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The Application of Cross Correlation Technique for Estimating Impulse Response and Frequency Response of Wireless Communication Channel

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***Abstract*-- Cross correlation technique is one of the most commonly used methods to estimate the channel impulse response such as a wireless communication channel. This paper looks at the use of two different types of signals that is applied in the cross correlation technique. The signals are Phase Shift Keying (PSK) and linear FM signals (Chirps). Pseudo random sequence (PRS) is used in the generation of the PSK signals. The parameters of the PRS are varied according to the sequence lengths, sampling frequency and bit rate. From the cross correlation functions and the cross spectrum, the channel impulse response and channel frequency response are respectively estimated. Besides PSK signals, linear FM signal is also used as an input to estimate the channel impulse response. Both PSK and linear FM signals are chosen due to their auto correlation properties that are similar to an impulse function. Thus, the channel impulse response can be estimated based on the cross correlation between the output and the input signals. The desired and the estimated channel impulse responses and channel frequency response are respectively estimated and compared based on the mean squared error (MSE).**

***Index Terms*-- Cross Correlation, Phase Shift Keying (PSK), Linear FM (chirps), Impulse response, channel characterization**

I. INTRODUCTION

HF (High Frequency) spectrum is a communication medium which has been used extensively for more than 80 years. It provides reliable communications which is inexpensive to operate and is used globally by the government, non-profit organizations, the military, small and large organizations and individuals. Multipath fading in HF channels causes time-delay spread that limits the maximum symbol rate and the frequency selective fading that attenuates signals at selected frequencies within a given communication channel. Measurement of the time delay and the frequency selectivity is important to ensure reliable data transmission within minimum loss of information and distortion.

The estimation of the channel impulse response utilizing cross correlation techniques has been well covered in papers [1],[2],[3],[4] and [5]. The cross correlation relationship between the input and output signals makes use of an impulse function properties of an auto correlation function of the input signal in estimating the channel or system impulse response (CIR) / (SIR). To produce an impulse function of an auto correlation function, the input must have the noise like character. To achieve this purpose, [1]- [4] proposed the use of pseudo random sequence (PRS). They used PRS in characterizing a wideband channel. PRS can be generated using several methods among them are the linear feedback shift register and Barker sequence. They have proved extremely popular in communication, navigation and ranging systems [6]. Estimating the CIR by means of cross correlation function make used of two types of signals: i) PRS in phase modulating the HF pulses and ii) linear FM signal.

This paper discusses the method in estimating the channel impulse response via the use of cross correlation method. Two types of input signal namely PSK and linear FM signal are used for this purpose. The estimated channel impulse response (CIR) and channel frequency response (CFR) are compared with their respective desired values using mean squared error (MSE) method to see the coherent between the two values.

II. CHANNEL MODEL

High Frequency signals or known as sky wave between 3 – 30 MHz can propagate from the transmitter to the receiver through the ionosphere hundreds of miles away by being repeatedly reflected from both the ionosphere and the earth. This signal experiences a typical perturbation which is affected by one or a combination of these factors: noise, Doppler Effect, phase instability, multipath propagation, mode dispersion and attenuation [3]. These factors contribute to the destructive signal fading. Therefore, the identification and the characterization of the HF channel are essential for data communication through the spectrum. This identification and the characterization of CIR will be used in mitigating channel impairment.

Many narrowband channels have been developed based on the model originated by Watterson et al [7] and [8]. However, this model has several limitations in terms of modeling with

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wider bandwidth channels, restricted to channel having low ray path, time and frequency stationary, negligible delay dispersion and a bandwidth of less than 12 kHz [8]. This model is adopted by the ITU-R (International Telecommunication Union-Radio) and is referred by the CCIR 520-1 Recommendations. Table 1 gives guidelines on practical values for frequency spread and delay times between ray components. These values are used to validate average and extreme conditions during simulation as well as during actual hardware testing [9].

TABLE I
CCIR RECOMMENDATION TABLE 520-I

Condition	Frequency spread (Hz)	Delay (ms)
Flat fading	0.2	0
Flat Fading (Extreme)	10.	0
Good	0.1	0.5
Moderate	0.5	1.0
Poor	1.0	2.0

For this paper, the estimation of the system impulse response is based on a channel having a time delay of 2 msec with equal amplitude. This value is chosen since it falls under poor channel condition as stated by the CCIR 520-1 Recommendation.

III. THEORY

A. Cross Correlation Technique

The channel is assumed to behave as a linear time invariant system [8] having an impulse response $h(t)$ and its frequency response $H(f)$. From [10], the system or channel impulse response, $h(t)$ for the linear time invariant system can be characterized using correlation function as shown in Figure 1.

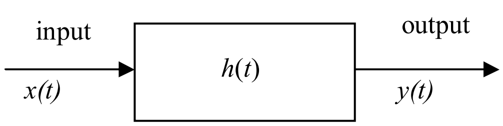

Figure 1: Linear Time Invariant System

To estimate the system impulse response, the cross correlation function between the output, $y(t)$ and input, $x(t)$ is estimated. Since the input-output relationship is a convolution, the relationship in terms of the cross correlation function is given as follows

$$R_{yx}(\tau) = h(\tau) * R_{xx}(\tau) \quad (1)$$

where $h(\tau)$ is the channel impulse response and $R_{xx}(\tau)$ is the autocorrelation function of the input signal.

Since the convolution in time domain is equivalent to multiplication in the frequency domain, then relationship in terms of the cross spectrum is

$$S_{yx}(f) = H(f).S_{xx}(f) \quad (2)$$

where $H(f)$ is the frequency representation of the channel or the channel frequency response (CFR) and $S_{xx}(f)$ is the power spectrum of the input signal.

If the auto correlation function $R_{xx}(\tau)$ approaches an impulse function, its power spectrum $S_{xx}(f)$ becomes a constant in value in frequency. Thus, the cross correlation function is equal to the system impulse response and the cross spectrum equals to the frequency representation of the channel. This is illustrated as follows

$$R_{yx}(\tau) = h(\tau) * p_0\delta(\tau) = p_0 h(\tau) \quad (3)$$

$$S_{yx}(f) = H(f)p_0 \quad (4)$$

where p_0 is the power of input signal. Two different types of inputs are employed in estimating the channel impulse response via cross correlation technique, which are Phase Shift Keying (PSK) and Linear FM signal.

B. Phase Shift Keying (PSK)

In general, PSK signal is defined as the following equations:

$$x(t) = \cos 2\pi f_1 t \text{ , binary bit `1'}$$
$$x(t) = -\cos 2\pi f_1 t \text{ , binary bit `0', } 0<t<T_b \quad (5)$$

where f_1 is the signal frequency and T_b is the bit duration of the signal. Assuming a pseudo random sequence is used, then the autocorrelation function and power spectrum are

$$R_{xx}(\tau) = \frac{1}{2}\left[1 - \frac{|\tau|}{T_b}\right]\cos 2\pi f_1 \tau \quad (6)$$

$$S_{xx}(f) = \frac{1}{4}\left[\frac{\sin 2\pi(f_1 - f)T_b}{2\pi(f_1 - f)T_b}\right]^2 + \frac{1}{4}\left[\frac{\sin 2\pi(f_1 + f)T_b}{2\pi(f_1 + f)T_b}\right]^2 \quad (7)$$

In this work, PSK signal is modulated based on several lengths of pseudo random sequence (PRS). PRS is known to have an auto correlation function of an impulse function. Two different sampling frequencies and their respective number of bit rate are used in the modulation of PSK signal. A binary pseudo random sequence is generated using a linear feedback shift register (LFSR). The output sequence of LFSR is purely random and is used to modulate PSK signal as mentioned by [5, 10]. The number of sequence length of a LFSR depends on the number of stages used. For this particular purpose, the LFSR defined by a polynomial in the GF(2) is

$$f(x) = 1 + x^3 + x^{10} \quad (8)$$

and the period of the sequence is 2^{10}-1=1023 bits.

For the comparison purposes between the desired and the estimated CIR and its respective CFR, a sampling frequency of 8 kHz and 16 kHz are used, where the sequence length is set for 256, 512 and 1024 bits. Overall, there are six different conditions of modulated PSK input signal to be analyzed. The comparison between the desired and estimated CIR are made based on the mean squared error (MSE). This is done by taking the sum of the difference squared between the estimated system impulse response and their desired values. Similarly comparison is made between the desired and the estimated CFR. The estimated CFR is the cross correlation spectrum of the CIR. MSE gives an indication on how good is this method in estimating the system impulse response. Furthermore, through the correlation method, the auto and cross correlation of the channel show the characteristic of the PSK and linear FM signals being tested.

C. Linear FM

A linear FM signal or known as chirp is a signal where its frequency increases or decreases with respect to time. It is commonly used in sonar, radar and spread spectrum communications.

Linear FM signal is defined as

$$x(t) = \exp\left(j\theta(t)\right)$$
$$\theta(t) = 2\pi(f_0 + \frac{\alpha}{2}t)t \quad (9)$$

where f_0 is the starting frequency and α is a chirp rate.

In this work, a linear FM signal is generated with a bandwidth of 3500 Hz, duration of 50msec and a zero delay. Since the auto-correlation function of a linear FM signal (chirp) is also an impulse function, the application of the linear FM to the system impulse response will give the same analysis as of the PSK signals. The spectrum of an impulse function is constant for the whole frequency spectrum. Therefore, the CIR and CFR can be determined the same manner as for the PSK signal.

D. Performance Measure

Mean square error (MSE) is used to quantify the difference between measured and actual data. Since the signal is processed in discrete-time and frequency, the MSE for the impulse response comparison is

$$E_t = \frac{1}{N}\sum_{n=0}^{N-1} e_t(n)^2 = \frac{1}{N}\sum_{n=0}^{N-1}(\hat{h}(n) - h_d(n))^2 \quad (10)$$

where $\hat{h}(n)$ is the estimated CIR and $h_d(n)$ is the desired CIR.

For the CFR, the MSE is defined as

$$E_f = \frac{1}{N}\sum_{k=0}^{N-1} e_f(k)^2 = \frac{1}{N}\sum_{k=0}^{N-1}(\hat{H}(k) - H_d(k))^2 \quad (11)$$

where $\hat{H}(k)$ is the estimated CFR and $H_d(k)$ is the desired CFR. Generally, MSE is used to quantify how good the estimated value is compared to the desired values and whether the method used is reliable and justifiable.

IV. RESULT AND DISCUSSION

As shown in Section III, the auto correlation function of a random sequence and linear FM signal approaches an impulse function. Therefore, the cross correlation function can estimate the channel impulse response (CIR) and its channel frequency response (CFR) is estimated from its power spectrum. Figure 2 depicts the desired CIR and its CFR for 8000 Hz sampling frequency. Table 2 summarizes the estimation of MSE for all the six (6) conditions of PSK signals while Table 2 gives the estimation of MSE for linear FM signal. Based on Table 2, the value of MSE estimated is dependent on two factors, which are the sequence length and sampling frequency. As the sequence length decreases from 1024 bits to 256 bits, MSE found is increased for both sampling frequencies. Figure 3a and 3b show the plot of CIR and CFR for different sequence length having same sampling frequency. Better performance is found for higher value of random sequence. Another factor that affects the estimation of the CIR and CFR is the sampling frequency. It shows that from Table 2, as the sampling frequency is increased from 8000 Hz to 16000 Hz, and by maintaining the same bit rate and sequence length, the value of MSE decreases. This indicates that better performance if the sampling frequency is higher. Theoretically this is true because as the sampling frequency increases, the bandwidth will get wider, which will give better spectrum. However, another question arises. Do we need that high frequency to perform our task? There is a choice to be made. The use of higher sampling frequency increases the bandwidth and the complexity of circuitry. Figure 3b and 3c compare the plot of CIR and CFR for both sampling frequency having the same sequence length.

The determination of MSE in the estimation of the CIR and CFR using linear FM signal is shown in Table 3. The relationship between MSE and sampling frequency agrees with the one found in PSK signal. The increase in sampling frequency gives lower MSE. Figure 4 shows the plot of CIR and CFR of linear FM signal, which shows better spectrum compared to any of the PSK signals as agrees with the values of MSE in Table 3. Overall, the estimation of CFR is better in linear FM signal compared to any of the PSK signals as indicated in Table 1 and 2. In generally, the MSE estimated for both PSK and Linear FM signal is much below unity. This indicates that the cross correlation function can be used to estimate the system impulse response.

TABLE 2
MEAN SQUARE ERROR OBTAINED FOR PSK SIGNALS

Sequence Length (SL)	PSK			
	CIR		CFR	
	fs=8kHz	fs=16kHz	fs=8kHz	fs=16kHz
L=1024	2.43 e-4	2.423e-4	0.1371	0.0311
L= 512	4.835e-4	4.805e-4	0.1460	0.0368
L=256	9.609e-4	9.466e-4	0.1650	0.0476

TABLE 3
MEAN SQUARE ERROR OBTAINED FOR LINEAR FM SIGNAL

CHIRPS			
CIR		CFR	
fs=8kHz	fs=16kHz	fs=8kHz	fs=16kHz
0.0024	0.0012	0.0689	0.0232

V. Conclusion

Based on the comparison between the estimated and desired CIR and CFR, it is concluded that the correlation function can be used as the estimation technique since the mean square error (MSE) estimated for both frequencies and different sequence length is very minimal. Furthermore, there are two parameters: sequence length and sampling frequency which play an important role in estimating the CIR and CFR if the signal used is PSK. Generally, linear FM signal give better estimates on the CFR compared to PSK signal.

VI. Appendix: Result

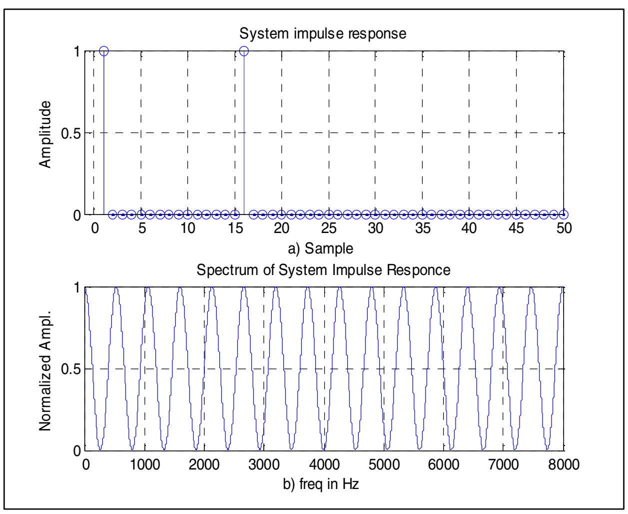

Figure 2: The desired CIR and CFR for fs = 8000 Hz

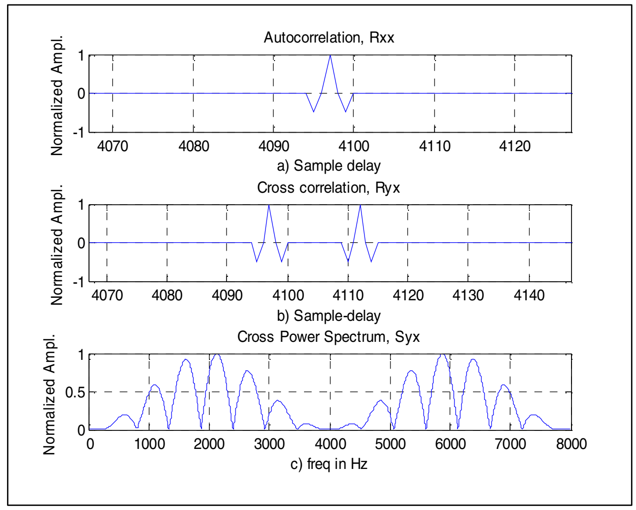

Figure 3a: Estimated CIR and its CFR of PSK signal having fs=8000 Hz and SL (Sequence Length) =1024

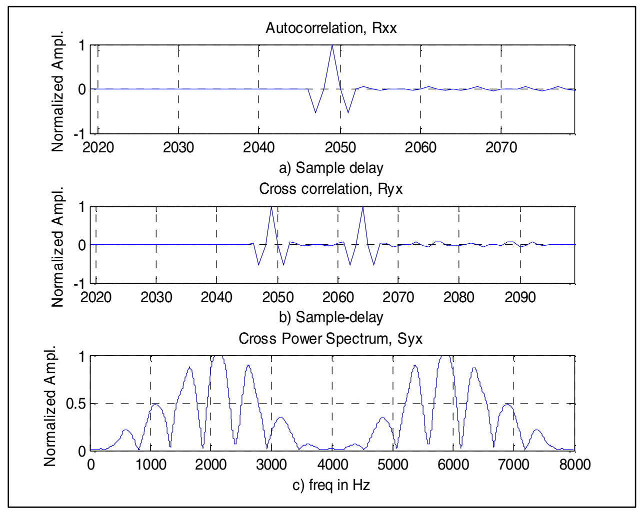

Figure 3b: Estimated CIR and its CFR of PSK signal having fs=8000 Hz and SL=512

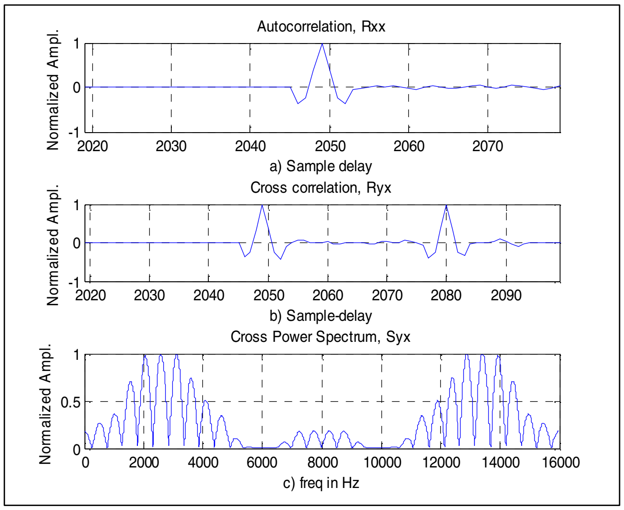

Figure 3c: Estimated CIR and its CFR of PSK signal having fs=16000 Hz and SL=512

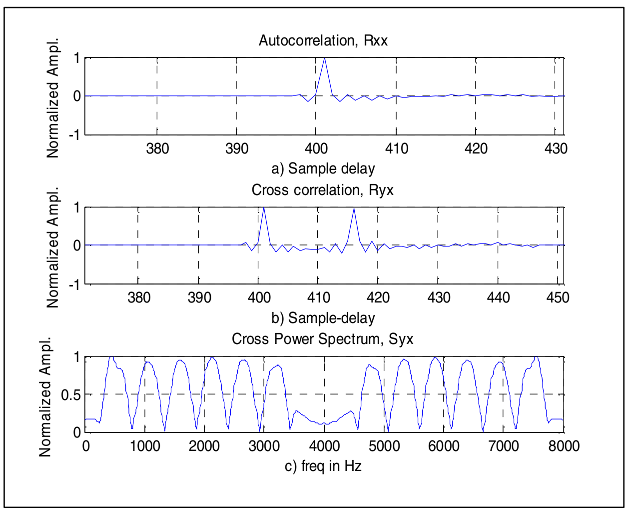

Figure 4: Estimated CIR for Lineear FM signal having fs=8000 Hz

VII. REFERENCES

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