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Recursive Subspace Identification Algorithm using the Propagator Based Method

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Abstract

Subspace model identification (SMI) method is the effective method in identifying dynamic state space linear multivariable systems and it can be obtained directly from the input and output data. Basically, subspace identifications are based on algorithms from numerical algebras which are the QR decomposition and Singular Value Decomposition (SVD). In industrial applications, it is essential to have online recursive subspace algorithms for model identification where the parameters can vary in time. However, because of the SVD computational complexity that involved in the algorithm, the classical SMI algorithms are not suitable for online application. Hence, it is essential to discover the alternative algorithms in order to apply the concept of subspace identification recursively. In this paper, the recursive subspace identification algorithm based on the propagator method which avoids the SVD computation is proposed. The output from Numerical Subspace State Space System Identification (N4SID) and Multivariable Output Error State Space (MOESP) methods are also included in this paper.

Keywords*: Subspace Identification, QR Decomposition, Propagator Method*

1. Introduction

Subspace model identification (SMI) is a method used in identifying system matrices of the state space model directly from the input and output data measurements. The advantage of SMI method is the reliable numerical tools- based which are the QR decomposition and singular value decomposition (SVD). Furthermore, this method can be implemented for multiple inputs and multiple outputs (MIMO) system identification. There are several algorithms commonly used for subspace identification which are Canonical Variate Analysis (CVA) proposed by [1, 2], Multivariable Output Error State Space (MOESP) by [3, 4] and Numerical Subspace State Space System Identification (N4SID) by [5, 6]. Basically, subspace identification algorithms are based on the concepts from different branches which are system theory, numerical linear algebra and statistics [7].

Subspace identification methods for linear time invariant systems initially can be divided into two groups. The first group aim for obtaining the column space of the extended observability matrix,Γ and subsequently use the shift invariant structure of this matrix to estimate **A** and **C** matrices. Then, the estimation of **B** and **D** matrices is done by recursive least squares method. The MOESP method is considered in this group. Meanwhile, the second group consists of method that aim at approximating the state sequence of the system and consequently use the approximate state to estimate the system matrices **A**, **B**, **C** and **D**. The methods that consider in the second group are the N4SID methods [5], [8] and CVA methods.

The difference in each subspace identification methods is CVA applies the canonical correlation analysis to estimate the state variables and fit them to the state space model. An ordinary MOESP algorithm transforms a Hankel matrix of $[\mathbf{U}_f : \mathbf{Y}_f]$ into QR decomposition and then does an SVD on submatrix of **R**. The singular matrix obtained from the SVD is taken as $\mathbf{\Gamma}_f$, based on which **A** and **C** matrices are estimated, while **B** and **D** are obtained based on least square fitting. Meanwhile N4SID projects $\mathbf{Y}_f$ onto $[\mathbf{Y}_p;\mathbf{U}_p;\mathbf{U}_f]$ and does an SVD on the part

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corresponding to the past data. The right singular vectors are estimated as state variables and fit them to the state space model [9].

In general, the subspace methods are based on robust numerical tools which are QR decomposition and SVD where these tools are applicable for batch processing. However, because of the computational complexity and the storage costs, they are not suitable for online identification. Therefore, an online subspace identification consists a recursive algorithm is needed. The most important thing in order to develop a recursive subspace state space system identification algorithm is to online update the estimation of the extended observability matrix when the new data received.

New technique for updating LQ decomposition through Givens rotation is proposed by [10]. The algorithm implemented forgetting factor in Recursive Stochastic Subspace Identification (RSSI) and applied to structural damage diagnosis. Then, based on the optimized version of the predictor-based subspace identification (PBSID) method for batch data, an algorithm so-called $PBSID_{opt}$ method is presented in [11]. The recursive implementation of the $PBSID_{opt}$ method applied propagator method to identify linear time invariant models from data measured in open or closed loop. The algorithm was applied to 2D airfoil system. By using this algorithm, the computational complexity is reduced by exploiting the structure in data equations and by using array algorithms to solve the main linear problem.

In [12], a recursive subspace identification algorithm for autonomous underwater vehicles (AUV) has been proposed. The AUV model is constructed as a Hammerstein model with nonlinear feedback in the linear part. The identification procedure is under general noise assumption and the propagator method (PM) based subspace approach is extended into errors in variable framework in order to make the algorithm recursively.

2. Research Method

2.1. Subspace Identification

Consider the linear time invariant state space model:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in \Re^n, \mathbf{y}_k \in \Re^{n_y}$ and $\mathbf{u}_k \in \Re^{n_u}$ are the system state, output and input respectively. Meanwhile $\mathbf{w}_k \in \Re^n$ and $\mathbf{v}_k \in \Re^{n_y}$ are additional unknown noise sequences. The goal is to estimate system matrices **A**, **B**, **C** and **D** with appropriate dimensions. In subspace identification, it is assumed that the number of available data point goes to infinity and the data is ergodic [8]. The main problem is the estimation of the column space of the extended observability matrix,Γ which defined as:

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \mathbf{CA}^2 & \cdots & \mathbf{CA}^{i-1} \end{bmatrix}^T \tag{2}$$

One important equation in the derivation of subspace state space system identification algorithms is the block Hankel matrices constructed from the input and output data. The input and output block Hankel matrices, **U** and **Y**, and the block triangular Toeplitz matrix, **H** are defined as:

$$\mathbf{U} = \left[\begin{array}{ccccc} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_j \\ \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4 & \cdots & \mathbf{u}_{j+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{u}_i & \mathbf{u}_{i+1} & \mathbf{u}_{i+2} & \cdots & \mathbf{u}_{i+j-1} \\ \hline \mathbf{u}_{i+1} & \mathbf{u}_{i+2} & \mathbf{u}_{i+3} & \cdots & \mathbf{u}_{i+j} \\ \mathbf{u}_{i+2} & \mathbf{u}_{i+3} & \mathbf{u}_{i+4} & \cdots & \mathbf{u}_{i+j+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{u}_{2i} & \mathbf{u}_{2i+1} & \mathbf{u}_{2i+2} & \cdots & \mathbf{u}_{2i+j-1} \end{array}\right] = \left[\begin{array}{c} \mathbf{U}_p \\ \hline \mathbf{U}_f \end{array}\right] \tag{3}$$

$$\mathbf{Y}=\left[\begin{array}{ccccc} \mathbf{y}_1 & \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_j \\ \mathbf{y}_2 & \mathbf{y}_3 & \mathbf{y}_4 & \cdots & \mathbf{y}_{j+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{y}_i & \mathbf{y}_{i+1} & \mathbf{y}_{i+2} & \cdots & \mathbf{y}_{i+j-1} \\ \hline \mathbf{y}_{i+1} & \mathbf{y}_{i+2} & \mathbf{y}_{i+3} & \cdots & \mathbf{y}_{i+j} \\ \mathbf{y}_{i+2} & \mathbf{y}_{i+3} & \mathbf{y}_{i+4} & \cdots & \mathbf{y}_{i+j+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{y}_{2i} & \mathbf{y}_{2i+1} & \mathbf{y}_{2i+2} & \cdots & \mathbf{y}_{2i+j-1} \end{array}\right]=\left[\frac{\mathbf{Y}_p}{\mathbf{Y}_f}\right] \tag{4}$$

$$\mathbf{H}=\begin{bmatrix} \mathbf{D} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{CB} & \mathbf{D} & \cdots & \mathbf{0} \\ \mathbf{CAB} & \mathbf{CB} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{CA}^{i-2}\mathbf{B} & \mathbf{CA}^{i-3}\mathbf{B} & \cdots & \mathbf{D} \end{bmatrix} \tag{5}$$

The starting point of some subspace identification algorithms for the estimation of the column space of $\mathbf{\Gamma}$ is given by:

$$\mathbf{Y}_f(t)=\mathbf{\Gamma}_f\mathbf{x}(t)+\mathbf{H}_f\mathbf{U}_f(t)+\mathbf{b}_f(t) \tag{6}$$

Hence, a typical state space subspace system identification algorithm involves two steps:

a) Identification of the extended observability matrix, Γ and a block triangular Toeplitz matrix, **H**.
b) Estimation of the system matrices **A**, **B**, **C** and **D** from the identified observability matrix and Toeplitz matrix.

2.2. Recursive Subspace Identification

As time goes on in online identification, it is very important to update the model with a reduced computational time and cost. The batch subspace identifications are not appropriate for online implementation because of the computational complexity of SVD. Hence, it is necessary to find the SVD alternatives in order to apply the subspace algorithm in a recursive framework [13]. In this paper, we will adapt a method from array signal processing which is the propagator method.

All propagator based techniques are built up of two important steps which are:

a) Online update the observation vector, **z**.
b) Recursive estimation of the extended observability matrix from the online update of the observation vector.

The fundamental principle of using the relationship between array signal processing and subspace identification is to apply the propagator method which initially obtained in array signal processing in order to track the subspace spanned by the extended observability matrix. In array signal processing, the considered subspace tracking problem consists in recursively determine the direction of arrival (DOA), θ, by online estimating the column subspace of the steering matrix, $\Gamma(\theta)$, from the following data generation model [13]:

$$\mathbf{z}(t)=\mathbf{\Gamma}(\boldsymbol{\theta})\mathbf{s}(t)+\mathbf{b}(t)$$

where **z** is the output of the sensors, **s** is the vector of the signal waveforms and **b** is the additive noise.

The goal of recursive subspace identification methods is to estimate online recursively the system matrices which are [**A**, **B**, **C**, **D**] at each new input and output data acquisition. The algorithms developed are based on the MOESP approach where the main problem is the consistent estimation of the extended observability matrix.

Thus, the relationship between subspace identification and array signal processing can be written as:

$$\mathbf{z}_f = \mathbf{y}_f(t) - \mathbf{H}_f\mathbf{u}_f(t) = \mathbf{\Gamma}_f\mathbf{x}(t) + \mathbf{b}_f(t) \tag{7}$$

From Eqn. (7), there are two steps required to recursively estimate the extended observability matrix. The first step is to update of the observation vector, $\mathbf{z}_f$, from the input and output measurements:

$$\mathbf{z}_f = \mathbf{y}_f(t) - \mathbf{H}_f\mathbf{u}_f(t). \tag{8}$$

Consider the QR decomposition, part of the MOESP method.

$$\mathbf{M} = \begin{bmatrix} \mathbf{U}_f \\ \mathbf{W}_p \\ \mathbf{Y}_f \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_{21} & \mathbf{R}_{22} & \mathbf{0} \\ \mathbf{R}_{31} & \mathbf{R}_{32} & \mathbf{R}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1^T \\ \mathbf{Q}_2^T \\ \mathbf{Q}_3^T \end{bmatrix} \tag{9}$$

When a new input and output couple is available, the QR decomposition on the right hand side of Eqn.(9) will becomes

$$\left[\sqrt{\lambda} \begin{bmatrix} \mathbf{R}_{11}(\psi) & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_{21}(\psi) & \mathbf{R}_{22}(\psi) & \mathbf{0} \\ \mathbf{R}_{31}(\psi) & \mathbf{R}_{32}(\psi) & \mathbf{R}_{33}(\psi) \end{bmatrix} \begin{matrix} \mathbf{u}_f(\psi+1) \\ \mathbf{w}_p(\psi+1) \\ \mathbf{y}_f(\psi+1) \end{matrix} \right] \begin{bmatrix} \mathbf{Q}_1(\psi) & \mathbf{0} \\ \mathbf{Q}_2(\psi) & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}$$

In this paper, the Gauss elimination method which applies the elementary row operation (ERO) procedures is performed in order to transform the transpose of R matrix together with the new input output data in the last row into lower triangular matrix form. This approach has advantage to show the good numerical performances in terms of round of error.

$$\left[\lambda \begin{bmatrix} \mathbf{R}_{11}(\psi) & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_{21}(\psi) & \mathbf{R}_{22}(\psi) & \mathbf{0} \\ \mathbf{R}_{31}(\psi) & \mathbf{R}_{32}(\psi) & \mathbf{R}_{33}(\psi) \end{bmatrix} \begin{matrix} \mathbf{u}_f(\psi+1) \\ \mathbf{w}_p(\psi+1) \\ \mathbf{y}_f(\psi+1) \end{matrix} \right]^T \rightarrow \begin{bmatrix} \mathbf{R}_{11}(\psi+1) & \begin{matrix} \mathbf{0} & \mathbf{0} \end{matrix} & \mathbf{0} \\ \mathbf{R}_{21}(\psi+1) & \lambda\begin{bmatrix} \mathbf{R}_{22}(\psi) & \mathbf{0} \\ \mathbf{R}_{32}(\psi) & \mathbf{R}_{33}(\psi) \end{bmatrix} & \xi(\psi+1) \\ \mathbf{R}_{31}(\psi+1) & & \mathbf{z}_1(\psi+1) \end{bmatrix}^T$$

$$\rightarrow \begin{bmatrix} \mathbf{R}_{11}(\psi+1) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_{21}(\psi+1) & \mathbf{R}_{22}(\psi+1) & \mathbf{0} & \mathbf{0} \\ R_{31}(\psi+1) & \mathbf{R}_{32}(\psi+1) & \lambda\mathbf{R}_{33}(\psi) & \mathbf{z}_2(\psi+1) \end{bmatrix}^T$$

After obtaining the observation vector, the second step of estimating the extended observability matrix is the estimation of a basis of $\mathbf{\Gamma}_f$ from the observation vector by using

$$\mathbf{z}_f = \mathbf{\Gamma}_f\mathbf{x}(t) + \mathbf{b}_f(t) \tag{10}$$

This can be done by applying the propagator method. Assume that [**A, C**] is observable. Then, since $\mathbf{\Gamma}_f \in \Re^{n_x \times n_x}$ with $n_y f > n_x$, thus the observability matrix has at least, n_x linearly independent rows. With the assumption that the order of n_x is a priori known, it is possible to construct a permutation matrix, $\mathbf{L} \in \Re^{n_y f \times n_y f}$ from an identity matrix such that the extended observability matrix can be decomposed as:

$$\mathbf{L\Gamma}_f = \begin{bmatrix} \mathbf{\Gamma}_{f_1} \\ \mathbf{\Gamma}_{f_2} \end{bmatrix} \begin{matrix} \} \ \Re^{n_x \times n_x} \\ \} \Re^{n_y f - n_{n_x} \times n_x} \end{matrix} \tag{11}$$

where Γ_{f_1} is the submatrix of n_x independent rows and Γ_{f_2} the submatrix of the $n_y f - n_x$ others. Thus, Γ_{f_2} can be expressed as a linear combination of Γ_{f_1}. More specifically, there is a unique matrix, $\mathbf{P}_f \in \Re^{n_x \times n_y f - n_x}$ named as propagator where $\Gamma_{f_2} = \mathbf{P}_f^T \Gamma_{f_1}$. Hence, it is easy to verify that

$$\mathbf{L}\Gamma_f = \begin{bmatrix} \Gamma_{f_1} \\ \Gamma_{f_2} \end{bmatrix} = \begin{bmatrix} \Gamma_{f_1} \\ \mathbf{P}_f^T \Gamma_{f_1} \end{bmatrix} = \begin{bmatrix} I_{n_x} \\ \mathbf{P}_f^T \end{bmatrix} \Gamma_{f_1} . \quad (12)$$

3. Results and Analysis

In this paper, we conducted simulations using data from Daisy data (CD player arm) [11] with two inputs and two outputs. The inputs for this simulation are the forces of the mechanical actuators. Meanwhile the outputs are related to the tracking accuracy of the arm. Figure 1 shows the output for N4SID method. The percentage of the variance accounted for (VAF) output 1 is 83.0633% and for output 2 is 97.2517%. The VAF for output 2 shows a good tracking performance but the range between VAF of output 1 and 2 is quite large that is 14.1884. Figure 2 shows the poles for **A** matrix which tabulated in a unit circle.

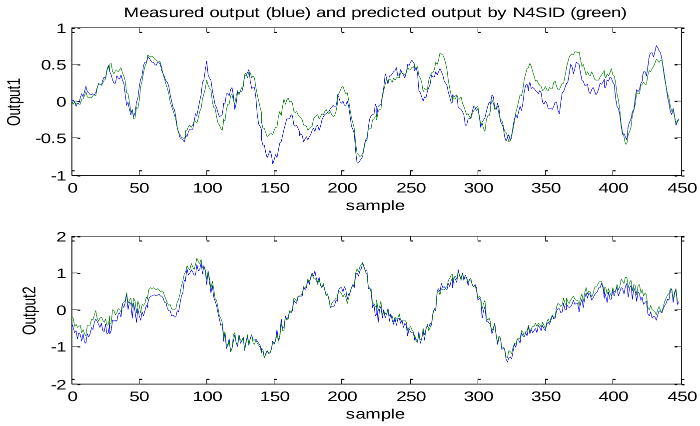

Figure 1. Measured and predicted output using N4SID method.

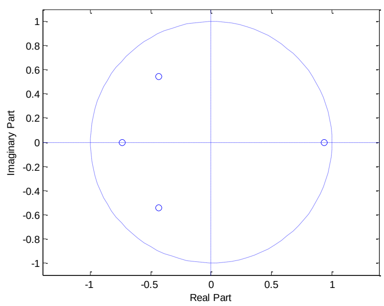

Figure 2. Poles for N4SID method

Figures 3 and 4 show the output for MOESP method. The percentage of the variance accounted for output 1 is 91.3743% and for output 2 is 96.3767% where the range of VAF between outputs 1 and 2 is 5.0024. From both algorithms, MOESP gives higher value but smaller range of variance accounted for both outputs 1 and 2 compared to N4SID algorithm.

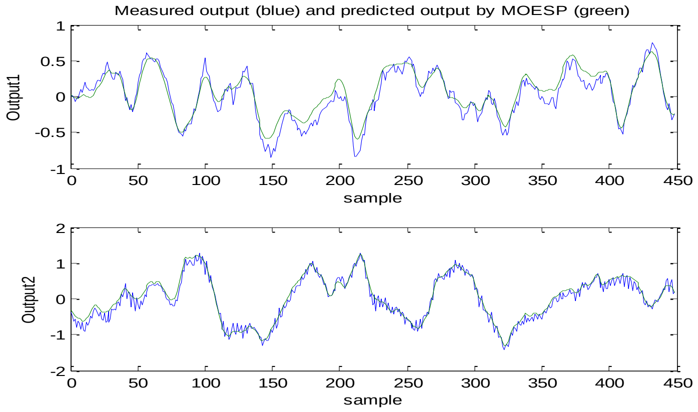

Figure 3. Measured and predicted output using MOESP method.

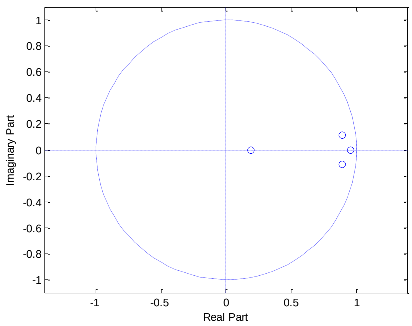

Figure 4. Poles for MOESP method

In this paper also, a simulation example has been carried out to show the performances of the developed algorithm which apply the ERO and propagator based method. Consider the 3rd order system:

$$\mathbf{x}_{k+1} = \begin{bmatrix} 0.8 & -0.4 & 0.2 \\ 0 & 0.3 & -0.5 \\ 0 & 0 & 0.5 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} 0 & 0 \\ 0 & -0.6 \\ 0.5 & 0 \end{bmatrix} \mathbf{u}_k$$

$$\mathbf{y}_k = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k$$

where the input is assumed to be a zero mean white noise with variance I_2 where I_2 is the 2 by 2 identity matrix. The number of data N taken for this simulation is 1000, and the number of block rows used is 15. Figure 5 shows the output for N4SID method. The percentage of the variance accounted for output 1 and output 2 are 99.98% and 99.92% respectively. This shows the high accuracy of the model given by N4SID.

The recursive MOESP approach with the application of ERO to zero out the new input output data is also tested. The output from the recursive method is shown in Figure 6. The variance accounted for the output 1 from this method is 99.9850 and the variance accounted for output 2 is 99.9927.

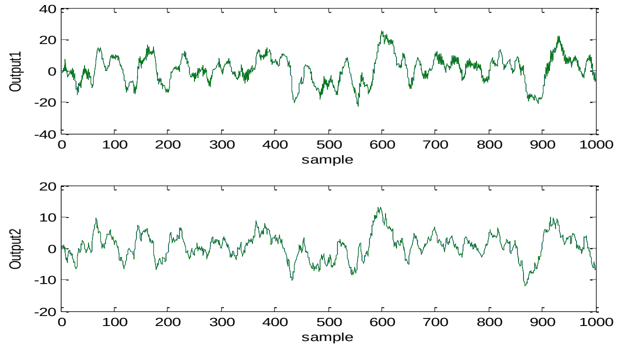

Figure 5. Measured and predicted output using N4SID method from simulation example.

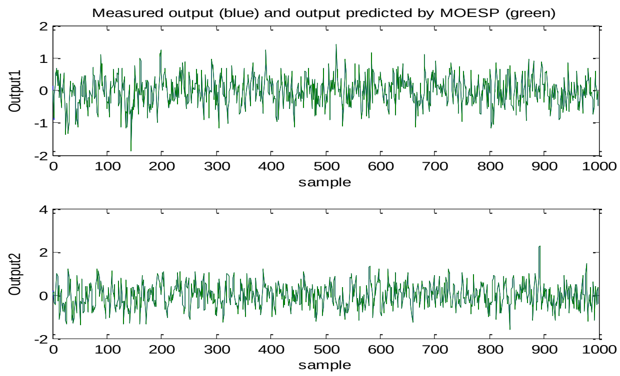

Figure 6. Measured and predicted output by recursive method.

From the system model, the eigenvalues of matrix **A** are 0.8, 0.3 and 0.5. Figure 7 illustrates the eigenvalues of the estimated matrix $\mathbf{A}_i$ using recursive MOESP propagator method. It shows that the eigenvalue of the estimated matrix $\mathbf{A}_i$ approaching those of the eigenvalues of the system.

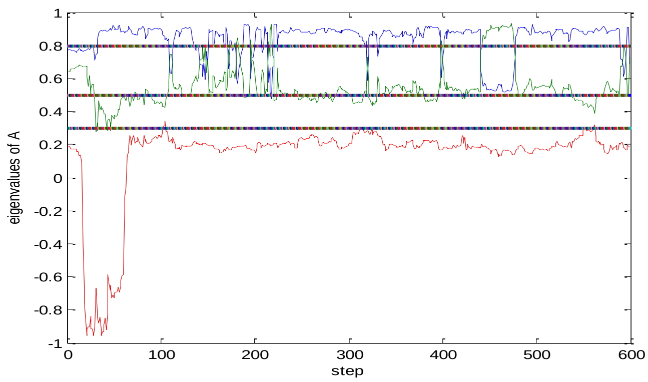

Figure 7. Eigenvalues of $\mathbf{A}_i$

4. Conclusion

A recursive subspace identification algorithm using the propagator method is proposed in this paper. The algorithm developed is based on the MOESP approach where the extended observability matrix is obtained from the observation vectors using propagator algorithm which adapted from array signal processing. The output shows the good tracking performance between the measured and predicted outputs.

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