Modeling and Proportional Integral Sliding Mode Control of Hydraulic Manipulators

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***Abstract* - This paper is concerned with the mathematical modeling and the application of a new position tracking control technique for hydraulic manipulators. The integrated model takes into account both the manipulator linkage as well as the actuator dynamics to represent a closer dynamic behaviour of the real system, thus providing a more suitable model for the purpose of advanced controller synthesis and analysis. Although hydraulic manipulators provide large torque and fast response, they possess highly nonlinear dynamics, parameter variations, uncertain load disturbances and strong couplings among various joints. Therefore, a robust control approach based on Proportional Integral Sliding Mode Control (PISMC) technique is adopted to provide position tracking for the system. It will be shown that the proposed controller is practically stable and is successful in forcing the robotic system to track the predefined desired trajectory at all time. A 3 DOF revolute robot manipulator is used in this study.**

Index terms – Robot Manipulator, Hydraulic System, Hydraulic Manipulator, Sliding Mode Control, Proportional Integral Sliding Mode Control

I. INTRODUCTION

Hydraulically actuated manipulators are widely used in a number of applications. Manipulators in construction, industry, and heavy load motion control and mobile equipment applications take the advantage of the high power to weight ratio, stiffness and short response time of hydraulic drives in performing their tasks [1]. In order to increase the hydraulic manipulators' productivity and performance, it is essential to be able to control the system well. However, in spite of their advantages, and unlike electric manipulators, the modeling and control of such system is a challenging task both theoretically and experimentally since hydraulic robots are more complex, due to their nonlinear mechanical linkage dynamics, dependence of the effective driving torque on joint angle, payload uncertainties, strong couplings among various joints and nonlinearities present in the hydraulic actuators themselves.

The majority of the previous work in the synthesis of control law for manipulators deals with the electrically actuated manipulators. Comparatively less work has been done for hydraulic actuated robot [1]. Previous research has spanned from both modeling and control of pure hydraulic servo systems to the control of hydraulic robot with no robotic manipulator dynamics considered in the model such as in [2] and [3]. Adaptive Control Technique was proposed in [2] to control hydraulic cylinders with the application to robot manipulators, but none of the mechanical linkage dynamics are incorporated in the model. The dynamics of the actuator alone is not sufficient to represent the hydraulic manipulator, since it does not include the arm dynamic forces such as inertia forces and gravity effects that the controller needs to compensate [1]. Thus tracking performance of the system can be improved by considering also the robot dynamics in the controller design since it is part of hydraulic servo actuating system. This approach has been successfully shown in many electrical robots in the past.

The majority of current industrial approaches to the robot control arm design treat each joint of the manipulator as a simple linear servomechanism with proportional plus integral plus derivative (PID) or Computed Torque (CTC) controllers [4].The problem with PID controllers is that they are not adequate for the cases when the robot moves at high speed and in situations requiring a precise trajectory tracking since the hydraulic manipulator is nonlinear, time varying, coupled and uncertain in nature. On the other hand, the problem with CTC is that it is essentially based on exact robot arm dynamic model, where the explicit use of an incorrect robot model will deteriorate the control performance. Hence, a robust controller is proposed to drive such a system.

A robust control technique based on Proportional Integral Sliding Mode Control (PISMC) method has been successfully designed for electrically driven robot manipulator as presented in [5]. This technique takes the advantages of zero steady error due to the integral term and robustness offered by the Sliding Mode Control (SMC). Once the system is in the sliding mode, its response depends thereafter on the gradient of the surface and remains insensitive to variations in the system parameters and external disturbances. Different from conventional SMC,

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the proposed technique avoids the need of transformation on the original plant into canonical form or reduced form. Therefore, the order of the motion equation in PISMC is equal to the order of the original system. In [5], a three DOF revolute electric robot is used in the simulations. It is verified that the proposed control law is effective in providing the tracking control and efficient in compensating the nonlinear, coupled and time varying inertia, coriolis, centrifugal and gravitational forces of the mechanical manipulator linkage.

This paper is concerned with the modeling and control of hydraulically driven robot manipulator. In terms of modeling, this paper presents the formulation of dynamic model for an N DOF electrohydraulic robot manipulator that integrates both the actuator dynamics and the mechanical arm dynamics in state space representation. The proposed model is believed to provide a better and much more suitable mathematical representation for the purpose of controller synthesis and analysis. In terms of control strategy for this particular system, this paper extends the control approach as proposed in [5] to provide trajectory tracking control of a hydraulically actuated robot manipulator. The stability proof based on Lyapunov theory is also presented. A 3 DOF electrohydraulic robot is used in the simulation study.

This paper is organized as follows: The system dynamics, including mechanical linkage and hydraulic dynamics are presented in Section II. In Section III, the adopted control approach is described. Simulation results are presented in Section IV. Finally, conclusions are drawn in Section V.

II. DYNAMIC MODELING OF THE HYDRAULIC ROBOT MANIPULATOR

A. Manipulator Mechanical Linkage Dynamics

The dynamic equation of an N DOF mechanical linkage of a robot manipulator with rigid links is governed by [4]:

$$M(\theta(t),\xi)\ddot{\theta}(t) + D(\theta(t),\dot{\theta}(t),\xi) + G(\theta(t),\xi) = T(t) \quad (1)$$

Where;

- $M(\theta(t),\xi)$: $N \times N$ Inertia matrix
- $D(\theta(t),\dot{\theta}(t),\xi)$: $N \times 1$ vector of coriolis and centrifugal forces
- $G(\theta(t),\xi)$: $N \times 1$ vector of gravitational forces
- $T(t)$: $N \times 1$ vector of driving torques applied by the actuators
- $\theta(t),\dot{\theta}(t),\ddot{\theta}(t)$: $N \times 1$ vector of generalized joint displacements, velocities and accelerations respectively
- ξ : uncertain parameters of the mechanism (payload mass)

The terms $D(\theta(t),\dot{\theta}(t),\xi)$ and $G(\theta(t),\xi)$ in (1) can also be written as:

$$D(\theta(t),\dot{\theta}(t),\xi) = \widehat{D}(\theta,\dot{\theta},\xi)\dot{\theta} \quad (2)$$

$$G(\theta(t),\xi) = \widehat{G}(\theta,\xi)\theta \quad (3)$$

B. Electrohydraulic Actuator Dynamics

The physical model of an ith nonlinear hydraulic actuator dynamics can be described in state space representation as in [3], [6], and [7]:

$$\dot{X}_i(t) = A_i X_i(t) + B_i U_i(t) + F_i T_{Li}(t) + W_i \dot{T}_{Li}(t) + N_i \quad (4)$$

where the state variables are:

$$X_i(t) = \left[\theta_{mi}(t) \;\; \dot{\theta}_{mi}(t) \;\; \ddot{\theta}_{mi}(t)\right]^T \quad (5)$$

and:

$$A_i = \begin{bmatrix} 0 & 1 & .0 \\ 0 & 0 & 1 \\ -\dfrac{4\beta_e K_t G}{V_t J_m} & -\left[\dfrac{4\beta_e K_t B_m}{V_t J_m} + \dfrac{G}{J_m} + \dfrac{4\beta_e D_m^2}{V_t J_m}\right] & -\left[\dfrac{4\beta_e K_t}{V_t} + \dfrac{B_m}{J_m}\right] \end{bmatrix} \quad (6)$$

$$B_i = \begin{bmatrix} 0 \\ 0 \\ \dfrac{4\beta_e D_m K_q K_v}{V_t J_m n_g} \end{bmatrix} \quad (7) \qquad W_i = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{1}{J_m n_g^2} \end{bmatrix} \quad (8)$$

$$F_i = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{4\beta_e K_t}{V_t J_m n_g^2} \end{bmatrix} \quad (9)$$

$$N_i = \begin{bmatrix} 0 \\ 0 \\ \dfrac{4\beta_e K_t G_n G n_g^2}{V_t J_m}\theta_i^3 + \dfrac{3G_n G n_g^2}{J_m}\theta_i^2\dot{\theta}_i \end{bmatrix} \quad (10)$$

$$K_q = C_d w\sqrt{(P_s - P_L\ \mathrm{sgn}(X_v))/\rho} \quad (11)$$

$$X_v = K_v U(t) \quad (12)$$

where X_v is the displacement of the spool in the servo valve, P_L is the load pressure, K_q is the flow gain which varies at different operating points, C_d is the discharge coefficient, w is the area gradient, ρ is the fluid mass density, P_s is the supply pressure, D_m is the volumetric displacement, θ_{mi} is the angular displacement of the ith motor shaft, $\dot{\theta}_{mi}$ is the angular velocity of the ith motor shaft, $\ddot{\theta}_{mi}$ is the angular acceleration of the ith motor shaft, K_t is the total leakage coefficient of the hydraulic system, V_t is the total compressed volume, is the effective bulk modulus of the oil, J_m is the motor inertia, B_m is the viscous damping coefficient, T is the load torque due to the joint of the ith manipulator on the ith motor, $\dot{T}$ is the derivative of the load torque due to the joint of the ith manipulator on the ith motor, G is the torsional spring constant, $G_n\theta_{mi}^3$ is the nonlinear stiffness of the spring, and n_g is the inverse of the gear ratio.

The equation is derived based on the assumptions that:

a. the piston is centered such that the volume of the fluid trapped at the sides of the actuator are equal,
b. the valve is an ideal critical center valve with matched and symmetrical orifices,
c. return line pressure is zero

For an N DOF robot manipulator (N actuators), the augmented dynamic equation of the actuators can be written in the following compact form as follows [4]:

$$\dot{X}(t) = AX(t) + BU(t) + FT(t) + W\dot{T}(t) + N \quad (13)$$

where;

$$X(t) = \left[X_1^T(t), X_2^T(t), \ldots, X_N^T(t)\right]^T$$
$$U(t) = \left[U_1(t), U_2(t), \ldots, U_N(t)\right]^T$$
$$T(t) = \left[T_1(t), T_2(t), \ldots, T_N(t)\right]^T$$
$$\dot{T}(t) = \left[\dot{T}_1(t), \dot{T}_2(t), \ldots, \dot{T}_N(t)\right]^T$$
$$A = \text{diag}\left[A_1, A_2, \ldots, A_N\right]$$
$$B = \text{diag}\left[B_1, B_2, \ldots, B_N\right]$$
$$F = \text{diag}\left[F_1, F_2, \ldots, F_N\right]$$
$$W = \text{diag}\left[W_1, W_2, \ldots, W_N\right]$$

and $X(t)$ is a $3N \times 1$ vector, $U(t)$ is an $N \times 1$ input vector, $T(t)$ is the $N \times 1$ mechanical link torque, and $\dot{T}(t)$ is its time derivative.

C. Integrated Electrohydraulic Manipulator Dynamics

The integrated model of the hydraulic manipulator can be obtained by first defining the following transformations [4]:

$$\begin{aligned} \ddot{\theta}(t) &= Z_B X(t) \\ \dot{\theta}(t) &= Z_{B1} X(t) \\ \theta(t) &= Z_{B2} X(t) \end{aligned} \quad (14)$$

Then, the manipulator torque in (1) is differentiated with respect to time to obtain its derivative [4]:

$$\begin{aligned} \dot{T}(t) = M(\theta(t), \xi)\dddot{\theta}(t) + \widetilde{C}\left(\theta(t), \dot{\theta}(t), \xi\right)\ddot{\theta}(t) \\ + \widetilde{D}\left(\theta(t), \dot{\theta}(t), \xi\right)\dot{\theta}(t) \end{aligned} \quad (15)$$

Substituting (1) and (15) into (13) and utilizing (14) yields the integrated model of the hydraulic robot manipulator as:

$$\dot{X}(t) = A_N(X, \xi, t)X(t) + B(X, \xi, t)U(t) \quad (16)$$

where;

$$A_N(X,\xi,t) = \left[I_{3N} - WM(X_B,\xi,t)Z_B\right]^{-1} \begin{Bmatrix} A_N + [FM(X,\xi,t) \\ + W\widetilde{C}(X,\xi,t)]Z_B \\ + [F_B\widehat{D}(X,\xi,t) \\ + W\widetilde{D}(X,\xi,t)]Z_{B1} \\ + F\widehat{G}(X,\xi,t)Z_{B2} \end{Bmatrix} \quad (17)$$

$$B(X, \xi, t) = \left[I_{3N} - WM(X, \xi, t)Z_B\right]^{-1} B \quad (18)$$

The system matrix, $A_N(X,\xi,t)$ and the input matrix, $B(X,\xi,t)$ are of dimensions $3N \times 3N$ and $3N \times N$ respectively. Therefore, for a 3 DOF revolute hydraulic robot manipulator, the size of its system matrix is 9×9 while its input matrix is of size 9×3. Each nonzero element of these matrices is a function of the instantaneous position, velocity and payload mass of the manipulator. From (16), (17) and (18), it is clear that the resulting dynamics description of the robotic system is analytically complex. The equations are time varying, highly nonlinear and coupled due to the nonlinear mechanical linkage as well as hydraulic dynamics. These equations also contain parameter uncertainty which is the varying payload mass. Therefore, a more robust controller that is capable of catering these plant characteristics as presented in next section is required.

III. PROPORTIONAL INTEGRAL SLIDING MODE CONTROLLER

The controller design involves two stages. The first part deals with the design of a sliding surface for which the system dynamics to slide and remain on it. The second part aims in deriving a control signal to maintain the system dynamics on this particular surface.

The bounds on elements of the matrices $A_N(X,\xi,t)$ and $B(X,\xi,t)$ can be computed and specified since the physical parameters of the manipulator mechanical linkage and hydraulic actuator are known (specified by the manufacturer). Therefore (16) can be decomposed into nominal and uncertain matrices and rewritten as:

$$\dot{X}(t) = [\overline{A} + \Delta A(x, \xi, t)]X(t) + [\overline{B} + \Delta B(x, \xi, t)]U(t) \quad (19)$$

$\overline{A}$ and $\overline{B}$ are the time-invariant, nominal values of $A_N(X,\xi,t)$ and $B(X,\xi,t)$ respectively, with

$$\overline{A} = \frac{A_{N\,MAX} + A_{N\,MIN}}{2} \quad (20)$$

$$\overline{B} = \frac{B_{MAX} + B_{MIN}}{2} \quad (21)$$

ΔA and ΔB are uncertainty value of values of $A_N(X,\xi,t)$ and $B(X,\xi,t)$ respectively, with

$$\Delta A = \overline{A} - A_{NMIN} \quad (22)$$

$$\Delta B = \overline{B} - B_{MIN} \quad (23)$$

Define the state vector of the system as

$$X(t) = [x_1(t), x_2(t),..., x_n(t)]^T \quad (24)$$

Let a continuous function $X_d(t) \in R^n$ be the desired state trajectory, where $X_d(t)$ is defined as:

$$X_d(t) = [x_{d1}(t), x_{d2}(t),..., x_{dn}(t)]^T \quad (25)$$

Define the tracking error, $Z(t)$ as

$$Z(t) = X(t) - X_d(t) \quad (26)$$

In this study, the following assumptions are made:

a. The state vector $X(t)$ can be fully observed;

b. There exist continuous functions $H(t)$ and $E(t)$ such that for all $X(t) \in R^n$ and all t:

$$\Delta A(t) = \overline{B}H(t) \ ; \quad \|H(t)\| \le \alpha \quad (27)$$

$$\Delta B(t) = \overline{B}E(t) \ ; \quad \|E(t)\| \le \beta \quad (28)$$

c. There exist a Lebesgue function $\Omega(t) \in R$, which is integrals on bounded interval such that

$$\dot{X}_d(t) = \overline{A}X_d(t) + \overline{B}\Omega(t) \quad (29)$$

d. The pair $(\overline{A}, \overline{B})$ is controllable.

Equations (27) and (28) in Assumption (b) ensures that the matching condition is satisfied, that is the uncertainties $\Delta A(X,\xi,t)$ and $\Delta B(X,\xi,t)$ lie in the range space of the nominal input matrix B. This assumption is needed so that the control signal, $U(t)$ which enters the system through the input matrix, $B(X,\xi,t)$ can compensate the parameter variations and uncertainties present in the system.

Define the Proportional-Integral (PI) sliding surface as [5]:

$$\sigma(t) = CZ(t) - \int_0^t [C\overline{A} + C\overline{B}K]Z(\tau)d\tau \quad (30)$$

where $Z(t)$ is defined as the tracking error:

$$Z(t) = X(t) - X_d(t) \quad (31)$$

The structure of the matrix C is as follows:

$$C = diag[c_1 \ \ c_2 \ \ \cdots c_{n_i}] \quad (32)$$

where n_i is the nth state variable associated to the ith input of the system. The matrix C is chosen such that $C\overline{B} \in R^{m\times m}$ is nonsingular. The matrix K is designed to satisfy [5]:

$$\lambda_{\max}(\overline{A} + \overline{B}K) < 0 \quad (33)$$

(33) guarantees that the system is stable by placing the desired poles in the left half plane. The elements of matrix K can be determined by pole placement technique with pre-specified poles locations[5].

Next, the control problem is to design a tracking controller using the PI sliding mode given by equation (30) such that the robotic system state trajectory $X(t)$ tracks the desired state trajectory $X_d(t)$ as closely as possible for all t in spite of the uncertainties and nonlinearities present in the system.

The manifold of equation (30) is asymptotically stable in the large, if the following hitting condition is held [5]:

$$(\sigma^T(t) / \|\sigma(t)\|)\,\dot{\sigma}(t) < 0 \quad (34)$$

As a proof, let the positive definite function be

$$V(t) = \|\sigma(t)\| \quad (35)$$

Differentiating equation (35) with respect to time, t yields

$$\dot{V}(t) = (\sigma^T(t)\,\dot{\sigma}_i(t)) / \|\sigma(t)\| \quad (36)$$

Following the Lyapunov stability theory, if equation (34) holds, then the sliding manifold $\sigma(t)$ is asymptotically stable in the large.

***Theorem*:** The hitting condition (34) of the manifold given by (30) is satisfied if the control $u(t)$ is governed by [5]:

$$u(t) = -(C\overline{B})^{-1}[\gamma_1\|Z(t)\| + \gamma_2\|X_d(t)\| + \gamma_3\|\Omega(t)\|]SGN(\sigma(t)) + \Omega(t) \quad (37)$$

where

$$\gamma_1 > (\alpha\|C\overline{B}\| + \|C\overline{B}K\|)/(1+\beta) \quad (38)$$

$$\gamma_2 > (\alpha\|C\overline{B}\|)/(1+\beta) \quad (39)$$

$$\gamma_3 > (\beta\|C\overline{B}\|)/(1+\beta) \quad (40)$$

Let (38), (39) and (40) hold, then the global hitting condition (34) is satisfied. The proof of this theorem is given in the Appendix.

IV. RESULTS AND DISCUSSIONS

A computer simulation is performed on the developed control system to evaluate its performance in compensating the plant's nonlinearities, parameter variations and uncertainties. Three DOF revolute hydraulically driven robot manipulator is used in this study. The desired trajectory, $\theta_{di}(t)$ for each of the joints is specified as a smooth function represented by [4]:

$$\theta_{di}(t) = \begin{cases} \theta_i(0) + \frac{\Delta_i}{2\pi}[\frac{2\pi t}{\tau} - \sin(\frac{2\pi t}{\tau})], & 0 \le t \le \tau \\ \theta_i(\tau), & \tau \le t \end{cases} \quad (41)$$

where,

$$\Delta_i = \theta_i(\tau) - \theta_i(0),\ i = 1,2,3 \qquad \tau = 2s \qquad (42)$$

The joint trajectories are set to start at the initial position of $[\theta_1(0) \quad \theta_2(0) \quad \theta_3(0)]^T = [-0.8 \quad -1.5 \quad -0.5]^T$ radians, to a desired final position of $[\theta_1(\tau) \quad \theta_2(\tau) \quad \theta_3(\tau)]^T = [1 \quad 0.2 \quad 1.2]^T$ radians in time $\tau = 2$ seconds.

For performance comparison, a linear control approach based on Independent Linear Joint Controller (IJC) technique is presented. IJC which is normally used in most industrial robot is designed with the dynamics of the mechanical linkage completely ignored. Each joint of the robot arm is treated as an independent servomechanism problem. The linear state feedback controller employed in each of the joint is described as:

$$U_i(t) = K_i Z_i(t) + \Omega_i(t) \qquad (43)$$

where, K_i is the linear state feedback gain, $\Omega_i(t)$ is the control component to eliminate the steady state error and

$$Z_i(t) = X_i(t) - X_{di}(t) \qquad (44)$$

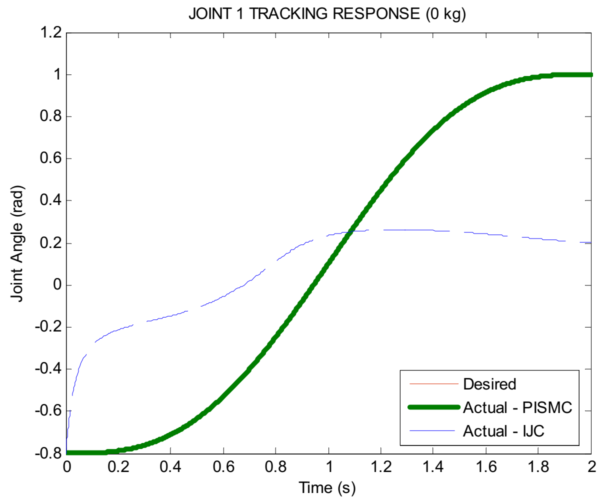
The simulation results for the manipulator operating under no load condition as can be observed from Figures 1 – 3 show that the system has successfully tracks the desired trajectory.

Figure 1 illustrates the tracking performance of PISMC and IJC on all the three joints with the robot operating at minimum payload mass or at its lower bound (0 kg). The result demonstrates that PISMC performance is by far better than IJC, in which it has successfully force the robotic system to track the desired trajectory very closely at all times. The tracking error as shown in Figure 2 verifies that the control system is insensitive against the plant's nonlinearities, parameter variations and couplings.

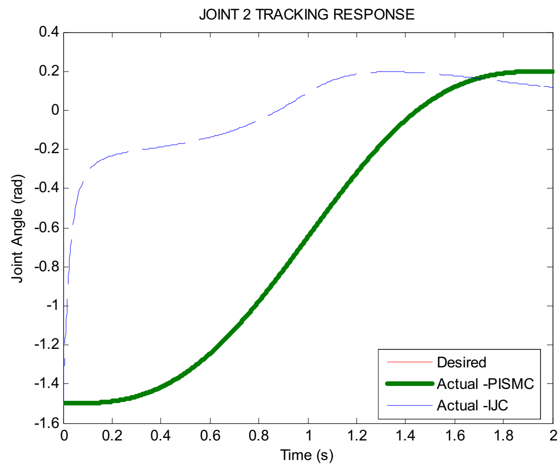
To validate the controller robustness against load variation (uncertainty), the simulation is repeated with the manipulator handling 10 kg load (upper bound). From Figure 3 it can be clearly observed that the manipulator efficiently tracks the desired trajectory with almost negligible error although the payload mass in increased. Therefore, it can be deduced that the control system is robust against uncertainty (payload mass variation).

V. CONCLUSION

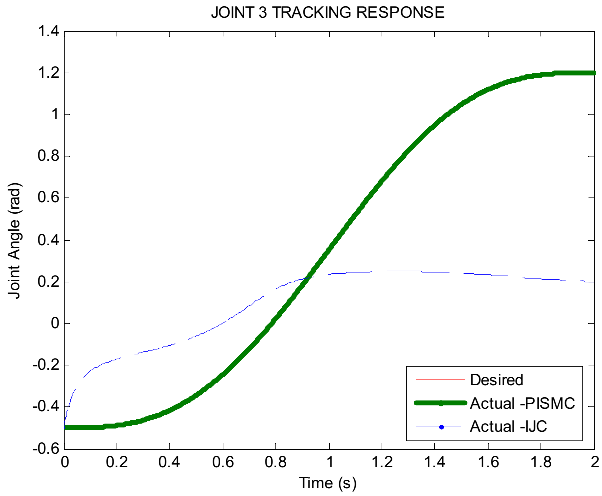
An integrated mathematical model of a hydraulically driven robot manipulator in state space representation is formulated and a robust control technique based on Proportional Integral Sliding Mode Control (PISMC) algorithm to control the system is presented in this paper. The developed integrated model is more feasible for controller synthesis and analysis, in which it does not only represent a closer dynamic nature of the real system, but also is formulated such that the matching condition that is required by sliding mode control is satisfied. Dissimilar from conventional SMC, the adopted technique avoids the need of original plant transformation into reduced form by including an integral term in the sliding surface. Simulation results show that the proposed approach has successfully compensate the manipulator's inertia, coriolis forces, centrifugal forces, gravitational forces and varying payload mass originated from the robot mechanical linkage; as well as the nonlinearities originated from the hydraulic actuator, in which the control system has effectively tracks the pre-specified desired joints position trajectory with zero error at all times.



(a)



(b)



(c)

Fig. 1: Joints 1, 2 and 3 Tracking Response by PISMC and IJC with the Manipulator Handling No Load

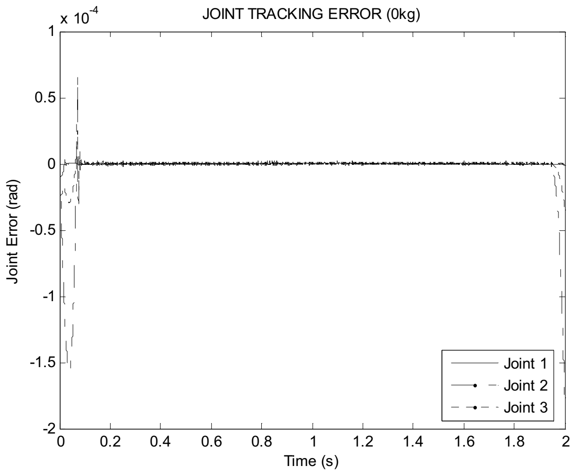


Fig. 2: Joints Tracking Error with Manipulator Handling No Load

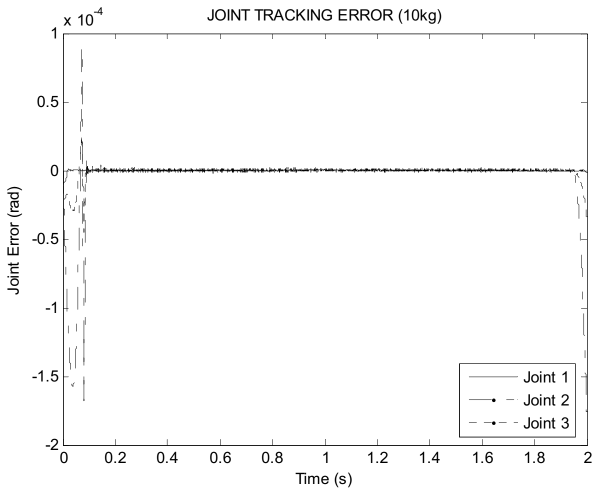


Fig. 3: Joints Tracking Error with Manipulator Handling 10 kg Load

APPENDIX A

The proof for the theorem is as in [5] which are briefly presented in the following:

Differentiating (30) and substituting (37) gives:

$$\begin{aligned}\dot{\sigma}(t) &= C\overline{B}[H(t) - K]Z(t) \\ &- (CB)[I_n + E(t)](C\overline{B})^{-1} \\ &\{\gamma_1\|Z(t)\| + \gamma_2\|X_d(t)\| \\ &+ \gamma_3\|\Omega(t)\|\}SGN(\sigma(t)) \\ &+ C\overline{B}H(t)X_d(t) + C\overline{B}E(t)\Omega(t)\end{aligned} \tag{45}$$

Substituting (45) into (36) gives the rate of change of the Lyapunov function:

$$\begin{aligned}\dot{V}(t) = (\sigma^T(t)/\|\sigma(t)\|)\{&-CB[H(X,\xi,t) - K]Z(t) \\ &- CB[I_n + E(X,t)](CB)^{-1}\gamma_1\|Z(t)\|SGN(\sigma(t)) \\ &+ CBH(X,\xi,t)X_d(t) \\ &- (CB)[I_n + E(X,\xi,t)](CB)^{-1}\gamma_2\|X_d(t)\|SGN(\sigma(t)) \\ &+ CBE(X,\xi,t)\Omega(t) \\ &- (CB)[I_n + E(X,t)](CB)^{-1}\gamma_3\|\Omega(t)\|SGN(\sigma(t))\}\end{aligned} \tag{46}$$

Alternatively, equation (46) can be broken down into:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \tag{47}$$

where;

$$\begin{aligned}\dot{V}_1(t) = \frac{\sigma^T(t)}{\|\sigma(t)\|}\{&CB[H(t) - K]Z(t) \\ &- (CB)[I_n + E(t)](CB)^{-1}\gamma_1\|Z(t)\|SGN(\sigma(t))\}\end{aligned} \tag{48}$$

$$\begin{aligned}\dot{V}_2(t) = \frac{\sigma^T(t)}{\|\sigma(t)\|}\{&CBH(t)X_d(t) \\ &- (CB)[I_n + E(t)](CB)^{-1}\gamma_2\|X_d(t)\|SGN(\sigma(t))\}\end{aligned} \tag{49}$$

$$\begin{aligned}\dot{V}_3(t) = \frac{\sigma^T(t)}{\|\sigma(t)\|}\{&CBE(t)\Omega(t) \\ &- (CB)[I_n + E(t)](CB)^{-1}\gamma_3\|\Omega(t)\|SGN(\sigma(t))\}\end{aligned} \tag{50}$$

Then, each of the Lyapunov terms can be simplified as follows. First term of equation (48):

$$\begin{aligned}&\frac{\sigma^T(t)}{\|\sigma(t)\|}\{CB[H(t) - K]\}Z(t) \\ &\le \frac{\|\sigma^T(t)\|}{\|\sigma(t)\|}\{\|CB\|\|H(t)\| + \|CBK\|\}\|Z(t)\| \\ &= \{\alpha\|CB\| + \|CBK\|\}\|Z(t)\|\end{aligned} \tag{51}$$

Noting that:

$$\frac{\sigma^T(t)}{\|\sigma(t)\|}SGN(\sigma(t)) = \frac{\sigma^T(t)\sigma(t)}{\|\sigma(t)\|^2} = \frac{\sigma^T(t)\sigma(t)}{\left(\sqrt{\sigma^T(t)\sigma(t)}\right)^2} = 1 \tag{52}$$

Then, the second term of (48) can be simplified as:

$$\begin{aligned} -\frac{\sigma^T(t)}{\|\sigma(\mathrm{t})\|}&\{(CB)[I_n+E(t)](CB)^{-1}\gamma_1\|Z(t)\|SGN(\sigma(t))\} \\ &\leq -\|CB\|[\|I_n\|+\|E(t)\|]\|(CB)^{-1}\|\gamma_1\|Z(t)\| \\ &= -[1+\beta]\gamma_1\|Z(t)\| \end{aligned} \quad (53)$$

Using (51) and (53), (48) becomes:

$$\dot{V}_1(t) \leq -\{(1+\beta)\gamma_1 - [\alpha\|CB\|+\|CBK\|]\}\|Z(t)\| \quad (54)$$

Similarly, equation (49) and (50) can be simplified in a same manner and the results are summarized as follows:

$$\dot{V}_2(t) \leq -\{(1+\beta)\gamma_2 - \alpha\|CB\|\}\|X_d(t)\| \quad (55)$$

$$\dot{V}_3(t) \leq -\{(1+\beta)\gamma_3 - \beta\|CB\|\}\|\Omega(t)\| \quad (56)$$

VI. REFERENCES

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