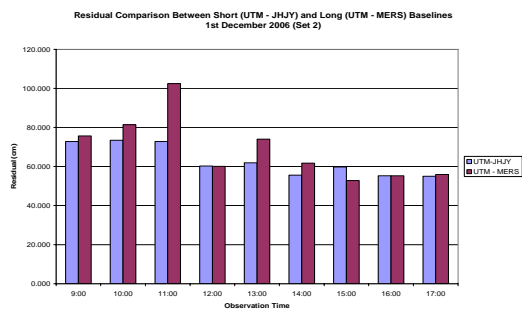


Fig. 18. Residual Comparison Between Short (UTM - JHJY) and Long (UTM - MERS) Baselines of 2nd campaign of 1st December 2006

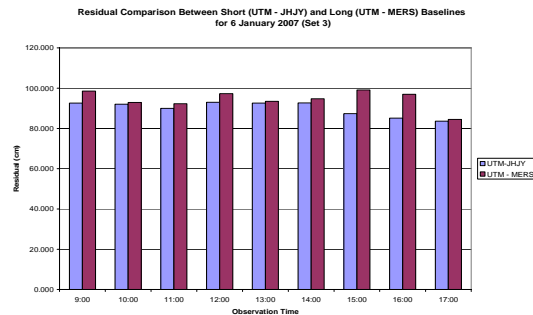


Fig. 19. Residual Comparison Between Short (UTM - JHJY) and Long (UTM - MERS) Baselines of 3rd campaign of 6th January 2007

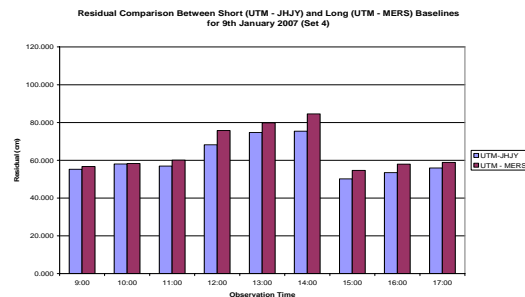


Fig. 20. Residual Comparison Between Short (UTM - JHJY) and Long (UTM - MERS) Baselines of 4th campaign of 9th January 2007

The result reveals that tropospheric error increases with the increases in the baseline length between two stations. For long baseline of UTM-MERS, the difference in tropospheric refraction will primarily be a function of the difference in the weather condition. This is due to the fact that signals transmitted from a satellite need to propagate through different amount of atmospheric content such as gases and water vapour within the troposphere due to large difference in baseline length before arriving to both receivers on the ground.

However, for short baseline, signal paths from satellite to both receivers are essentially identical. This is because the errors common to both stations tend to cancel during double differencing with the tropospheric correction decomposing into the common station parts and the satellite-dependent part [11]. Therefore, better result in the derived position is expected compared to long baseline.

C. Estimation of GPS Signal Propagation

Within the troposphere, the propagation speed of signals transmitted from GPS satellites are equally reduced with respect to free-space propagation. To determine signals propagation delay of each available satellite, a computer program called TROPO.exe was developed based on refined Saastamoinen model. A total of four available satellites were used in this study. The satellites include; SV 1, 7, 22 and 27. they were observed from UTM-JHJY baseline on 29th August 2006. The estimated delay recorded in UTM-JHJY baseline on 29th August 2006 for each satellite is shown in Figure 21-24.

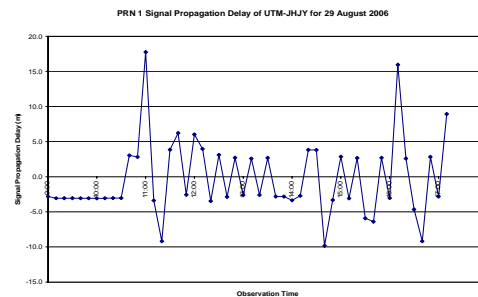


Fig. 21. Signal Propagation Delay of SV 1 UTM-JHJY Baseline for 29th August 2006

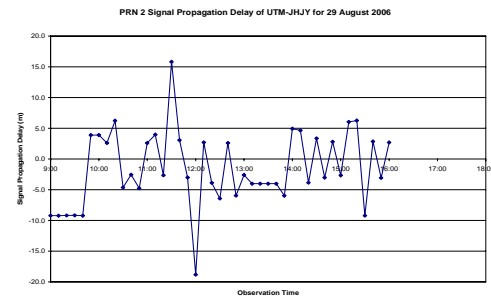


Fig. 22. Signal Propagation Delay of SV 2 UTM-JHJY Baseline for 29th August 2006

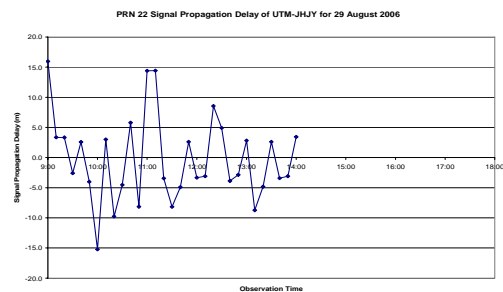


Fig. 23. Signal Propagation Delay of SV22 UTM-JHJY Baseline for 29th August 2006

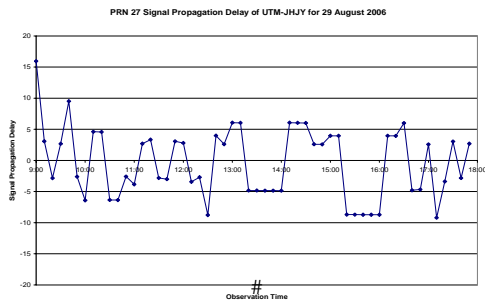


Fig. 24. Signal Propagation Delay of SV 27 UTM-JHJY Baseline for 29th August 2006

Figures 21 to 24, shows inconsistency in the delay variation. Reaching maximum delay up to 18 meters in pseudo range, the peak of the delay was detected at 11 am for SV 1. For SV 2, the occurrence time is at 12 pm. Maximum latency of signal propagation for SV 22 was detected at 10 am followed by 9 am for SV 27.

D. Tropospheric Delay on differences in antenna height

From the results obtained from Figure 2 to 13 increments on the antenna height at 0.5 m per session shows no significant effects or improvement towards the accuracy of computed ellipsoidal height obtained from each baseline. This might be due to the fact that 0.5 m increment is very small compared to the range of coverage of the troposphere medium above the earth surface (16 km above equator).

To study in which way the delay are influenced by differences in station height above mean sea level, a test was conducted using seven sets of simulated data. While both ground local meteorological condition (temperature, pressure and relative humidity) and satellite elevation angle being kept constant, signal propagation delay at each condition was computed using different value of station heights. List of simulated data used in this study is shown in Table 10.

TABLE 10
SIMULATED COMPUTATIONAL DATA

Set	Temp. (C)	Pressure (Hpa)	R.Humidity (%)	Sat. Elev. (deg)	Stn Height (m)
1	32.3	1010.2	56	60.00	0.00*
2					5.00
3					50.00
4					100.00
5					1000.00
6					10000.00
7					50000.00

* at mean sea level (MSL)

TABLE 11
AMOUNT OF SIGNAL PROPAGATION DELAY

Set	Signal Propagation Delay (m)	Differences (m)
1	2.6863	CONSTANT
2	2.6850	0.0013
3	2.6729	0.0134
4	2.6595	0.0268
5	2.4294	0.2569
6	0.9929	1.6934
7	0.2714	2.4149

Based on these simulated data, Table 11 shows the amount of signal propagation delay computed using TROPO.exe for each set of data.

Theoretically, the lesser the amount of signal propagation delay, the better the derived position results can be obtained using GPS. It is obvious therefore, that the higher station, the smaller amount of signal propagation delay can be detected. The amount of signal propagation delay for station at MSL is 2.6863 m whereas at 5 m above MSL is 2.6850 m. This shows 5 m of differences in height can only give an effect or improvement around 0.0013 m or 1.3 mm in signal propagation delay. Changes up to 1 cm can only been seen if differences in station height range up to at least 50 m above the mean sea level.

V. CONCLUSION

In order to mitigate the tropospheric delay effect, a priori tropospheric models such as Saastamoinen, Hopfield, Davis et al, etc. are often employed. In this research, a TROPO.exe programme was developed based on the refined Saastamoinen global tropospheric delay model in estimating the amount of signal propagation delay as presented in Figures 21-24. This is followed with simulation test as shown in Table 11.

From the results obtained in this study, it is obvious that neglecting the use of a standard tropospheric model leads to variations in height component of GPS measurement. The tropospheric refraction varies with changes on meteorological condition. Tropospheric delay is also distance-dependent error that increases when the baseline length between two stations increases. Based on a test using simulated data; the amount of tropospheric delay decrease with increase on the antenna height.

ACKNOWLEDGMENT

The authors would like to acknowledge the Geodesy Section, Department of Surveying and Mapping Malaysia (DSMM) for providing the data used in this study.

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