An Approach to Determine a Pair of Power-Flow Solutions Related to the Voltage Stability of Unbalanced Three-Phase Networks

Mamdouh Abdel-Akher, Mahrous E. Ahmad, Rabindra N. Mahanty, and Khalid Mohamed Nor, *Senior Member, IEEE*

***Abstract*—This paper proposes an approach to determine a pair of power-flow solutions associated with the voltage stability of unbalanced three-phase networks. The approach is derived from the observations of the multiple three-phase power-flow solutions of a two-bus network. It is found that there are two pairs of solutions at the load bus. The plot of the voltage magnitude of each pair against the power demand at the load bus shows that two possible PV curves can be constructed for each phase. Each of the two curves is a combination of the two pairs of solutions. One of these curves is associated with the voltage stability of the system whereas the other is associated with the imbalance of the three-phase network. Based on the above observations, a constant impedance load model is utilized to calculate the solution associated with the voltage stability of the study system. Then the equivalent complex power load demand is used to calculate the two pairs of solutions, i.e., the multiple three-phase power-flow solutions. Simulation studies have been carried out for the multiple solutions. The results show that there is a point which is directly proportional to the imbalance in the power demand at the load bus. This point is used to set a criterion to differentiate between the two PV curves. Hence, the PV curve which is related to the voltage stability can be determined without the assumption of the linear load model at the start of the study.**

***Index Terms*—Multiple solutions, neutral voltage, PV curves, three-phase power-flow, voltage stability.**

I. Introduction

VOLTAGE STABILITY is an important aspect of power systems operation and control. The continuous growth of loads with a limited reserve capacity and economical constraints force both transmission and distribution networks to operate close to their voltage stability limits. Traditionally, the voltage stability problem involves the calculation of the multiple power-flow solutions at all possible loading conditions. This is referred to as the continuation power-flow method [1]–[6].

In balanced three-phase or positive-sequence networks, the voltage stability problem involves the calculation of a pair of power-flow solutions [1]–[3]. One of the two power-flow solutions is the stable one whereas the other is the unstable one. Recently, the continuation power flow method has been used to study the voltage stability in the case of unbalanced three-phase networks [4]–[6]. In the proposed methods, multiple solutions which are associated with the imbalance in the system [7], [8] are not considered, and hence only a pair of solutions is calculated at each phase as in case of balanced networks.

However, in unbalanced power systems, with zero-sequence blocking transformers [7] or ungrounded loads [8], additional pair of solutions exists which is related to the degree of the imbalance of the system. Hence, it will be challenging to study the voltage stability due to the existence of more than one pair of the power-flow solutions. The multiple solutions phenomenon requires further understanding and investigation to differentiate between power-flow solutions related to the voltage stability and those related to the system imbalance [4]–[8].

The aim of the current paper is to investigate the multiple solutions phenomenon in case of unbalanced networks with zero-sequence blocking transformer [4] or ungrounded load [5]. The proposed study is performed for a two-bus network with an ungrounded unbalanced load. The study shows that there are two possible pairs of solutions for the terminal voltage at the load bus. Consequently, there are four possible solutions at the load bus. Two of these solutions are related to the degree of the imbalance in the load whereas the other two solutions are related to the classical voltage stability of the study system. Based on the two pairs of solutions, two PV curves can be constructed for each phase. Each PV curve is a combination of two solutions at the load bus. One of the two PV curves is related to the voltage stability of the system.

An important observation in course of the proposed study is that there is a unique point on the locus of the terminal voltage solutions at which two PV curves can be constructed. It has also been observed that this point is a function of the degree of imbalance of the load. Based on this observation, a criterion is set to differentiate between the two PV curves. The criterion is found out from the ratios among the complex power demands of the three phases.

The paper is organized as follows. In Section II, the load model and basic equations are presented. Determination of the two PV curves from the multiple solutions is discussed in Section III. A criterion to differentiate between the two PV curves is presented in Section IV. In Section V, the application to real problems is proposed. The conclusions are drawn in Section VI.

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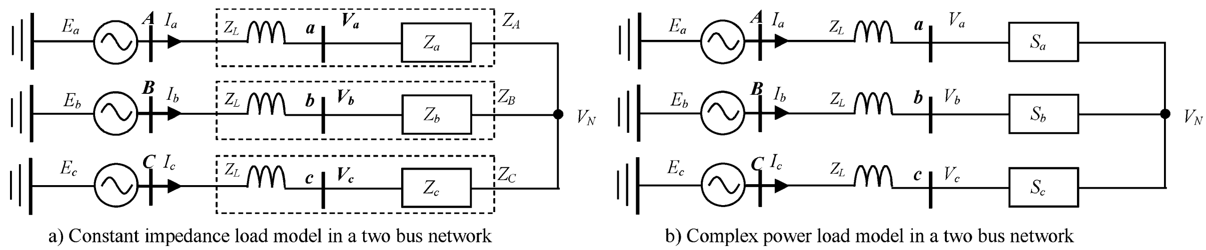

Fig. 1. Two-bus network. Ea $= 1.0\angle 0°$, Eb $= 1.0\angle -120°$ p.u., Ec $= 1.0\angle 120°$ p.u. $Z_L = j0.004$ p.u., and $Z_a = Z_b = Z_c = 0.0467 + j0.0204$ p.u..

II. Load Model, Test System, and Basic Equations

It is well known that the multiple solutions exist in case of the constant complex power load model [8]. However, if a constant impedance load model is assumed, a direct solution for the voltage at the neutral point is possible.

The two-bus networks shown in Fig. 1(a)-(b) are for constant impedance and constant complex power models respectively, each of them represent a practical system. Hence, the voltage at the neutral node in both the networks will be the same. The series impedance Z_L may represent the source, line, or Thévenin's equivalent of a network [9].

The calculation of the neutral voltage in each network is as follows.

A. Constant Impedance Load Model

The following Kirchhoff's current law equation can be obtained at the neutral node:

$$I_a + I_b + I_c = 0. \tag{1}$$

Substituting the loads and source voltages yields

$$\frac{E_a - V_N}{Z_A} + \frac{a^2 E_a - V_N}{Z_B} + \frac{aE_a - V_N}{Z_C} = 0. \tag{2}$$

where $Z_K = Z_k + Z_L$, k refers to nodes "a," "b," or "c," K refers to nodes "A," "B," or "C," Z_k is the load impedance, X_L is the line impedance, Z_K is the total impedance, and $a = 1.0\angle -120°$. Rewriting (2), the voltage at the neutral node is calculated as follows:

$$V_{N0} = \left(\frac{a\sigma + a^2\mu + \mu\sigma}{\sigma + \mu + \mu\sigma}\right) E_a. \tag{3}$$

where the subscript "0" in V_{N0} refers to solution "0," and

$$\mu = \frac{Z_B}{Z_A},\ \sigma = \frac{Z_C}{Z_A}.$$

The solution "0" of the line current I_{k0} and the terminal voltage V_{k0} can be calculated as follows:

$$I_{k0} = \frac{E_k - V_{N0}}{Z_K} \tag{4}$$

$$V_{k0} = E_k - I_{k0} Z_L. \tag{5}$$

B. Constant Complex Power Load Model

Equation (1) shown above is valid in case of this model also. Therefore

$$\frac{S_a}{V_{a0} - V_N} + \frac{S_b}{V_{b0} - V_N} + \frac{S_c}{V_{c0} - V_N} = 0 \tag{6}$$

where

$$S_k = |I_{k0}|^2 Z_k. \tag{7}$$

Equation (6) can be reduced to the following equation:

$$L\left(\frac{V_N}{V_{a0}}\right)^2 + M\left(\frac{V_N}{V_{a0}}\right) + N = 0 \tag{8}$$

where

$$\begin{aligned} L &= 1 + \alpha + \beta \\ M &= -(\gamma + \omega + \alpha + \alpha\omega + \beta + \beta\gamma) \\ N &= \gamma\omega + \alpha\omega + \beta\gamma \\ \alpha &= \frac{S_b}{S_a},\ \beta = \frac{S_c}{S_a},\ \gamma = \frac{V_{b0}}{V_{a0}},\ \text{and } \omega = \frac{V_{c0}}{V_{a0}}. \end{aligned}$$

Equation (8) can be solved directly as follows:

$$V_{N1,2} = \left(\frac{-M \pm \sqrt{M^2 - 4LN}}{2L}\right) V_{a0}. \tag{9}$$

Equation (9) gives two solutions for the voltage at the neutral node. Consequently, there are two solutions for the line currents and terminal voltages, which can be calculated using (4)–(5). Table I illustrates the results obtained from the solutions of the models shown in Fig. 1.

III. Multiple Power-Flow Solutions

The aforementioned models are used to calculate the profiles of terminal voltages when the load impedance is varied from short to open circuit states. The linear load model in Fig. 1(a) will give a single solution which is referred to as solution "0" in Table I. However, the nonlinear load model shown in Fig. 1(b) gives two solutions of the terminal voltages, which are referred to as solution "1" and solution "2," respectively. One of these two solutions, or a combination of them, should be identical to solution "0."

A. Procedure to Calculate the Multiple Solutions

The power-flow solutions shown in Table I are calculated according to the process illustrated in Fig. 2. The solutions loci

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TABLE I
MULTIPLE POWER-FLOW SOLUTIONS DUE TO THE UNBALANCE IN THE NETWORKS SHOWN IN FIG. 1

Serial	Variable	Constant impedance load model	Constant complex power load model	
		Result of solution 0	Result of solution 1	Result of solution 2
Neutral voltage	V_N	$V_{N0}=\left(\dfrac{a\sigma+a^2\mu+\mu\sigma}{\sigma+\mu+\mu\sigma}\right)E_a$	$V_{N1}=\left(\dfrac{-M+\sqrt{M^2-4LN}}{2L}\right)V_{a0}$	$V_{N2}=\left(\dfrac{-M-\sqrt{M^2-4LN}}{2L}\right)V_{a0}$
Line current	I_k	$I_{k0}=\dfrac{E_k-V_{N0}}{Z_K}$	$I_{k1}=\left(\dfrac{S_k}{V_{k0}-V_{N1}}\right)^*$	$I_{k2}=\left(\dfrac{S_k}{V_{k0}-V_{N2}}\right)^*$
Terminal voltage	V_k	$V_{k0}=E_k-jI_{k0}X_L$	$V_{k1}=E_k-jX_LI_{k1}$	$V_{k2}=E_k-jX_LI_{k2}$

Where k refers to phases 'a', 'b', or 'c', K refers to phases 'A, 'B', or 'C'. The superscript '* 'refers to the conjugate of a complex number

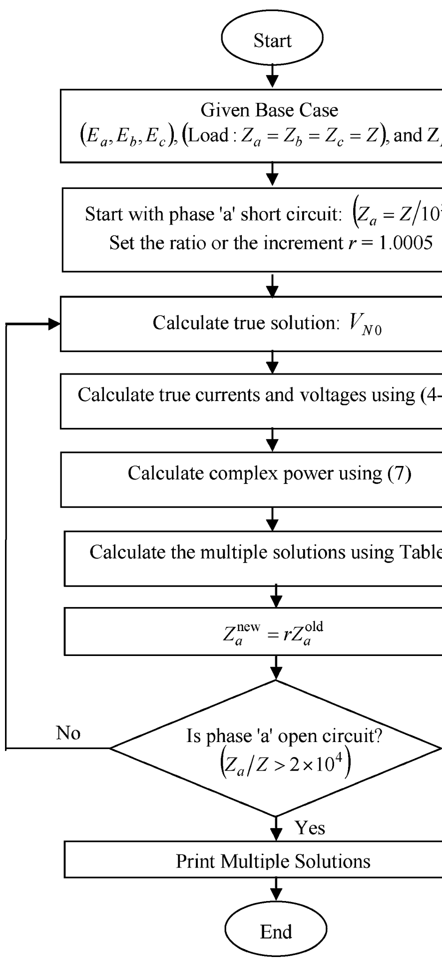

Fig. 2. Procedure for multiple solutions.

can be obtained by varying the impedance Z_a from 0 to infinity. This means that the complex power demand at phase "a" will vary from zero to its maximum value. The impedances of phases "b" and "c" are kept constant at their nominal values. The variation of the impedance Z_a will in turn cause variation in the total complex power demand for the three-phases. This means $Z_b = Z_c$ with varying Z_a will not guarantee similar changes in the complex power demands S_a, S_b, and S_c due to the nonlinearity in the system.

The solution "0" locus of the load terminal voltage, shown in Fig. 3, is calculated from the linear impedance load model. This

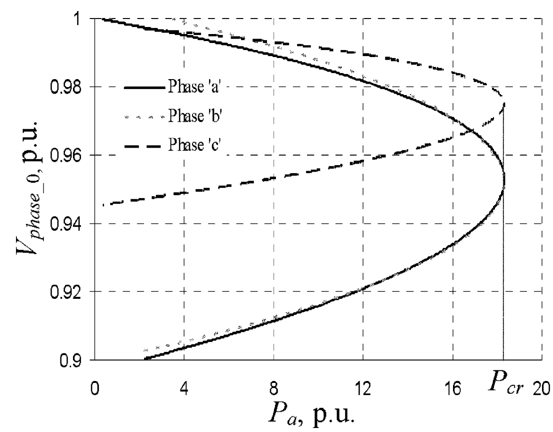

Fig. 3. Locus of the solution "0" at the load terminal.

locus represents the PV curve of phase "a." Similar results are obtained for phases "b" and "c" as shown in Fig. 3.

B. Loci of Multiple Power-Flow Solutions

The loci of the terminal voltage solutions "1" and "2" in Table I are shown in Fig. 4. The locus for each solution comprises of two solutions as shown in Fig. 4(a) and (b), respectively, for the three-phase voltages. Consequently, in all, there will be four possible solutions for the terminal voltages. Two out of these solutions are related to the degree of the imbalance in the system whereas the other two solutions are related to the voltage stability problem.

C. Observations on the Loci of Multiple Power-Flow Solutions

The points x_1 to x_7 on the loci of the terminal voltage are shown in Fig. 4 for the three-phases. Amongst these seven points, there are three distinct points: x_2, x_3, and x_6, and they are marked for each phase. At these points, two of the four possible power-flow solutions merge into one solution. The three points represent the boundaries of the study system. Therefore, they are classified as bifurcation points [10]–[12].

1) Points x_3 and x_6: The points x_3 and x_6 represent the nose curve of the traditional voltage stability limits. At these points, the critical power P_{cr}, i.e., the maximum power demand, is the same in case of the two solutions: solution "1" and solution "2," with different critical voltage values. Consequently, for

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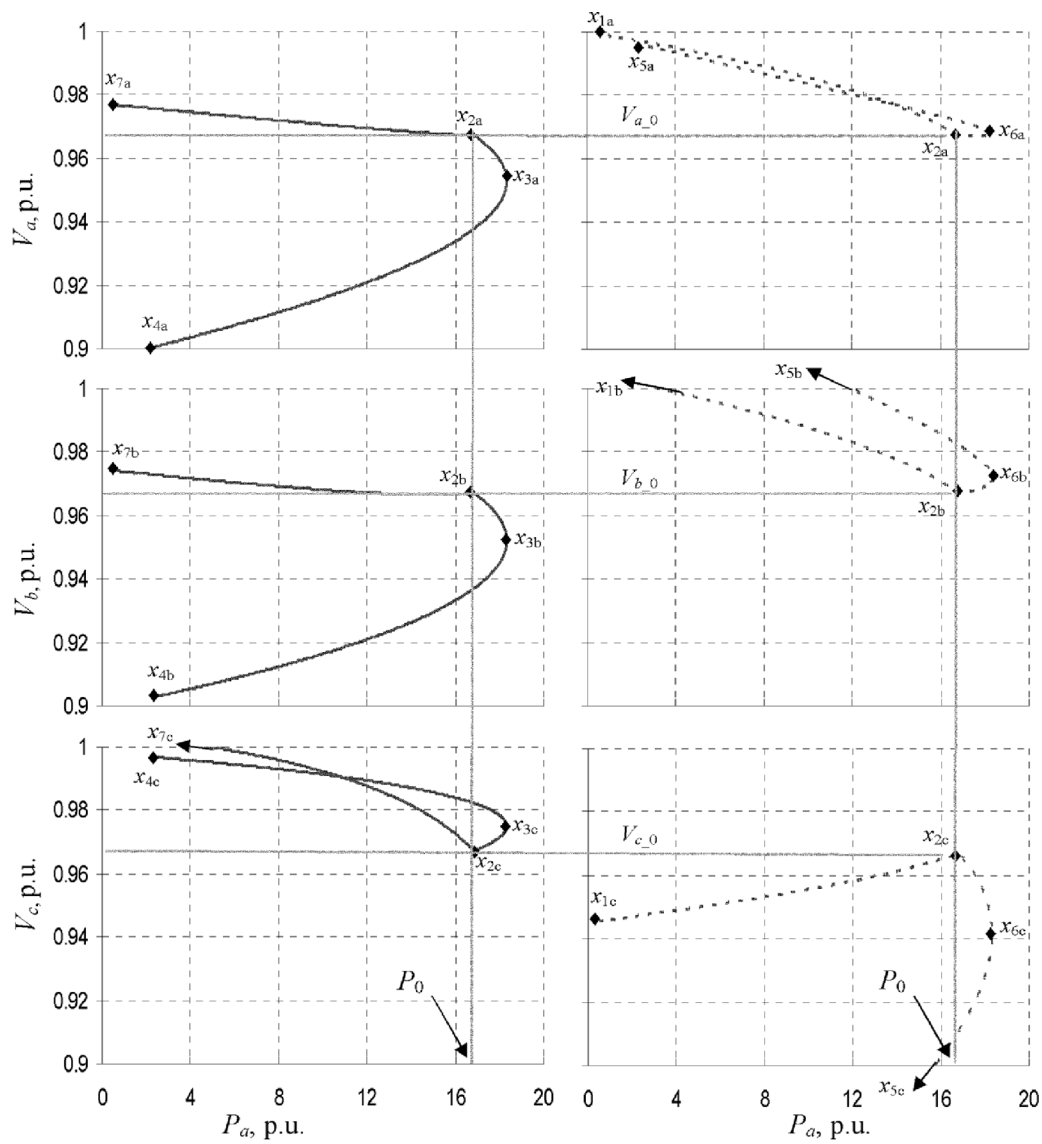

a) Solution 1, corresponds to V_{N1} b) Solution 2, Corresponds to V_{N2}

Fig. 4. Multiple three-phase power-flow solutions, locus of terminal voltage at the load bus.

each phase, the two critical voltages occur at the same maximum power demand. The coordinates for the points x_3 and x_6 are as follows:

$$x_{3k} : (V_k = V_{k\_cr1}, P_k = P_{cr}) \tag{10a}$$

$$x_{6k} : (V_k = V_{k\_cr2}, P_k = P_{cr}). \tag{10b}$$

where k refers to the phases "a," "b," and "c."

2) Point x_2: The third point of importance is x_2. At this point, two of the four possible solutions merge into one. For example, at the locus of phase "a," the point x_{2a} in Fig. 4(a) is the same as the point x_{2a} in Fig. 5(b). This is also valid for phases "b" and "c." The coordinates for the point x_2 are expressed as follows:

$$x_{2k} : (V_k = V_{k0}, P_k = P_0) \tag{11}$$

where P_0 is the power at which the two different solutions, i.e., solution "1" and solution "2" merge into one.

As has been discussed in the previous section, the points x_3 and x_6 exhibit the traditional stability limits, the third point x_2 needs further investigation. The importance as well as the derivation of the point x_2 will be discussed in detail in the following sections.

IV. Three-Phase PV Curves

Based on the aforementioned distinct points, two independent PV curves can be constructed as shown in Fig. 5. The two PV curves, for the three phases, have two different noses, viz.: the points x_3 and x_6. In addition, the two PV curves should pass through the point x_2. The first PV curve has the path $x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4$, while the second PV curve has the path $x_5 \rightarrow x_6 \rightarrow x_2 \rightarrow x_7$ as shown in Fig. 5.

A. Combined Solution 1: The Path $x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4$

The tracing direction of the first PV curve, shown in Fig. 5(a) for phase "a" and phase "b," is clockwise similar to the classical positive sequence network PV curve. Therefore, the upper part

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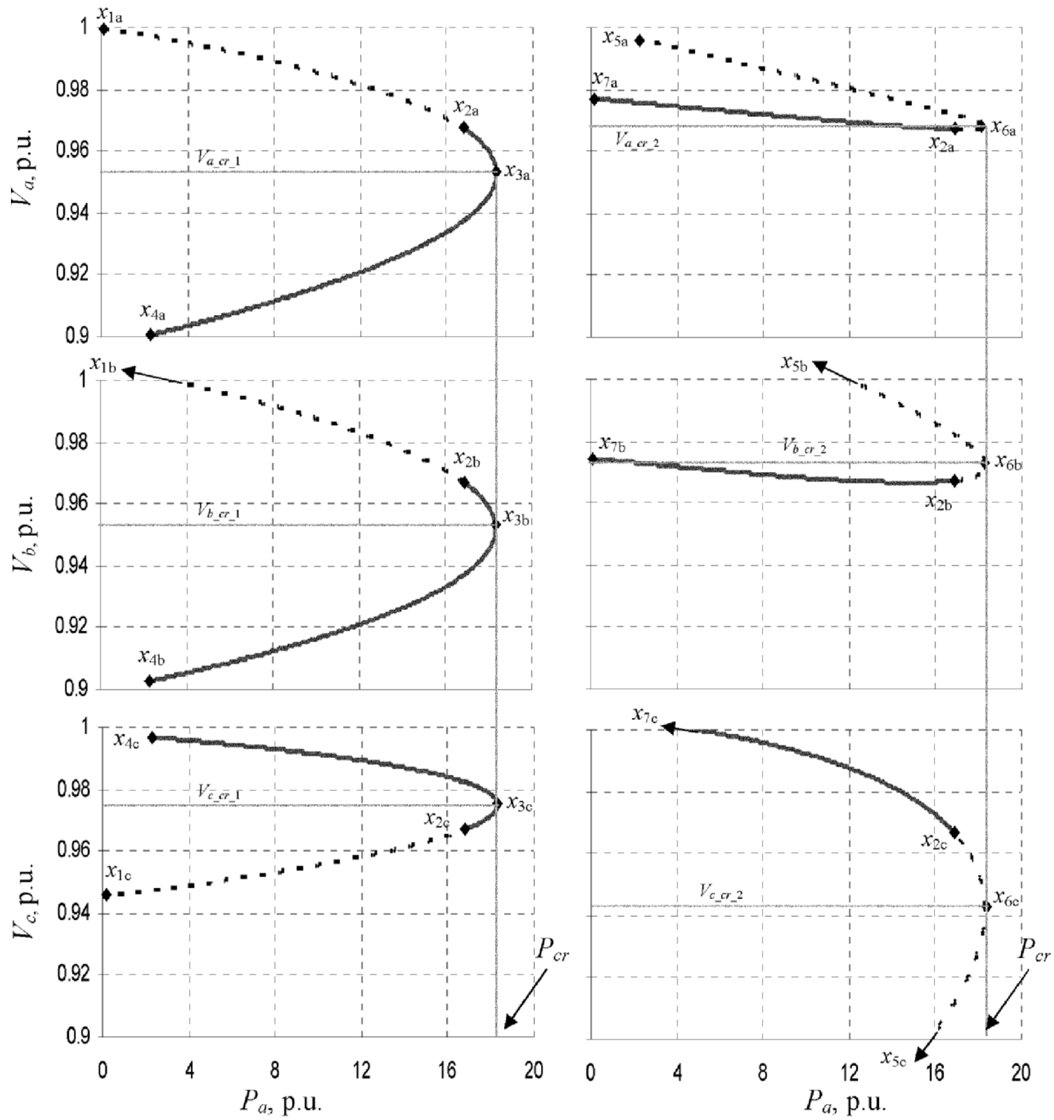

a) Combined Solution 1, Path: $x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4$ b) Combined Solution 2, Path: $x_5 \rightarrow x_6 \rightarrow x_2 \rightarrow x_7$

Fig. 5. PV curves of three-phase system "a."

of this curve corresponds to the stable power-flow solutions and the lower part corresponds to the unstable power-flow solutions. On the other hand, the tracing direction of the PV curve, shown in Fig. 5(a) for phase "c," is anti-clockwise. In this case, the lower part corresponds to the stable power-flow solutions and the upper part corresponds to the unstable power-flow solutions [4].

B. Combined Solution 2: The Path $x_5 \rightarrow x_6 \rightarrow x_2 \rightarrow x_7$

Similar to the above case, Fig. 5(b) shows the combined solution "2" which exhibits three-phase PV curves. The tracing direction of the PV curves of phases "a" and "b" is clockwise whereas it is anti-clockwise for the PV curve of phase "c."

C. Comparison With the Solution "0"

The above discussion shows that it is difficult to decide which of the two PV curves shown in Fig. 5 is related to the voltage stability of the system. However, a comparison with the terminal voltage locus calculated using the linear load model, solution "0" given in Fig. 3, shows that it is identical to the combined solution shown in Fig. 5(a). This shows that the solution "0" locus can be obtained from the distinct points x_2, x_3, and x_6 without the assumption of a linear model at the start of the study.

V. Discrimination Between the Two PV Curves

As has been mentioned above, the knowledge of the point x_2 is very important to construct the two PV curves from the multiple solutions shown in Fig. 4. The point x_2 has similar characteristics to the points x_3 and x_6. At the points x_3 and x_6 two solutions merge into one, and they also differentiate between two solutions: one solution is the stable one whereas the other is the unstable one. Coming back to the point x_2, also there are two solutions merging into one, and at this point two PV curves can be constructed. The similarity in the characteristics of point x_2 with the points x_3 and x_6 shows that the point x_2 can be used to set a criterion to choose the PV curve shown in Fig. 5 that represents the solution "0" shown in Fig. 3. Consequently, the point x_2 can be used to set the criterion to differentiate between

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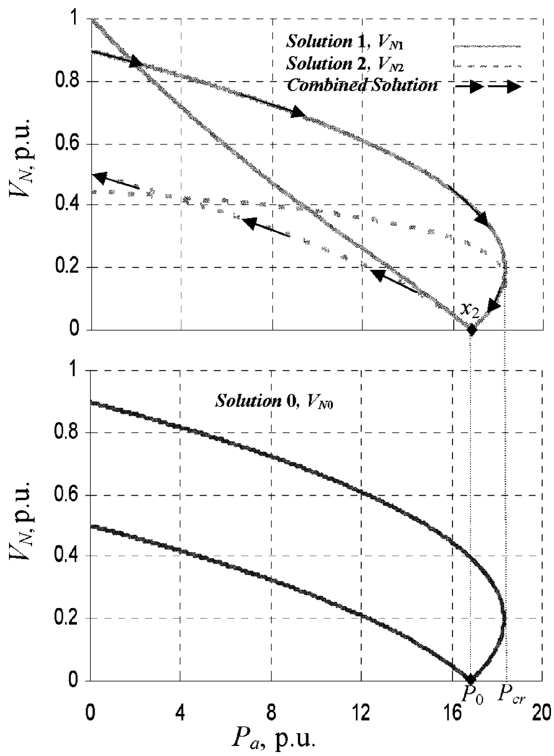

Fig. 6. Locus of the magnitude voltage at the neutral node.

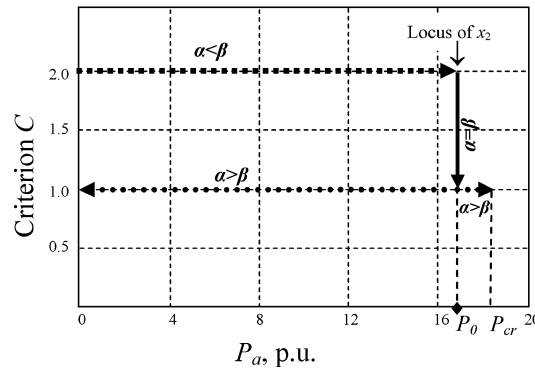

Fig. 7. Multiple solutions criterion due to unbalanced three-phase load.

the two PV curves, i.e., to identify which curve is related to the voltage stability study.

Fig. 6 shows the locus of the solution "0" of the neutral voltage as well as that of solution "1" and "2." The locus shows the point x_2 at which the two solution "1" and "2" merges into one, which is also the solution "0." The point x_2 can be calculated using (9) as follows:

$$V_{N1} = V_{N2} = \frac{-M}{2L} V_{a0}. \tag{12}$$

Equation (12) is valid when the distinguished of (9) equals to zero, i.e.,

$$f(\alpha, \beta, \omega, \gamma) = M^2 - 4LN = 0. \tag{13}$$

It is difficult to solve (13) in a closed form since f is a function of both the load imbalance factors α and β and the terminal voltage imbalance factors ω and γ. The variables α, β, ω, and γ are complex numbers. Therefore, f will be a function of eight variables representing the real and imaginary parts of the system unbalance factors.

For the two-bus network shown in Section II, a direct solution can be obtained for (13), if the terminal voltages at the load bus are assumed to be balanced, the factors ω and γ are equal to $1.0\angle 240^\circ$ and $1.0\angle 120^\circ$, respectively. As it has been mentioned in Section II, the power factor of the load has been considered to be constant. This implies that the imbalance factors α and β are real numbers.

The solution of (13), given in the Appendix , gives four points at which the two solutions merge into one. The solution is of interest when both the imbalance factors α and β are equal to one. The other solutions are not visible. This is because the generation in power system is balanced and the imbalance usually arises from the load or the asymmetrical elements in the network. Consequently, the point x_{2k} in Fig. 6 corresponds to the balanced load condition as follows:

$$\alpha = \beta = 1.0. \tag{14}$$

The other three solutions are not shown in Fig. 6. In the system under study, the load impedance is being changed from open circuit to short circuit. This change in the impedance will cause variation in the absorbed complex power from zero to a maximum value at the load bus.

So far the point x_2 has been considered to be the point that represents the balanced load; also, at this point, the solution "0" shifts from one solution to the other. Consequently, the relation between the factors α and β can directly determine which of the two solutions is identical to the solution "0." On the basis of the above discussion, a criterion has been found for determining the solution "0" from the multiple solutions as follows:

$$\alpha > \beta \quad \text{Solution 1 is the same as solution 0} \tag{15a}$$
$$\alpha < \beta \quad \text{Solution 2 is the same as solution 0} \tag{15b}$$
$$\alpha = \beta \quad \text{Solution 1} = \text{solution 2}. \tag{15c}$$

This can be expressed by the factor C as a criterion as follows:

$$\text{If solution 1 is the same as solution 0 set} \quad C = 1.0 \tag{16a}$$
$$\text{If solution 2 is the same as solution 0 set} \quad C = 2.0. \tag{16b}$$

where the factor C refers to the solution identification number

The factor C is plotted in Fig. 7 which shows that at point x_2, the solution is shifted from one solution to the other according to (16).

In summary, the significance of the study can be stated as follows:

1) Two pairs of possible power-flow solutions exist in the case of unbalanced networks with a zero sequence blocking transformer or with an ungrounded unbalanced load.
2) The locus of each pair is not meaningful. However, a combination can be constructed based on these pairs of solutions to construct two PV curves. The key point to construct the two PV curves is the point at which the two PV

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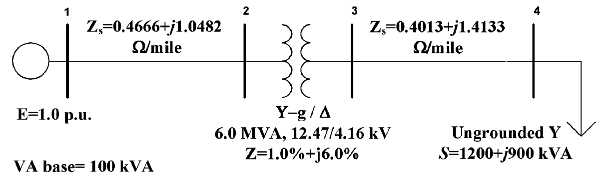

Fig. 8. Modified IEEE four-node feeder with an ungrounded load.

curves pass, i.e., the point x_2. One of these curves must represent the actual solution which is calculated from the linear load model.

3) The point x_2 is a kind of a bifurcation point at which equilibrium occurs [10]–[12]. Hence, the point x_{2k} is similar to the nose of the PV curve in the voltage stability studies. The nose of the PV curve differentiates between two solutions. One solution is the stable solution whereas the other is the unstable solution. In comparison, the point x_2 also differentiates between two PV curves. One PV curve is related to the voltage stability of the system whereas the other is related to the imbalance of the system.
4) The solution which is related to the imbalance of the system is probably not realistic and appears due to the complex power load model assumption.
5) The point x_2 was derived for the two-bus network, to demonstrate the aforementioned facts. Based on the assumed model at the start of the study, a criterion has been set to select the actual PV curve or that corresponding to the network voltage stability curve.

VI. Application to Real Problems

A. Application to Radial Distribution Networks

A distribution network is characterized by its radial structure and is usually fed from a single power source. For a distribution network it is possible to perform the voltage stability analysis based on the Thévenin's equivalent two node model. For an ungrounded load connected at node "i," the Thévenin's equivalent can be calculated as follows [9].

1) Disconnect the ungrounded load at node "i" and then run the conventional power-flow program to calculate the no load voltage at node "i."

2) Calculate the Thévenin's impedance at node "i" by formulating the system impedance matrix or by any alternative approach.

As the two-node network model is constructed similar to that shown in Fig. 2(b), the PV curve corresponding to the voltage stability can be determined according to the selection criterion given by (15).

The IEEE four-node feeder, shown in Fig. 8, is used to test out the proposed approach. The feeder comprises of two line segments, Y-g/D step down transformer and unbalanced and ungrounded load connected at bus "4." The original feeder data are modified such that the unbalance is only due to the unbalanced load.

The power-flow (6) at bus "4" is solved numerically according to the power-flow procedure given in Appendix B. A balanced load $S = 1200 + j\ 900$ kVA per phase at bus "4" is considered as the base case for the analysis. The load of phases "a" and "c" are kept constant as the base value whereas the load of phase "b" is changed in steps such that all possible loading conditions are considered.

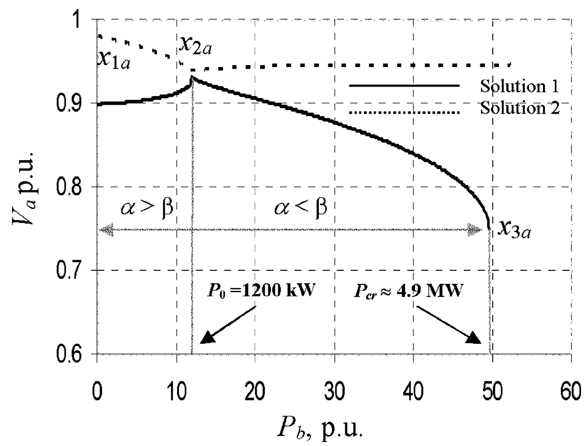

Fig. 9. Power-flow solutions at bus "4" of the system shown in Fig. 8.

1) Selection of the Real Power-Flow Solution: The solution of the terminal voltage at phase "a" is shown in Fig. 9 for all possible loading conditions of phase "b." There are two solutions corresponding to the neutral voltages calculated from (9). These two solutions represent two halves of the PV curves obtained in Section IV. As the demand of phase "b" becomes close to the demand of the other phases, i.e., phases "a" and "c," the convergence becomes difficult. This is mainly due to the characteristics of the point x_2 as a bifurcation point. The conventional power-flow methods must be modified in order to obtain reliable results near the bifurcation point x_2 [10].

Although the results shown in Fig. 9 may not be highly accurate due to the simple power-flow adopted, as illustrated in Appendix B, the results show that both solution "1" and solution "2" move towards the point x_2 as the demand of phase "b" increases. Then the slope of each solution completely changes. Hence a combination of the two solutions is required to find out the actual behavior of the system with the increase in the demand.

The selection criterion given by (15) shows that the voltage profile consists of solution "2" when $\alpha > \beta$, solution "1" when $\alpha < \beta$ and the two solutions merge into one at the point x_2. Consequently, the real behavior of the terminal voltage with the increase in the demand of phase "b" is determined by the path $x_1 \rightarrow x_2 \rightarrow x_3$ as shown in Fig. 9.

2) Effect of the Balanced Load Base Case on the Stability Limit of the Study System: When the balanced load base case is changed, the positions of the points x_2 and x_3 also change. This is demonstrated in Fig. 10. The figure shows both solution "1" and solution "2" at different balanced load base cases. For each balanced load base case the point x_2 is different and also stability limit x_3 is different. The total maximum demand for any of the tested cases in Fig. 10 can be determined as follows:

$$P_{\max} = P_a + P_b + P_c = 2P + P_{cr}. \tag{17}$$

where $P_a = P_c = P$ and P_b is varied from zero up to the maximum value P_{cr}.

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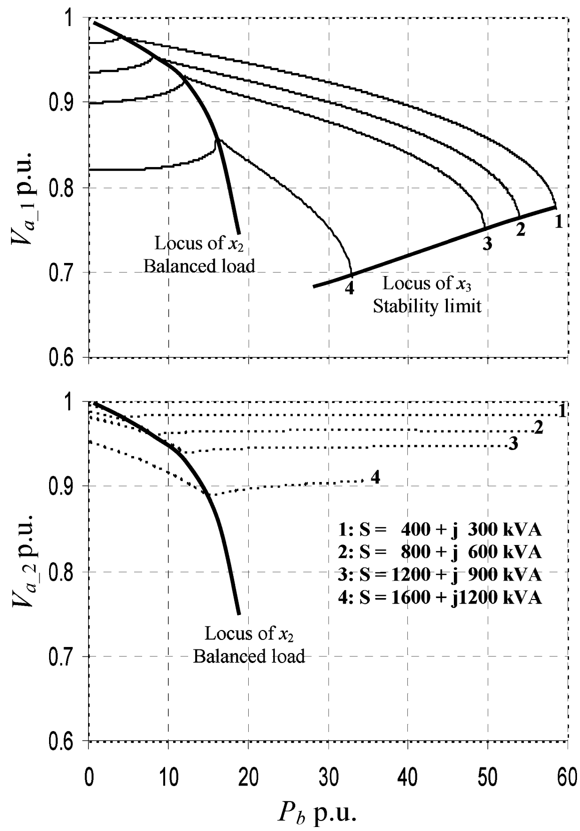

Fig. 10. Effect of the balanced load base case on the maximum power transferred to phase "b" of the load.

The total maximum power transferred through the three-phase network can be estimated based on the results shown in Fig. 10 and (17) as follows:

Case 1: $P_{\max} = 2 \times 400 + 5800 = 6600$ kW
Case 2: $P_{\max} = 2 \times 800 + 5500 = 7100$ kW
Case 3: $P_{\max} = 2 \times 1200 + 4900 = 7300$ kW
Case 4: $P_{\max} = 2 \times 1600 + 3300 = 6500$ kW

The above calculations show that the contribution of the load among the phases affects the stability limit of three-phase networks.

B. Application to Meshed Transmission Networks

Due to the existence of many generators in transmission networks, the application of the equivalent Thévenin's network is not suitable for these cases and a full continuation power-flow must be used. However, the existing three-phase power-flow algorithms in the available literature cannot handle systems with isolated neutral points or grounding problems [7]. The conventional power-flow should be extended first to account for neutral voltages. Hence, a suitable continuation power-flow method can be applied [10]–[12]. Due to the complexity of power system modeling requirements, this portion has been left for future research.

VII. Conclusions

The paper has presented an approach which can be used to differentiate between the multiple power-flow solutions. The approach is helpful to study the voltage stability problem for unbalanced networks with multiple solutions. In this paper, two identical two-bus networks with constant impedance and with constant complex power load models have been used. The solution of the first network is used to determine the locus of the PV curve related to the voltage stability analysis. The solution of the second network leads to four possible solutions. It is found that there is a unique point at which two out of the four solutions merge into one and two different PV curves can be constructed. This point is proportional to the degree of the imbalance of the system. One of the PV curves is identical to the PV curve constructed from the constant impedance load model. Based on this observation, a criterion has been set to differentiate between the two PV curves. The criterion is found out from the ratios among the complex power demands of the three phases.

Appendix A
Derivation of the Solution at the Point x_2

The imbalance factors due to the source voltage are defined as follows:

$$\omega = a^2 = 1.0\angle 240^\circ = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \qquad \text{(A.1a)}$$

$$\gamma = a = 1.0\angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}. \qquad \text{(A.1b)}$$

From (7)

$$L = 1 + \alpha + \beta \qquad \text{(A.2a)}$$

$$M = -(\gamma + \omega + \alpha + \alpha\omega + \beta + \beta\gamma) \qquad \text{(A.2b)}$$

$$N = \gamma\omega + \alpha\omega + \beta\gamma. \qquad \text{(A.2c)}$$

Substituting (A.1) in (A.2), the following equations are obtained:

$$L = 1 + \alpha + \beta \qquad \text{(A.3a)}$$

$$M = 1 + a^2\alpha + a\beta \qquad \text{(A.3b)}$$

$$N = 1 + a\alpha + a^2\beta. \qquad \text{(A.3c)}$$

The imbalance factors α and β are assumed to be real numbers. This is because of the assumption of a constant power factor. Then substituting (A.3) into (13) yields

$$f = \left(1 + a^2\alpha + a\beta\right)^2 - 4(1 + \alpha + \beta)\left(1 + a\alpha + a^2\beta\right). \qquad \text{(A.4)}$$

Substituting (A.1) in (A.3) yields

$$\operatorname{Re}(f) = \alpha^2 + \beta^2 + 4\alpha\beta - 2\alpha - 2\beta - 2 = 0 \qquad \text{(A.5a)}$$

$$\operatorname{Im}(f) = -\alpha^2 + \beta^2 - 2\alpha + 2\beta = 0. \qquad \text{(A.5b)}$$

Solution of (A.5) gives the following possible locations for the point x_2:

Solution 1: $\alpha = 1$ $\beta = 1$
Solution 2: $\alpha = 1$ $\beta = -3$
Solution 3: $\alpha = -3$ $\beta = 1$
Solution 4: $\alpha = -1/3$ $\beta = -1/3$.

Appendix B
Power-Flow Solution of Two-Bus Network With an Ungrounded Star Connected Load

The power-flow (6) at the load bus is solved numerically to calculate both the terminal voltages as well as the neutral voltage of the load. The iterative solution process involves the following steps:

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1) Assume initial terminal voltage at the load bus,

2) Calculate the unbalance factors α, β, ω, and γ. Then, calculate the two solutions of the neutral voltage according to (8).

3) Calculate the line currents in terms of the specified load for each phase "k"

$$I_{k1} = \frac{S_a}{V_{k1} - V_{N1}} \quad \text{(B.1a)}$$

$$I_{k2} = \frac{S_a}{V_{k2} - V_{N2}}. \quad \text{(B.1b)}$$

4) Calculate the updated terminal voltages that correspond to both solution "1" and solution "2"

$$V_{k1} = E_{k,th} - Z_{kk,th} I_{k1} \quad \text{(B.2a)}$$

$$V_{k2} = E_{k,th} - Z_{kk,th} I_{k2}. \quad \text{(B.2b)}$$

5) Calculate the power mismatches at the load bus as follows:

$$\Delta S_{k1} = S_k - (V_{k1} - V_{N1})(I_{k1})^* \quad \text{(B.3a)}$$

$$\Delta S_{k2} = S_k - (V_{k2} - V_{N2})(I_{k2})^* . \quad \text{(B.3b)}$$

Check if the program has converged; if not, go to step 2.

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