Development of Equations Through Trajectories Linearization for an HEPWM Inverter

N. A Azli, *Member IEEE* and L. Y. Chan

***Abstract*--Harmonic elimination pulse width modulation (HEPWM) is an inverter control technique that eliminates specific harmonics of its output voltage by controlling switching signals at certain switching angles. This method offer less losses in the inverter switches since the switching frequency is 40% lower than the typical sinusoidal pulse width modulation (SPWM) technique. However, as the equations to calculate the switching angles based on the HEPWM technique are too complex to be solved online by microprocessors, simpler solutions for the switching angles are required. Simplified solutions through trajectories linearization for the HEPWM technique are proposed in this paper. A set of equations for the HEPWM technique switching angles solutions suitable for microprocessor implementation are derived and then tested for accuracy and performance through simulation using MATLAB/Simulink.**

***Index Terms*—Inverter, HEPWM, solutions trajectories**

I. Introduction

Control of an inverter output from the aspect of modulation techniques can generally be classified into two which are carrier modulated natural sampled and regular sampled sinusoidal pulse width modulation (SPWM) techniques and harmonic elimination pulse width modulation (HEPWM) technique. It is a well-known fact however that the HEPWM technique offers several distinct advantages over the carrier modulated SPWM type. In particular, an inverter employing the HEPWM technique is found to achieve great reduction in its effective switching frequency which contributes to reduced switching losses for the same amount of reduction in the lower order harmonics when compared to the one using the carrier modulated SPWM technique. The HEPWM technique involves determining the switching angles of a generalized PWM waveform using numerical minimization search techniques applied to a set of non-linear and transcendental equations that specifically eliminates certain lower order harmonics in an inverter output voltage.

The typical practical implementation of the HEPWM technique is by programming the precalculated switching angles for all the values of the amplitude of the fundamental of the inverter output voltage in per unit (ap1) into a microprocessor's memory. This in turn allows online generation of the switching angles by using look up tables and interpolation techniques to achieve a certain inverter control range and resolution.With a large number of possibilities of ap1 and frequency values, the memory requirement can be very large. To overcome the problem of memory requirement, several algorithms for generating near-optimal HEPWM waveforms have been proposed [3]-[10]. Previous papers have reported the use of a Curve Fitting Technique (CFT) for optimal pulse width modulation (OPWM) online control of a conventional and multilevel inverter topology [3]-[5]. The prototype for the later has been developed and the OPWM control of the multilevel inverter based on the CFT was implemented using a digital signal processor (DSP). Using the DSP to generate the multilevel inverter power devices gating signals, the lowest sampling time achievable by the DSP is 100 μ seconds, which corresponds to only 1.8° switching angles resolution [3][4]. This results in some missing pulses particularly for lower ap1 values where by there exist pulse widths that are less than one sampling interval.

The work presented in this paper involves the development of simple equations derived from trajectories linearization that can achieve much smaller sampling time in generating gating signals based on the HEPWM technique online using the DSP. The proposed linearization method can give accurate representation of the switching angles solution trajectories based on the HEPWM technique for its online implementation. The accuracy of the linearization method is then verified through error analysis as well as a simulation study on the operation of a single-phase bridge inverter utilizing the developed equations. The following section describes briefly the HEPWM technique. This is followed by the derivation of the equations through trajectories linearization. Then, some results from the simulation study will be presented and analyzed followed by the conclusion of the paper.

II. HEPWM Technique

According to [2], it is theoretically possible to eliminate as many harmonics as the chops per half cycle of a half-bridge inverter output waveform and to eliminate as many harmonics as the pulses per half-cycle of a full-bridge inverter output waveform. HEPWM technique basically carries out this theory through the construction of PWM waveforms whose switching angles have been precalculated so that certain harmonics can be specifically eliminated in its spectra. The advantages of HEPWM over the conventional SPWM are [1]:

• Reduction of about 40% in the inverter switching frequency is achieved when comparing with the conventional

N. A. Azli is with the Energy Conversion Department (ENCON), Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Malaysia (e-mail: naziha@ieee.org).

L. Y. Chan was a student at the Energy Conversion Department (ENCON), Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Malaysia in 2005.

1-4244-0273-5/06/$20.00 ©2006 IEEE

carrier-modulated sinusoidal PWM scheme.

• Higher voltage gain due to over-modulation is possible. This contributes to higher utilization of the power conversion process.

• Due to the high quality of the output voltage and current, the ripple in the DC link current is also small. Thus, a reduction in the size of the DC link filter components is achieved.

• The reduction in the switching frequency contributes to the reduction in the switching losses of the inverter and permits the use of GTO switches for high-power converters.

• Elimination of lower-order harmonics causes no harmonic interference such as resonance with external line filtering networks typically employed in inverter power supplies.

A. Generalized Method of Harmonic Elimination in Full-Bridge Inverter

Fig. 1 illustrates a generalized PWM waveform generated by a full-bridge inverter.

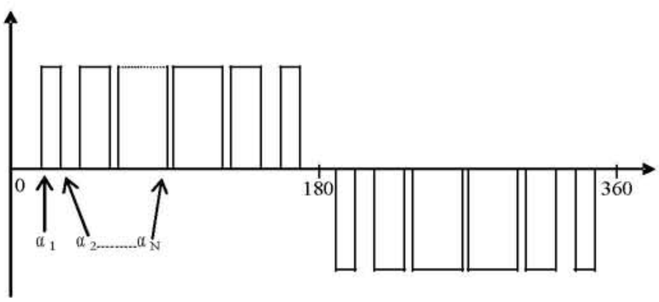


Fig. 1 Generalized PWM inverter waveform

The Fourier coefficients of the waveform are given by [1][2],

$$a_n = \frac{4}{n\pi}\left[\sum_{k=1}^{N}(-1)^{k+1}\cos(n\alpha_k)\right] \quad (1)$$

$$b_n = 0 \quad (2)$$

where,
n = harmonic order
N = number of switching angles per quarter cycle
α_k = k^{th} switching angle.

A set of solutions is obtainable by equating any N-1 harmonics to zero and assigning a specific value to the amplitude of the fundamental of the inverter output voltage (ap1). These equations are nonlinear as well as transcendental in nature. The equations to eliminate N-1 lower order harmonics such as 3,5,7 etc. are in the form of [1][2],

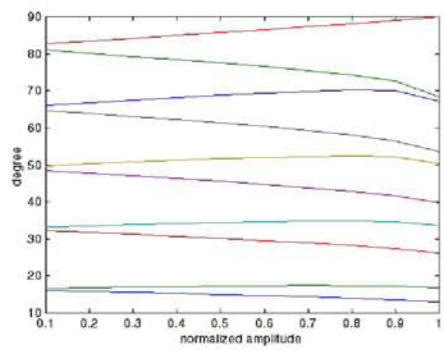
$$\begin{bmatrix} \cos\alpha_1 & -\cos\alpha_2 & \cdots\cdots(-1)^{N+1}\cos\alpha_N \\ \cos 3\alpha_1 & -\cos 3\alpha_2 & \cdots\cdots(-1)^{N+1}\cos 3\alpha_N \\ \vdots & & \\ \cos(x)\alpha_1 & -\cos(x)\alpha_2 & \cdots\cdots(-1)^{N+1}\cos(x)\alpha_N \end{bmatrix} = \begin{bmatrix} \frac{\pi\, ap1}{4} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3)$$

where,

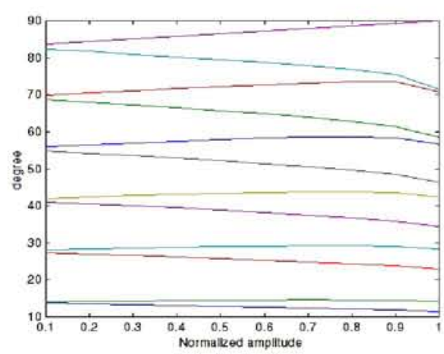
$$x = 2N - 1 \quad (4)$$

In this work, N= 10, 12, 14 and 16 are chosen to eliminate 9, 11, 13 and 15 lower order harmonics respectively. Fig. 2 illustrates the switching angles solutions trajectories for N=10, 12, 14 and 16. These PWM switching angles solutions are found by solving the nonlinear equations concerned using a numerical method subroutine available from Matlab's NAG Foundation Toolbox. This subroutine is capable of finding the solution of system of nonlinear equations using function values. With proper set up of the nonlinear equations and initial guessing values of the switching angles, a set of solutions is obtained at each assigned ap1 satisfying the criterion

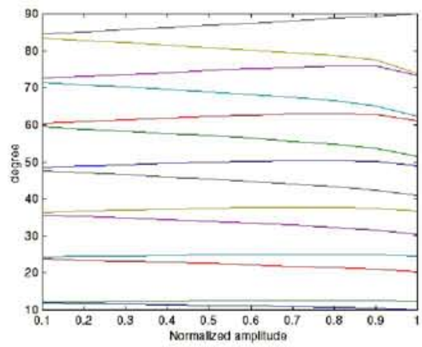
$$\alpha_1 < \alpha_2 < \alpha_3 < \text{---------}\alpha_N < \pi/2 \quad (5)$$



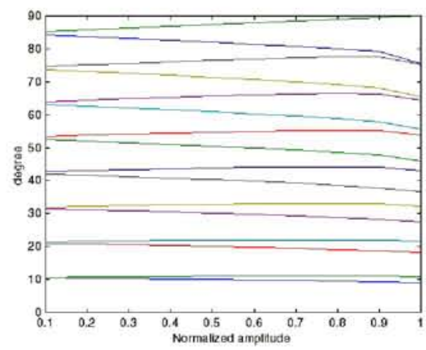
(a) N = 10



(b) N = 12



(c) N = 14



(d) N = 16

Fig. 2 Solution trajectories for different N

III. Development Of Equations Through Trajectories Linearization

The first step taken to develop the equations involve the use of simple regression method. Referring to Fig. 2, it can be observed in general that the slopes of the trajectories experience change from the point where ap1 is between 0.8 and 0.9. Thus each solution trajectory is divided into two segments that are found to be adequate in representing it. One linear equation is obtained from the first segment for ap1 < 0.85 while the other equation is obtained from the segment where ap1 > 0.85. Since the last trajectory for each N does not experience much slope change, only one equation is derived to represent it. Again, due to the nature of the trajectories shown in Fig. 2, the equations are derived separately for the odd and even angles respectively. Table 1 and Table 2 show the slope and intercept of the linear equations that can be used to calculate the odd and even switching angles respectively for N = 10.

The next step is to generalize the equations obtained using the software *CurveExpert* so that the number of equations required can be further reduced. The approach taken is to plot the slope (m) and intercept (c) of the trajectories for each k^{th} odd and even angle where k = 1,2 ... N. This is done for both segments defined earlier as shown in Table 1 and Table 2. Fig. 3 and Fig. 4 show the graphs plotted for the odd angles for the range ap1 < 0.85 and N = 10. The best quadratic equation representing the slope is then given by the software in the form of,

$$y = a + bx + cx^2 \tag{6}$$

where, a = -1.1015455
b = -1.9350459
c = 0.11225191

The same procedure is repeated for the intercept. In this case, a linear fit is found to be adequate in representing the graph as shown in Fig. 3(b). The intercept is thus represented by a linear equation in the form of,

$$y = d + ex \tag{7}$$

where, d = 8.3314807
e = 8.2039164

At this stage, a generalized equation for the odd angles with ap1 < 0.85 and N = 10 is obtained using the following equation in the form of,

$$Alpha_{k(odd)} = (M \times ap1) + C \tag{8}$$

where, $k(odd) = k^{th}$ odd angle
$M = 0.112252\,k^2 - 1.93505\,k - 1.10155$
$C = 8.20392\,k + 8.33148$

Table 1. Slope and Intercept for Linear Equations for Odd Switching Angles

Odd angle	ap1<0.85		ap1>0.85	
	m	c	m	c
Alpha1	-2.99885	16.47757	-5.25286	18.31169
Alpha3	-5.74879	32.95498	-10.8129	37.07819
Alpha5	-7.96633	49.41793	-17.1251	56.90301
Alpha7	-9.29968	65.82124	-24.9981	78.78576
Alpha9	-9.34867	82.0836	-35.5741	104.2031

Table 2. Slope and Intercept for Linear Equations for Even Switching Angles

Even angle	ap1<0.85		ap1>0.85	
	m	c	m	c
Alpha2	1.37925	16.51514	-3.47643	20.34839
Alpha4	2.845071	32.98819	-7.547	41.26935
Alpha6	4.472357	49.37587	-13.0837	63.62737
Alpha8	5.905917	65.7521	-21.1129	88.56219
Alpha10	8.048819	81.78812		

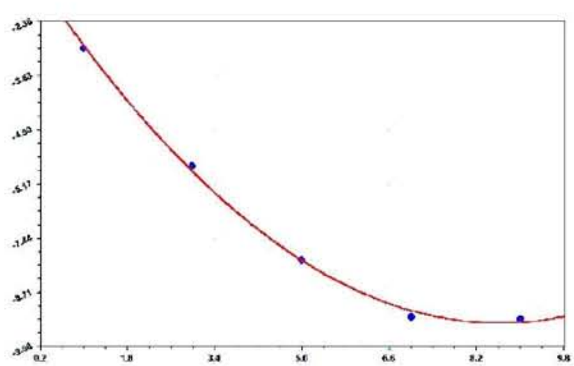


Fig. 3 Slope versus k^{th} odd angle (ap1 < 0.85, N = 10)

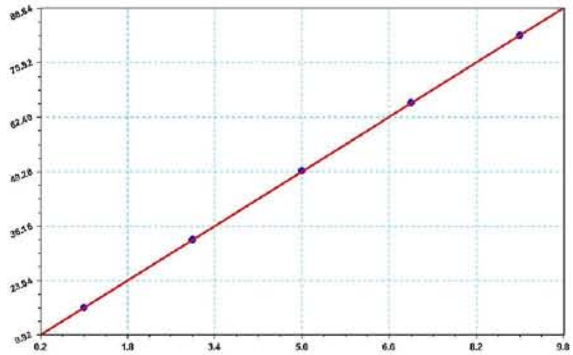


Fig. 4 Intercept versus kt[h] odd angle (ap1 < 0.85, N = 10)

The target however is to obtain more generalized equations that can cater for various values of N in calculating the switching angles at a given ap1. In this work, N = 10, 12, 14 and 16 are chosen. Thus the process above, that has been completed for N = 10, is repeated for the other values of N. Table 3 and Table 4 list the values of the coefficients a, b and c of (6) and coefficients d and e of (7) in fulfilling (8) for all values of N.

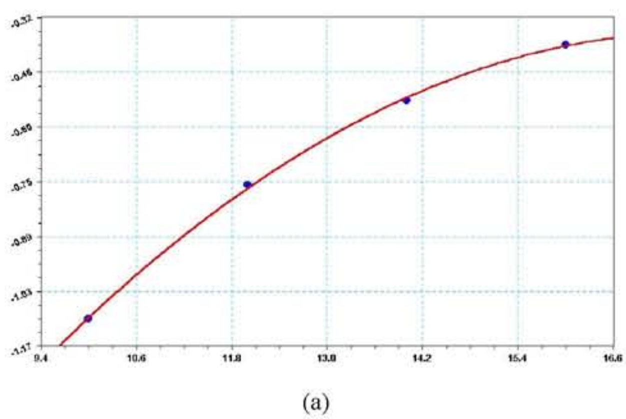
Table 3. Coefficients of (6) to calculate slope for various N

N	Coefficients		
	a	b	c
16	-0.39069	-0.86979	0.033232
14	-0.53588	-1.09816	0.047392
12	-0.75423	-1.42939	0.070802
10	-1.10155	-1.93505	0.112252

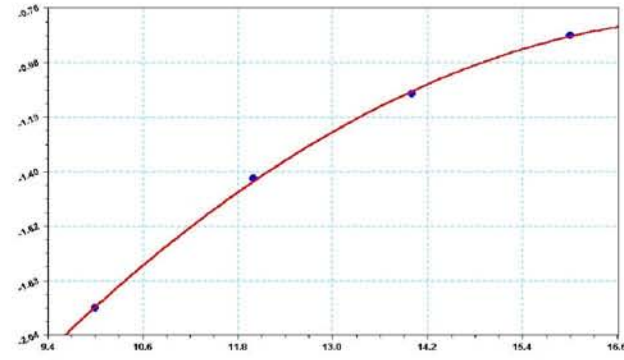
Table 4. Coefficients of (7) to calculate intercept for various N

N	Coefficients	
	d	e
16	5.417281	5.300723
14	6.132606	6.009303
12	7.065177	6.936897
10	8.331481	8.203916

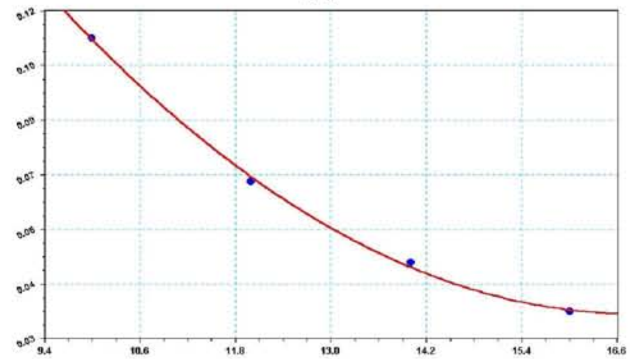
Based on the data obtained in Table 3 and Table 4, the graphs of the coefficient values versus N are plotted as shown in Fig. 5. and Fig. 6 respectively. Again the same software is utilized to obtain best fit representations of the coefficients in terms of N and k.



(a)

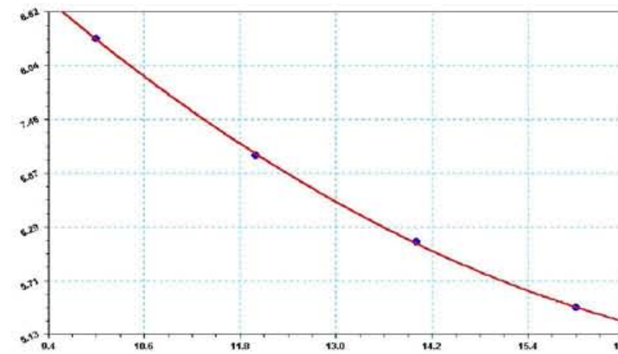


(b)

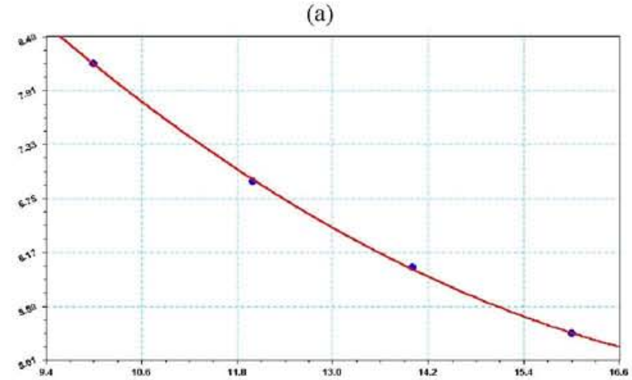


(c)

Fig. 5 Coefficients (a) a (b) b (c) c versus N for slope



(a)



(b)

Fig. 6 Coefficients (a) d (b) e versus N for intercept

Thus, for ap1 < 0.85, an equation in the form of (8) is obtained where,

$$M = (0.00170N^2 - 0.05736N + 0.5149)k^2 +$$

$$(-0.01733N^2 + 0.6269N - 6.467)k + (-0.01263N^2 + 0.44598N - 4.2953)$$
$$C = (0.034N^2 - 1.379N + 18.637)k + (0.035N^2 - 1.389N + 18.6)$$

The same process is repeated for odd angles with ap1 $\geq$ 0.85 where the equations obtained for M and C are,

$$M = (-0.00338N^2 + 0.11271N - 0.99525)k^2 + (-0.01054N^2 + 0.40364N - 4.65124)k + (-0.02225N^2 + 0.8565N - 9.90195)$$
$$C = (0.00429N^2 - 0.14387N + 1.2829)k^2 + (0.02905N^2 - 1.1943N + 16.964)k + (0.04237N^2 - 1.7182N + 23.2847)$$

For the even angles, the equations obtained are in the form of,

$$Alpha_{k(even)} = (M \times ap1) + C \quad (9)$$

where, k(even) = k^{th} even angle

For ap1 < 0.85, the equations for M and C are obtained as,

$$M = (0.00054N^2 - 0.0168N + 0.135)k^2 + (0.00389N^2 - 0.151N + 1.689)k + (0.00594N^2 - 0.188N + 1.469)$$
$$C = (0.0343N^2 - 1.368N + 18.413)k + (0.0019N^2 - 0.073N + 0.824)$$

Whereas for even angles with ap1 $\geq$ 0.85, the equations for M and C are,

$$M = (-0.00403N^2 + 0.13399N - 1.18368)k^2 + (-0.00248N^2 + 0.09905N - 1.18935)k + (-0.01035N^2 + 0.39015N - 4.50672)$$
$$C = (0.0041N^2 - 0.13638N + 1.203)k^2 + (0.03977N^2 - 1.5697N + 20.553)k + (0.0105N^2 - 0.3987N + 4.6544)$$

An error analysis is conducted to determine the accuracy of the switching angles calculated using the derived equations compared to the actual switching angles. Table 5 gives the results of the error analysis.

Table 5. Maximum and minimum errors

N	Maximum error (°)	Minimum error (°)
10	0.6536	0.0002
12	0.6123	0.0121
14	0.9605	0.0095
16	0.6071	0.0101

The table indicates that the errors between the actual switching angles and the switching angles calculated using the derived equations are between a maximum of 0.9605° and a minimum of 0.0002°. In general, the errors are higher with higher k, as ap1 approaches 1. The effect of the errors on the performance of the inverter based on the HEPWM technique will be further discussed in the next section.

IV. Results Of Inverter Simulation Using The Derived Equations

The single-phase full-bridge inverter operation is simulated using Matlab/Simulink. Fig. 7 shows the flow chart of the inverter simulation. The switching angle calculator represents a sub-system designed in Simulink, taking into account the derived equations that allow calculation of the switching angles based on the HEPWM technique, depending on the value of ap1 and N. The simulation is conducted using *fixed-step* solver with minimum step size of 10μ seconds which translates to 0.18° switching angle resolution. The results of the simulation is analyzed in terms of its output voltage harmonic spectrum. The simulation is run for several N and ap1 values.

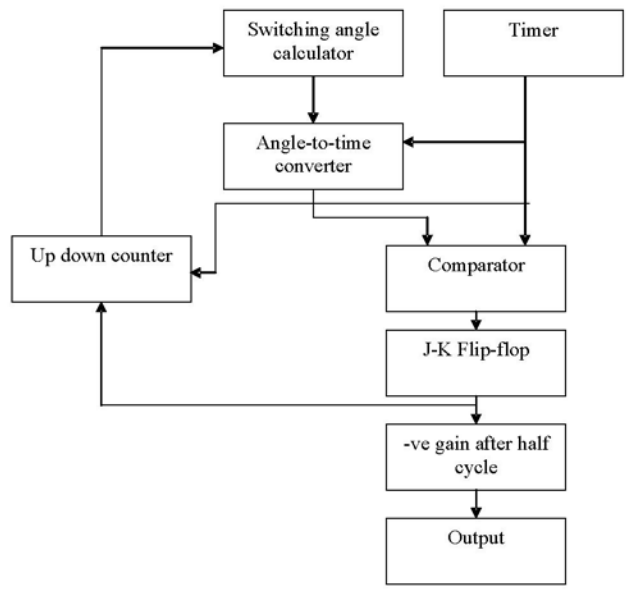
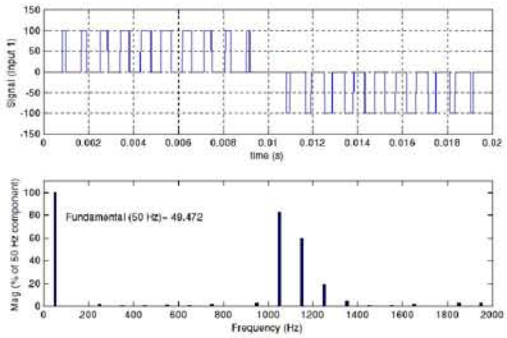


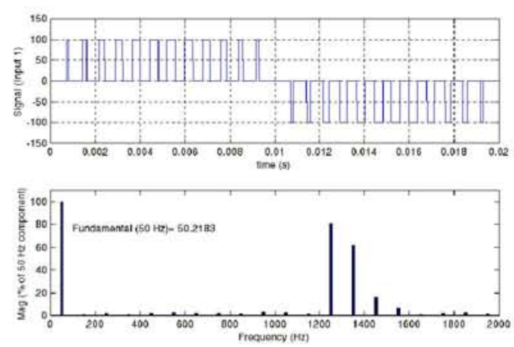
Fig. 7 Flow chart of the simulation

From the output voltage harmonic spectrums of Fig. 8, it can be observed that most of the N-1 harmonics have been eliminated or substantially reduced for each given case. The fundamental values are also generally compatible to the given ap1 value although in some cases minor inconsistencies are detected. This typically occurs at the condition where the error between the actual switching angles and the calculated switching angles are higher compared to the rests as highlighted in the previous section.

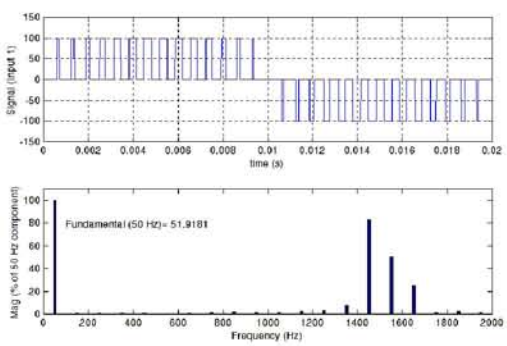
It is important to realize that implementation of HEPWM technique on an inverter requires accurate switching angle values in order to ensure that the specified harmonics will be eliminated according to the theory. With the use of microprocessors or other digital techniques, some deviations from the actual switching angle values are expected but has to be limited by ensuring that the sample time chosen is the smallest possible. With the proposed technique of trajectories linearization, although the step size (representing sample time) used is already quite small, improvement must also be made to further reduce the errors between the calculated and the actual switching angle values. Work is currently being done to introduce correction factors that can compensate the shortcomings of the derived equations.



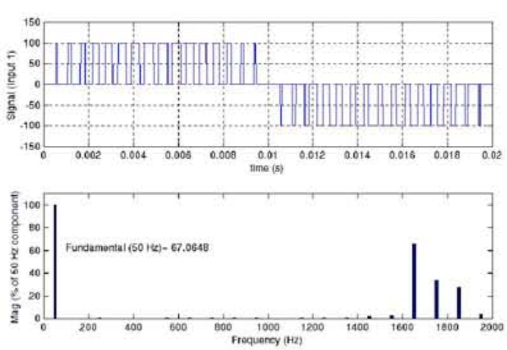
(a) N = 10, ap1 = 0.5



(b) N = 12, ap1 = 0.5



(c) N = 14, ap1 = 0.5



(d) N = 16, ap1 = 0.6

Fig. 7 Output voltage waveform and harmonic spectrum

V. Conclusions

The proposed linearization method is able to represent the trajectories for N = 10, 12, 14 and 16 with errors ranging between 0.0002 and 0.9605. The set of derived equation is long but consists of basic operation only, which may suit the requirement for microprocessor implementation. The total mathematical operation for the longest equation is 19 multiplications and 18 additions. The operation can simply be completed by doing function looping in microprocessor programming since the equation is formed from 8 quadratic equations.

According to the result from the simulation, most of the harmonics are eliminated while some exist but with negligible magnitude. For certain N and ap1, the magnitude of the fundamental showed inconsistencies with the ap1 values. This is probably due to the errors from the derived equations and step size of the simulation. Further improvement can still be made to the proposed linearization method but it is worth noting that with the derived equations, the switching angles for the HEPWM technique can be calculated online for any ap1 value and with a choice of four different N which can prove to be useful for inverter drive applications.

VI. References

[1] Enjeti, P. N., Ziogas, P.D and Lindsay, J.F., "Programmed PWM Techniques to Eliminate Harmonics: A Critical Evaluation", *IEEE Transactions on Industry Applications*, vol. 26, no. 2, 1990, pp.302-316.

[2] H.S. Patel and R.G. Hoft, " Generalized Harmonic Elimination and Voltage Control in Thyristor Inverters: Part I – Harmonic Elimination", *IEEE Transactions on Industry Applications*, vol. 9, no. 3, 1973, pp.310-317.

[3] Azli, N.A. and Yatim, A. H. M., "Optimal Pulsewidth Modulation (PWM) Online Control of a Modular Structured Multilevel Inverter (MSMI)." The 4th IEEE International Conference on Power Electronics and Drive Systems PEDS'01,22-25 October 2001, Bali, Indonesia.

[4] N. Ahmad Azli and A.H.M Yatim, "A Curve Fitting Technique (CFT) for Optimal PWM Online Control of a Modular Structured Multilevel Inverter (MSMI)." Australasian Universities Power Electronics Conference AUPEC2002, Melbourne, Australia. 29th Sept. – 2nd October 2002.

[5] Naziha Ahmad Azli and Abdul Halim Mohd Yatim, " Curve Fitting Technique for Optimal Pulsewidth Modulation (PWM) On-line Control of a Voltage Source Inverter (VSI)." IEEE TENCON 2000, 24-27 September 2000, Kuala Lumpur.

[6] Zainal Salam and Chew Tit Lynn, "A New Near Optimal Harmonics Elimination PWM Algorithm for Traction Drives", *Jurnal Teknologi*, 36(D): June 2002, pp. 111-128, UTM Publisher.

[7] Bowes, S. R., "Suboptimal Switching Strategies for Microprocessor-controlled PWM Inverter Drives", *IEE Proceedings Pt. B*. 132(3), 1985, pp 133-148.

[8] Sun, J., Beineke, S. and Grotstollen, H., "Optimal PWM Based on Real-time Solution of Harmonic Elimination Equations", *IEEE Transactions on Power Electronics*, 11 No. 4, 1996, pp 612-621.

[9] Taufiq, J. A., Mellitt, B. and Goodman, C. J., "Novel Algorithm for Generating Near Optimal PWM Waveforms for AC Traction Drives", *IEE Proceedings Pt. B*. 133 No. 2., 1986, pp 85-94.

[10] Jiang, Q., Holmes, G. and Giesner, D. B., "A Method of Linearising Optimal PWM Switching Strategies to Enable Their Computation On-line in Real-time", Conf. Rec. of IEEE/IAS Annual Meeting, 1991, pp 819-825.