Ground Receiving Station Reference Pair Selection Technique for a Minimum Configuration 3D Emitter Position Estimation Multilateration System

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Abstract. *Multilateration estimates aircraft position using the Time Difference Of Arrival (TDOA) with a lateration algorithm. The Position Estimation (PE) accuracy of the lateration algorithm depends on several factors which are the TDOA estimation error, the lateration algorithm approach, the number of deployed GRSs and the selection of the GRS reference used for the PE process. Using the minimum number of GRSs for 3D emitter PE, a technique based on the condition number calculation is proposed to select the suitable GRS reference pair for improving the accuracy of the PE using the lateration algorithm. Validation of the proposed technique was performed with the GRSs in the square and triangular GRS configuration. For the selected emitter positions, the result shows that the proposed technique can be used to select the suitable GRS reference pair for the PE process. A unity condition number is achieved for GRS pair most suitable for the PE process. Monte Carlo simulation result, in comparison with the fixed GRS reference pair lateration algorithm, shows a reduction in PE error of at least 70 % for both GRS in the square and triangular configuration.*

Keywords

Condition number, lateration algorithm, minimum configuration, multilateration, reference selection.

1. Introduction

Passive wireless positioning and navigation systems utilize aircraft transponder emission which is detected with the support of antenna of a Ground Receiving Station (GRS) for determining the position of the aircraft. Determining the position of an aircraft is a two-stage process [1]. The first stage involves the estimation of the position dependent signal parameter from the received aircraft transponder emission. Some examples of position dependent signal parameters are the Angle Of Arrival (AOA), the Time Of Arrival (TOA), the Time Difference Of Arrival (TDOA) and Receive Signal Strength (RSS). In the second stage, the position dependent signal parameter estimated at the first stage is input into a Position Estimation (PE) algorithm to determine the position of the aircraft. This is known as the PE process. Examples of the PE algorithm used at the PE process are angulation, fingerprinting and lateration. Multilateration system is an example of a wireless positioning system. The system estimates TDOA from the received aircraft transponder emission as its position dependent signal parameter and uses the lateration algorithm to determine the aircraft position [1], [2] and [3]. It consists of several specially placed GRSs all connected to a central processing unit. Depending on the number of GRS deployed, 2-Dimension (2D) or 3-Dimension (3D) position of the aircraft is resolved. For 3D PE, there is a minimum of four GRSs needed [3]. Many studies have described methods for TDOA estimation [4], [5], [6], [7] and [8]. For example, [4] compares TDOA estimation using cross-correlation and fast cross-correlation to determine which method is faster for practical and theoretical implementation. In [8], Signal-to-Noise Ratio (SNR) is used for the benchmark to compare the performance of five different TDOA estimation techniques.

The PE process is the scope of this work. Depending on the number of GRS (N), $N-1$ nonlinear hyperbolic equations are generated [9]. Several approaches developing the lateration algorithms have been proposed in articles [9], [10], [11], [12], [13], [14] and [15] which can be grouped into two as a linear and nonlinear approach [1] and [11]. The non-linear approach involves the use of linear approximation and iterative methods such as Taylor's series expansion to perform PE [9], [10] and [11]. The linear algorithm involves algebraically manipulating the hyperbolic equations to directly set an inverse problem that linearly relates the unknown aircraft position to the known TDOA measurements as described in [12], [13], [14] and [15]. Due to the convergence issue and the use of initial position estimated for the non-linear approach [10], this study focuses on using the linear approach to developing the lateration algorithm.

The linear lateration algorithm has been characterized with high PE error. Numerous researchers have proposed techniques such as weighting functions [16], total least squares [17] and Tikhonov regularization [9] for improving the PE accuracy of the lateration algorithm. These techniques efficiently to use but require at least five GRSs to be deployed. The minimum GRS deployed for 3D PE is four. Thus, these techniques cannot be used to improve PE accuracy of the lateration algorithm for 3D minimum configuration.The use of more than one GRS as a reference is suggested for improving the accuracy of the lateration algorithm [6] and [18]. The choice of the reference GRS has been reported to improve the PE accuracy of the lateration algorithm [19], [20] and [21]. In [20], a TDOA residual-based method was proposed to select the suitable GRS as a reference for PE with lateration algorithm in an active system. It was assumed that the emitter position is known but there was the need to continuously track the position of the system using another system. Using the known emitter position, each of the deployed GRS is used as a reference and the GRS that resulted in the least TDOA residual is chosen as a reference for subsequent estimation of the emitter position. An SNR based GRS reference selection method was proposed in [21]. With the assumption that all noise power at the GRS remains constant, the GRS with the highest received SNR is the closest to the emitter and it is the most suitable GRS to be used as a reference for the PE with the lateration algorithm. Using GRS pair as a reference for the lateration algorithm, this study suggests a technique to choose the suitable GRS reference pair for the lateration algorithm. The lateration algorithm considered is for the 3D minimum configuration multilateration system. The suggested technique involves calculating the condition number of a derived matrix and choosing the GRS pair with the least condition number from the derived matrix. The proposed GRS reference pair selection technique is validated by comparison with the fixed GRS reference pair lateration algorithm, used in [12], with selected emitter position and the GRSs deployed in square and triangular configurations.

The rest of the paper is organized as follows. Section 2. presents the multilateration PE methodology and PE condition number analysis. The proposed GRS reference pair selection technique is described in Sec. 3. The result and discussion are presented in Sec. 4. followed by the conclusion in Sec. 5.

2. Multilateration PE Methodology and Condition Number Analysis

This section describes the variable GRS reference pair PE lateration algorithm followed by the condition number analysis of the multilateration PE mathematical model for different GRS reference pair.

2.1. Variable GRS Reference Pair Multilateration PE Methodology

Let $\mathbf{x} = (x, y, z)$ be the coordinate of a stationary emitter in 3D Euclidean space and $S_i = (x_i, z_i, z_i)$. the coordinate of the i-th GRS The distance travelled by the electromagnetic emission from the emitter position to the i-th GRS is calculated as:

$$d_i = c \cdot \tau_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}, \quad (1)$$

where $c = 3 \cdot 10^8$ m·s^{-1} is the speed of light and τ_i is the propagation time of the signal from the emitter to the i-th GRS.

The Path Difference (PD) between i-th and m-th GRS pair is obtained as:

$$d_{i,m} = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - \\ + \sqrt{(x - x_m)^2 + (y - y_m)^2 + (z - z_m)^2}. \quad (2)$$

GRS pair is used as a reference for the lateration algorithm. Let the i-th and j-th GRSs to be chosen as reference pair with coordinates (x_i, j_i, k_i) and (x_j, j_j, k_j) respectively while the non-reference GRSs are labelled the m-th and n-th with coordinates

(x_m, j_m, k_m) and (x_n, j_n, k_n) respectively. Using the i-th GRS as a reference, two independent PD equations are obtained [12] and expressed in the following:

$$d_{i,n} = d_i - d_n, \tag{3}$$

$$d_{i,m} = d_i - d_m, \tag{4}$$

while for the j-th reference GRS, the two independent PD equations are obtained as:

$$d_{j,n} = d_j - d_n, \tag{5}$$

$$d_{j,m} = d_j - d_m. \tag{6}$$

Combining Eq. (3) and Eq. (4) after further simplification results into 3D plane equation which is presented in [12] as follows:

$$A_{i,n,m} = xB_{i,n,m} + yC_{i,n,m} + zD_{i,n,m}, \tag{7}$$

where the coefficients of Eq. (7) are:

$$A_{i,n,m} = 0.5\left(d_{i,m} - d_{i,n} + \frac{k_{i,m}}{d_{i,m}} - \frac{k_{i,m}}{d_{i,n}}\right), \tag{8}$$

$$B_{i,n,m} = \frac{X_{n,i}}{d_{i,n}} - \frac{X_{m,i}}{d_{i,m}}, \tag{9}$$

$$C_{i,n,m} = \frac{Y_{n,i}}{d_{i,n}} - \frac{Y_{m,i}}{d_{i,m}}, \tag{10}$$

$$D_{i,n,m} = \frac{Z_{n,i}}{d_{i,n}} - \frac{Z_{m,i}}{d_{i,m}}, \tag{11}$$

$$k_{i,w} = \left(x_i^2 + y_i^2 + z_i^2\right) - \left(x_w^2 + y_w^2 + z_w^2\right), \tag{12}$$

$$X_{i,w} = x_i - x_w,\ Y_{i,w} = y_i - y_w, \tag{13}$$

$$Z_{i,w} = z_i - z_w,\ w \in [m, n]. \tag{14}$$

In addition, combining Eq. (5) and Eq. (6) after further simplification results into another 3D plane equation as follows:

$$A_{j,n,m} = xB_{j,n,m} + yC_{j,n,m} + zD_{j,n,m}, \tag{15}$$

where the coefficients of Eq. (15) are:

$$A_{j,n,m} = 0.5\left(d_{j,m} - d_{j,n} + \frac{k_{j,m}}{d_{j,m}} - \frac{k_{j,m}}{d_{j,n}}\right), \tag{16}$$

$$B_{j,n,m} = \frac{X_{n,j}}{d_{j,n}} - \frac{X_{m,j}}{d_{j,m}}, \tag{17}$$

$$C_{j,n,m} = \frac{Y_{n,j}}{d_{j,n}} - \frac{Y_{m,j}}{d_{j,m}}, \tag{18}$$

$$D_{j,n,m} = \frac{Z_{n,j}}{d_{j,n}} - \frac{Z_{m,j}}{d_{j,m}}, \tag{19}$$

$$k_{j,w} = \left(x_j^2 + y_j^2 + z_j^2\right) - \left(x_w^2 + y_w^2 + z_w^2\right), \tag{20}$$

$$X_{j,w} = x_j - x_w,\ Y_{j,w} = y_j - y_w, \tag{21}$$

$$Z_{j,w} = z_j - z_w,\ w \in [m, n]. \tag{22}$$

Equation (7) and Eq. (15) when represented in matrix form is:

$$\begin{bmatrix} B_{i,n,m} & C_{i,n,m} & D_{i,n,m} \\ B_{j,n,m} & J_{j,n,m} & D_{j,n,m} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A_{i,n,m} \\ A_{j,n,m} \end{bmatrix}, \tag{23}$$

$$\mathbf{Q}_{ij} \cdot \mathbf{x} = \mathbf{a}_{ij}. \tag{24}$$

Equation (23) is known as the multilateration 3D PE mathematical model for minimum GRS configuration. The subscript "i, n, m" and "j, n, m" for the entries of the matrices $\mathbf{Q}_{ij}$ and $\mathbf{a}_{ij}$ indicate that the entry is obtained using the i-th and j-th GRS as a reference respectively with the m-th and n-th GRS as non-reference. The location of the aircraft (x, y, z) is obtained by finding the inverse matrix solution of Eq. (23) with TDOA or PD measurements and GRSs coordinates as input.

2.2. Multilateration PE Mathematical Condition Number Analysis

In the practical application, the PD measurements are obtained with errors which affect the solution obtained using matrix Eq. (23). The effect of the PD measurement error on the solution of matrix Eq. (23) is determined by the sensitivity of matrix $\mathbf{Q}_{ij}$ defined by the condition number value. The condition number of a square matrix indicates on how the error in the input variables is amplified to the solution obtained using the system. Matrix $\mathbf{Q}_{ij}$ in Eq. (24) is a rectangular matrix whose condition number cannot be determined. Assuming that the GRSs have insignificant height difference, that is:

$$Z_{i,w} = z_i - z_w \approx 0, \tag{25}$$

$$Z_{j,w} = z_j - z_w \approx 0, \tag{26}$$

Tab. 1: Matrix $\mathbf{A}_{ij}$ condition number for different GRS pair as reference. Yellow shade indicates the GRS pair with the least $K(\mathbf{A}_{ij})$.

Range (km)	Bearing (°)	Altitude (km)	GRS reference pair condition number					
			$i=1$ & $j=2$	$i=1$ & $j=3$	$i=1$ & $j=4$	$i=2$ & $j=3$	$i=2$ & $j=4$	$i=3$ & $j=4$
5	60	1	17	21	4	19	10	30
50		1	87	59	17	64	50	99
5	120	7	37	21	31	8	30	49
50		7	89	50	66	17	60	101

for all $1 \leq i \leq 4$, $1 \leq j \leq 4$, $1 \leq w \leq 4$ and $i \neq j \neq w$, matrix $\mathbf{Q}_{ij}$ can be reduced to a square matrix written as:

$$\mathbf{A}_{ij} = \begin{bmatrix} B_{i,n,m} & C_{i,n,m} \\ B_{j,n,m} & C_{j,n,m} \end{bmatrix}, \tag{27}$$

where $D_{i,n,m} = D_{j,n,m} = 0$.

Using the matrix $\mathbf{A}_{ij}$, the condition number can be obtained and used in determining the effect of PD measurement error on the PE accuracy of the lateration algorithm. The condition number of matrix $\mathbf{A}_{ij}$ in Eq. (27) denoted as $K(\mathbf{A}_{ij})$ is obtained as:

$$K(\mathbf{A}_{ij}) = \|\mathbf{A}_{ij}\|_2 \cdot \left\|\mathbf{A}_{ij}^{-1}\right\|_2, \tag{28}$$

where $\|\mathbf{A}_{ij}\|_2$ and $\left\|\mathbf{A}_{ij}^{-1}\right\|_2$ are the 2-norm of matrix $\mathbf{A}_{ij}$ and its inverse respectively.

The 2-norm of the matrix $\mathbf{A}_{ij}$ and its inverse are defined with respect to entries in Eq. (27) which have been expressed in [22].

$$\|\mathbf{A}_{ij}\|_2 = = \sqrt{|B_{i,m,n}|^2 + |C_{i,m,n}|^2 + |B_{j,m,n}|^2 + |C_{j,m,n}|^2}, \tag{29}$$

$$\left\|\mathbf{A}_{ij}^{-1}\right\|_2 = = \frac{\sqrt{|B_{i,m,n}|^2 + |C_{i,m,n}|^2 + |B_{j,m,n}|^2 + |C_{j,m,n}|^2}}{\det(\mathbf{A}_{ij})}, \tag{30}$$

where $\det(\mathbf{A}_{ij})$ is a determinant of matrix $\mathbf{A}_{ij}$ expressed mathematically as:

$$\det \mathbf{A}_{ij} = (B_{i,m,n} \cdot C_{j,m,n}) - (B_{j,m,n} \cdot C_{i,m,n}). \tag{31}$$

Substituting Eq. (29) and Eq. (30) into Eq. (28), the condition number the matrix $\mathbf{A}_{ij}$ as function of its entries can be written as Eq. (32).

Equation (32) represents the condition number of matrix $\mathbf{A}_{ij}$ in Eq. (27) whose entries are obtained using the i-th and j-th GRS pair as a reference with the m-th and n-th as non-reference GRSs. For an emitter at a stationary position with a fixed GRS configuration, different GRS pair (i, j) will produce different entries of matrix $\mathbf{A}_{ij}$. This will result in different condition number value in Eq. (32). Higher condition number values indicate greater error in the solution obtained using Eq. (23). Table 1 shows the condition number of the matrix $\mathbf{A}_{ij}$ using Eq. (32) for different GRS pair (i, j) at four emitter positions with GRS in the square configuration. Emitter positions are given in cylindrical coordinate system. The condition number differs for different emitter positions and GRS reference pairs. At fixed emitter position, different GRS pair produces different condition numbers. At emitter position (5 km, 60°, 1 km), GRS pair $i = 1$ and $j = 4$ has the least condition number value while GRS pair $i = 3$ and $j = 4$ has the highest condition number value. At emitter position (50 km, 120°, 7 km), GRS pair $i = 2$ and $j = 3$ has the least condition number value while $i = 3$ and $j = 4$ has the highest condition number value. For each emitter position, the pair with the least condition number value,used as a reference for the lateration algorithm, will result in the emitter position estimated with the least error. This means that for emitter positions (5 km, 60°, 1 km) and (50 km, 60°, 1 m), the suitable GRS pair as a reference are the $i = 1$ and $j = 4$. For emitter positions (5 km, 120°, 7 km) and (50 km, 120°, 7 km), the suitable GRS pairs as a reference are $i = 2$ and $j = 3$.

GRS reference selection for PE is carried out prior to the actual PE process. The available parameters related to the emitter position which can be used for selecting the suitable GRS pair as a reference are the PD measurements only. Thus, the condition number obtained from matrix $\mathbf{A}_{ij}$ in Eq. (27) cannot be used since it is a function of both PD measurements and GRS coordinate. In next section, the approach is developed to determine the suitable GRS reference pair to be selected for the PE process.

3. Proposed GRS Reference Pair Selection Technique

In this section, the technique for selection of the suitable GRS reference pair for the PE process is presented. In Subsec. 2.2. , it was concluded that using matrix $\mathbf{A}_{ij}$ to determine the suitable GRS pair as a reference for PE is not possible. Matrix $\mathbf{A}_{ij}$ can

$$K(\mathbf{A}_{ij}) = \frac{\left(|B_{i,m,m}|^2 + |C_{i,m,m}|^2 + |B_{j,m,m}|^2 + |C_{j,m,m}|^2\right)}{(B_{i,m,m} \cdot C_{j,m,m}) - (B_{j,m,m} \cdot C_{i,m,m})}. \tag{32}$$

$$\mathbf{A}_{ij} = \begin{bmatrix} \left(\frac{X_{m,i}}{d_{i,m}} - \frac{X_{n,i}}{d_{i,n}}\right) & \left(\frac{Y_{m,i}}{d_{i,m}} - \frac{Y_{n,i}}{d_{i,n}}\right) \\ \left(\frac{X_{m,j}}{d_{j,m}} - \frac{X_{n,j}}{d_{j,n}}\right) & \left(\frac{Y_{m,j}}{d_{j,m}} - \frac{Y_{n,j}}{d_{j,n}}\right) \end{bmatrix} = \begin{bmatrix} \frac{(X_{m,i}d_{i,n} - X_{n,i}d_{i,m})}{d_{i,m}d_{i,n}} & \frac{(Y_{m,i}d_{i,n} - Y_{n,i}d_{i,m})}{d_{i,m}d_{i,n}} \\ \frac{(X_{m,j}d_{j,n} - X_{n,j}d_{j,m})}{d_{j,m}d_{j,n}} & \frac{(Y_{m,j}d_{j,n} - Y_{n,j}d_{j,m})}{d_{j,m}d_{j,n}} \end{bmatrix} = \\ = \begin{bmatrix} (d_{i,m} \cdot d_{i,n})^{-1} & 0 \\ 0 & (d_{j,m} \cdot d_{j,n})^{-1} \end{bmatrix} \cdot \begin{bmatrix} (X_{m,i}d_{i,n} - X_{n,i}d_{i,m}) & (Y_{m,i}d_{i,n} - Y_{n,i}d_{i,m}) \\ (X_{m,j}d_{j,n} - X_{n,j}d_{j,m}) & (Y_{m,j}d_{j,n} - Y_{n,j}d_{j,m}) \end{bmatrix}, \tag{33}$$

be split into two matrices while one of the matrices is having only the PD measurements as its entries. From Eq. (27), the matrix $\mathbf{A}_{ij}$ is written as Eq. (33). Let

$$\mathbf{M}_{ij} = \begin{bmatrix} (d_{i,m} \cdot d_{i,n})^{-1} & 0 \\ 0 & (d_{j,m} \cdot d_{j,n})^{-1} \end{bmatrix}. \tag{34}$$

$$\mathbf{N}_{ij} = \begin{bmatrix} (X_{m,i}d_{i,n} - X_{n,i}d_{i,m}) & (Y_{m,i}d_{i,n} - Y_{n,i}d_{i,m}) \\ (X_{m,j}d_{j,n} - X_{n,j}d_{j,m}) & (Y_{m,j}d_{j,n} - Y_{n,j}d_{j,m}) \end{bmatrix}. \tag{35}$$

Then

$$\mathbf{A}_{ij} = \mathbf{M}_{ij} \cdot \mathbf{N}_{ij}. \tag{36}$$

Matrix $\mathbf{M}_{ij}$ and $\mathbf{N}_{ij}$ are both square matrices. The matrix $\mathbf{M}_{ij}$ is having only PD measurements obtained using any possible GRS pair (i, j) as its entries. This matrix can be used instead of matrix $\mathbf{A}_{ij}$ for condition number calculation to determine the suitable GRS pair as a reference for the PE process. The condition number of matrix $\mathbf{M}_{ij}$ as a function of the PD measurement is obtained as Eq. (37).

Further simplification of Eq. (37) will result in:

$$K(\mathbf{M}_{ij}) = \left(\frac{d_{j,m} \cdot d_{j,n}}{d_{i,m} \cdot d_{i,n}}\right) + \left(\frac{d_{i,m} \cdot d_{i,n}}{d_{j,m} \cdot d_{j,n}}\right). \tag{38}$$

Using Eq. (38), the condition numbers for all the possible GRS pairs are obtained. The pair with the least condition number is chosen as a reference for the PE process with the lateration algorithm. The summary of the procedure for selection of GRS reference pair for four numbers of GRSs is described as follows:

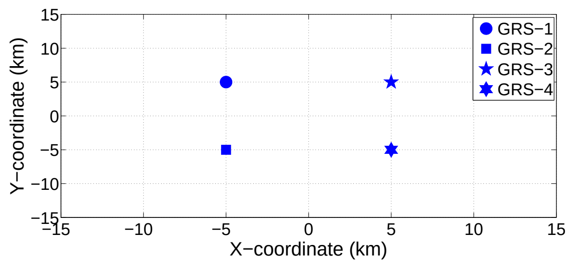
- Obtain the PD measurement set using Eq. (39) for each of the possible GRS pair (i, j) as references as shown below.

$$\mathbf{d}_{i,j,m,n} = [d_{i,m}, d_{i,n}, d_{j,m}, d_{j,n}]. \tag{39}$$

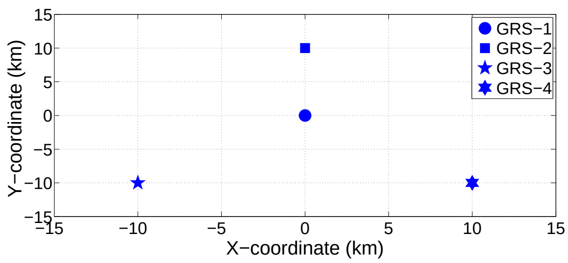
- Using the PD measurement set from (i) for each GRS pair, substitute into Eq. (38) and solve for $K(\mathbf{M}_{ij})$.
- Choose the GRS pair with the least $K(\mathbf{M}_{ij})$ value from step (ii) as the reference pair for the PE process with the lateration algorithm.

4. Results and Discussion

In this section, the technique for the selecting the suitable GRS reference pair based on the condition number which is calculated using Eq. (38) for the PE using the lateration algorithm is validated. This is done with a comparison of the condition number which has been obtained using Eq. (38) with the support of Eq. (32). Validation is carried out for some selected emitter positions with GRSs in the square and triangular configuration as shown in Fig. 1. It has been established



(a) Square GRS configuration.



(b) Triangular GRS configuration.

Fig. 1: Square and triangular GRS configuration with GRS separation of 10 km.

$$K\mathbf{M}_{ij} = \frac{1}{\det(\mathbf{M}_{ij})} \cdot \|\mathbf{M}_{ij}\|_2^2 = \frac{\left\| \begin{bmatrix} (d_{i,m} \cdot d_{i,n})^{-1} & 0 \\ 0 & (d_{j,m} \cdot d_{j,n})^{-1} \end{bmatrix} \right\|_2^2}{\det\left(\begin{bmatrix} (d_{i,m} \cdot d_{i,n})^{-1} & 0 \\ 0 & (d_{j,m} \cdot d_{j,n})^{-1} \end{bmatrix} \right)} =$$
$$= \frac{\left(\sqrt{(d_{i,m} \cdot d_{i,n})^{-2} + (d_{j,m} \cdot d_{j,n})^{-2}}\right)^2}{(d_{i,m} \cdot d_{i,n} \cdot d_{j,m} \cdot d_{j,n})^{-1}} = (d_{i,m} \cdot d_{i,n} \cdot d_{j,m} \cdot d_{j,n}) \cdot \left(\frac{1}{(d_{i,m} \cdot d_{i,n})^2} + \frac{1}{(d_{j,m} \cdot d_{j,n})^2} \right). \quad (37)$$

that the GRS pair with the least condition number is the suitable GRS pair as a reference for the lateration algorithm.

For each of the GRS configurations, the validation of the proposed GRS reference pair selection technique is carried out for the emitter positions which are defined in Tab. 2.

Tab. 2: Emitter positions for validation.

No.	Emitter position	Range (km)	Altitude (km)	Bearing (°)
1	A	5	7	30
2	B			120
3	C			220
4	D			320

Table 3 shows the condition number comparison of the matrix $\mathbf{A}_{ij}$ in Eq. (27) and $\mathbf{M}_{ij}$ in Eq. (34) using Eq. (32) and Eq. (37) respectively for the square GRS configuration. For the selected emitter positions considered, it is seen that the GRS pair with the least $K\mathbf{A}_{ij}$ also has the least $K\mathbf{M}_{ij}$. At emitter position A, the GRS pair with the least $K\mathbf{A}_{ij} = 8$ and least $K\mathbf{M}_{ij} = 1$ is the pair $i = 1$ and $j = 4$. At emitter location B, the GRS pair with the least $\mathbf{A}_{ij} = 8$ and least $K\mathbf{M}_{ij} = 1$ is the pair $i = 2$ and $j = 3$. It is also seen that the GRS pair with the least $\mathbf{A}_{ij}$ will have $K\mathbf{M}_{ij} = 1$. This means that the GRS pair suitable as a reference for PE process with the lateration algorithm at any given emitter position will have $K\mathbf{M}_{ij} = 1$.

Table 4 shows the condition number comparison of matrix $\mathbf{A}_{ij}$ in Eq. (27) and matrix $\mathbf{M}_{ij}$ in Eq. (34) using Eq. (32) and Eq. (37) respectively for the triangular GRS configuration. The same conclusion for the square GRS configuration is deduced for the triangular configuration. Even thought, at the emitter positions A and D the least $K(\mathbf{A}_{ij})$ is obtained by more than one GRS reference pair. One of the GRS pairs was chosen as the most suitable for the PE process of the emitter at the selected position which has $K(\mathbf{M}_{ij}) = 1$.

4.1. PE Accuracy Improvement

In this section, the PE accuracy of the lateration algorithm with the proposed reference selection technique in Sec. 3. is compared with the fixed GRS reference pair approach (GRS 1 and GRS 2) used in [12]. PE Root Mean Square Error (RMSE) is used as the performance measure for the comparison. Mathematically, the PE RMSE is obtained as:

$$PE_{rmse} = \sqrt{\frac{\sum_{i=1}^{N} \left[(\hat{x} - x)^2 + (\hat{y} - y)^2 + (\hat{z} - z)^2 \right]}{N}}, \quad (40)$$

where (x, y, z) are the known emitter coordinates and $(\hat{x}_i, \hat{y}_i, \hat{z}_i)$ are the estimated emitter coordinates at the i-th Monte Carlo simulation realization. The Monte Carlo simulation results were obtained after 500 realizations. The PD Estimation (PDE) error was modelled as $N(0, \sigma^2)$ and it was assumed to be the same at all the spatially placed GRSs.

By varying the PDE error standard deviation from 0 m to 2 m, the PE RMSE of the lateration algorithm with the proposed reference selection technique and that of the fixed GRS reference pair were obtained and compared. Figure 2 and Fig. 3 show the PE RMSE comparison between two approaches for emitter at position B using the square and triangular configuration respectively. The PE RMSE increases with increase in the PDE error standard deviation from 0 m to 2 m. Comparison between the PE RMSE of the lateration algorithm with the proposed reference selection technique for both square and triangular configuration shows that there is an improvement in the PE accuracy by the reduction in the PE RMSE. From Fig. 2, at PDE error standard deviation of 1 m, the PE RMSE of the lateration algorithm with the proposed technique is 6.25 m and that of using the fixed GRS reference pair is 21.78 m. This means a reduction in the PE RMSE of about 15.53 m (~ 71 %) was achieved with the proposed technique at emitter position A with the GRS in the square configuration. Extending the analysis to the triangular configuration, at PDE error standard devi-

Tab. 3: Square GRS configuration condition number comparison. Yellow shade indicates the GRS pair with the least $K\mathbf{A}_{ij}$ value while green shade indicates the GRS pair with the least $K\mathbf{M}_{ij}$ value.

Emitter position		GRS reference pair: $i=1$ & $j=2$	$i=1$ & $j=3$	$i=1$ & $j=4$	$i=2$ & $j=3$	$i=2$ & $j=4$	$i=3$ & $j=4$
A	$K(\mathbf{A}_{ij})$	21	49	8	31	37	30
	$K(\mathbf{M}_{ij})$	3	5	1	2	3	6
B	$K(\mathbf{A}_{ij})$	37	21	31	8	30	49
	$K(\mathbf{M}_{ij})$	3	3	2	1	6	5
C	$K(\mathbf{A}_{i}j)$	85	73	7	80	98	60
	$K(\mathbf{M}_{ij})$	17	9	1	2	16	9
D	$K(\mathbf{A}_{ij})$	85	98	80	7	73	60
	$K(\mathbf{M}_{ij})$	17	16	2	1	9	9

Tab. 4: Triangular GRS configuration condition number comparison. Yellow shade indicates the GRS pair with the least $K(\mathbf{A}_{i}j)$ value while green shade indicates the GRS pair with the least $K(\mathbf{M}_{i}j)$ value.

Emitter position		GRS reference pair: $i=1$ & $j=2$	$i=1$ & $j=3$	$i=1$ & $j=4$	$i=2$ & $j=3$	$i=2$ & $j=4$	$i=3$ & $j=4$
A	$K(\mathbf{A}_{i}j)$	14	2	2	2	2	15
	$K(\mathbf{M}_{ij})$	2	4	1	5	3	2
B	$K(\mathbf{A}_{ij})$	3	3	8	5	1	2
	$K(\mathbf{M}_{ij})$	3	2	3	3	1	2
C	$K(\mathbf{A}_{i}j)$	13	44	13	8	4	7
	$K(\mathbf{M}_{ij})$	15	3	13	5	1	5
D	$K(\mathbf{A}_{ij})$	10	1	2	1	4	10
	$K(\mathbf{M}_{ij})$	2	1	3	1	2	2

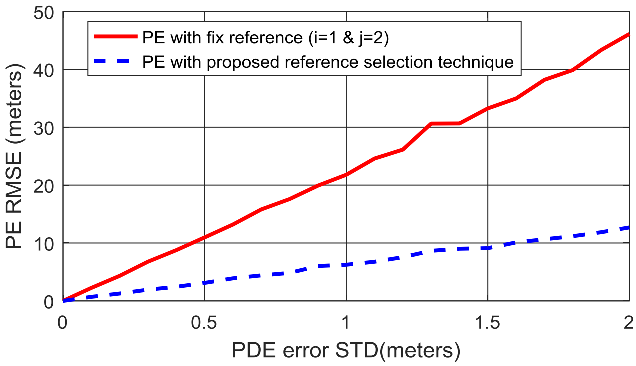


Fig. 2: PE RMSE comparison with the square GRS configuration.

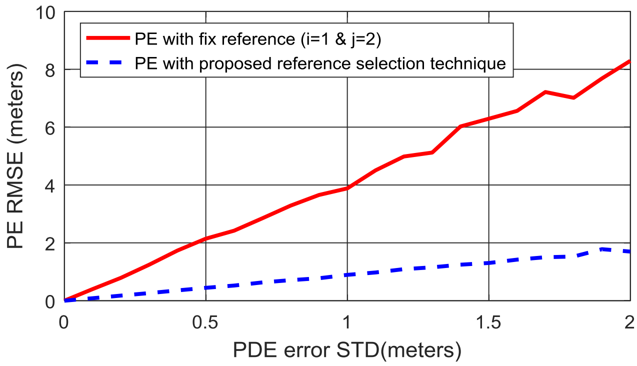


Fig. 3: PE RMSE comparison with the triangular GRS configuration.

ation of 1 m, the reduction in PE RMSE of about 3 m ($\sim$ 77 %) was obtained.

Furthermore, comparing the PE RMSE for the square and triangular GRS configuration, the triangular GRS configuration resulted in the least PE RMSE. This is due to the low condition number values obtained with the triangular configuration as shown in Tab. 4 compared to the square configuration as shown in Tab. 3. Thus, the triangular GRS configuration will result in higher PE accuracy compared to the square GRS configuration.

5. Conclusion

This research has accomplished a method to select the suitable GRS reference pair which is to be used for improving PE accuracy of the lateration algorithm for a minimum configuration 3D multilateration system. The technique was validated by condition number calculation and PE RMSE estimation comparison with a fixed GRS reference pair lateration algorithm. Condition number calculation results indicate that the most suitable GRS pair, used as a reference for the lateration algorithm, has the least condition number value. PE RMSE Monte Carlo simulation results comparison shows that the proposed reference selection technique improved the PE accuracy of the lateration algorithm by a reduction in the PE RMSE of at least 70 % for both square and triangular GRS configuration. Further work will focus on the extension of the technique for more than 4 GRSs.

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