



# INFLUENCE OF SLIP VELOCITY ON CONVECTIVE BOUNDARY LAYER FLOW OF JEFFREY FLUID UNDER CONVECTIVE BOUNDARY CONDITIONS

H. A. M. Al-Sharifi<sup>1</sup>, A. R. M. Kasim<sup>1</sup>, M. Z. Salleh<sup>1</sup>, N. Md. Sarif<sup>1</sup>, N. F. Mohammad<sup>2</sup>, S. Shafie<sup>3</sup> and A. Ali<sup>3</sup>

<sup>1</sup>Faculty of Industrial Sciences and Technology, Universiti Malaysia Pahang, Malaysia

<sup>2</sup>Kulliyah of Science, International Islamic University Malaysia

<sup>3</sup>Department of Mathematical Sciences, Universiti Teknologi Malaysia, Malaysia

E-Mail: [rahmanmohd@ump.edu.my](mailto:rahmanmohd@ump.edu.my)

## ABSTRACT

The problem on influence of slip velocity on convective boundary layer flow of Jeffrey fluid with convective boundary conditions together with the effects of magnetohydrodynamic is investigated. The governing equations correspond to the discussed problem are first transformed into ordinary differential equations using similarity transformations before the computation process is done by Keller box method. It is found that, the magnetic parameter enhanced the fluid temperature and lowers the velocity of the fluid flow while the growth of the values of Prandtl and Deborah number weaken the distribution of the fluid temperature.

**Keywords:** slip velocity, boundary layer, jeffrey fluid, convective boundary condition.

## INTRODUCTION

Extensive studied on the topic of heat transfer in viscous (Newtonian) fluids has been done by many researchers in the past few decades. However, the investigation regarding the problem on non-Newtonian fluids has gained momentum as well. Non-Newtonian fluids are normally defined as fluids that have retention of a fading “memory” of their flow history. Non-Newtonian fluids typically represent liquids which are formed either partly or wholly of macromolecules (polymers), or two phase materials. For a variety of reasons, non-Newtonian fluids are classified on the basis of their shear properties. There are many types of non-Newtonian fluids exist in this world [3, 4, 6, 1]. One of them is Jeffrey fluid which is considered as the simplest model of viscoelastic fluid yet interesting to research on [10, 5]. During the last few years, the problem regarding Jeffrey fluids has gained considerable importance because of its applications in various branches of science, engineering, and technology, particularly in material processing, chemical and nuclear industries, geophysics, and bio-engineering. The study of the flow of this fluid type is also significant in oil reservoir engineering.

Following Merkin [7] there are four common heating processes specifying the wall-to-ambient temperature distribution known as constant wall temperature (CWT), constant surface heat flux (CHF), Newtonian heating (NH) and convective boundary condition. NH is the transfer of heat from bounding surface with a finite heat capacity which is proportional to the local surface temperature and normally termed as conjugate convective flow while CBC is the supplies of heat through a bounding surface of finite thickness and finite heat capacity. All mentioned heating processes are very important and significant in the fluid flow problems and has been taking into account by many researchers in their investigation [9, 2, 6].

Therefore, this paper presenting the outcomes of the numerical solution of Jeffrey fluid model moving toward the stretching sheet under convective boundary condition with the influences of slip velocity and additional effects of magnetohydrodynamic (MHD). The reduced ordinary differential equation of the proposed model has been solved using Keller box method. It is worth to mention here, the numerical code for solving the respected problem has been try for the problem projected by Turkyilmazoglu and Pop [11] and the results shown a strong agreement. We are believed, the extended outcomes produced from this present model are accurate and precise.

## PROBLEM FORMULATION

The problem on the steady of two-dimensional convective boundary layer flow of Jeffrey fluid under convective boundary conditions with slip velocity has been considered. Besides that, the effects of magnetohydrodynamic has been includes into the formulation and the behaviors of the fluid flow characteristics has been investigated.

Under the boundary layer approximation, the governing equations of the discussed problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} - \frac{\sigma B_0^2}{\rho} u + \frac{\nu}{1 + \lambda_1} \left[ \frac{\partial^2 u}{\partial y^2} + \lambda_2 \left( u \frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} \right) \right] \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

where  $\nu$  is a kinematic viscosity,  $u$  and  $v$  are the velocity components,  $\sigma$  is the electrical conductivity



of the fluid,  $\lambda_1$  and  $\lambda_2$  are the ratio of relaxation to retardation times and the retardation time respectively,  $B_0$  is constant magnetic field,  $\rho$  is the density,  $T$  is temperature and  $\alpha$  is thermal conductivity. The velocity outside the boundary layer is notated as  $u_e$ .

The corresponding boundary conditions

$$u = u_w(x) = cx + g \frac{\partial u}{\partial y}, v = 0, -k \frac{\partial T}{\partial y} = h_f (T_w - T) \text{ at } y = 0, \quad (4)$$

$$u \rightarrow u_e(x) = ax, u_y \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty,$$

has been applied together with governing equations (1) - (3) where  $a$  is a positive external velocity,  $c$  is positive stretching or shrinking velocity,  $T_w$  is a constant wall temperature, and  $T_\infty$  is free stream temperature.

The similarity transformations (5) are introduced

$$\eta = y \sqrt{\frac{c(1+\lambda_1)}{\nu}}, u = cxf'(\eta), v = -\sqrt{\frac{c\nu}{1+\lambda_1}} f(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad (5)$$

and it shows that the equation (1) is identically satisfied and the equations (2) and (3) are reduced to the following equations,

$$f''' + ff'' - (f')^2 + \beta((f')^2 - ff^{(4)}) - M^2 f' + \delta^2 = 0 \quad (6)$$

$$\theta'' + Pr f \theta' = 0 \quad (7)$$

where  $\beta = \lambda_2 c$  is the Deborah number,  $M^2 = \frac{\sigma B_0^2}{c\rho}$  is

magnetic parameter,  $\delta = \frac{a}{c}$  is stretching strength

parameter, and  $Pr = \frac{\nu}{(1+\lambda_1)}$  is the prandtl number.

The boundary conditions (4) are changed to

$$f'(0) = 1 + kf''(0), f(0) = 0, \theta'(0) = -\gamma(1 - \theta(0)) \text{ at } y = 0, \quad (8)$$

$$f'(\infty) \rightarrow 0, f''(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0 \text{ as } y \rightarrow \infty.$$

where  $k = g \sqrt{\frac{c(1+\lambda_1)}{\nu}}$  is velocity slip parameter,

$\gamma = \frac{h_f}{k \sqrt{\frac{c(1+\lambda_1)}{\nu}}}$  is thermal slip parameter.

**RESULTS AND DISCUSSION**

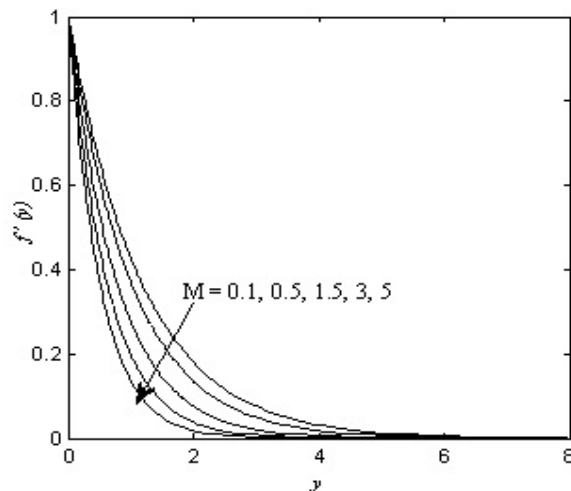
The ordinary differential Equations. (6) and (7) with correspond to the boundary condition (8) are solved using numerical scheme called Keller box method. The discussion of the results begins with the comparison results with the previous published outputs to ensure the correctness of the numerical codes applied to compute numerical results for the problem discussed in this articles. The results of the heat transfer coefficient as shown in Table-1 are consistent with the reference results.

Effects of magnetic parameter, M on velocity and temperature profiles have been presented in Figure-1 and Figure-2 respectively and its shows that, an increase in M lead to decrease the velocity and enhanced the temperature distribution of fluid. We can say that, the magnetic effects give significant impact to the transport of fluid. Besides, the presence of the magnetic field will boost the Lorentz force and automatically give more resistance to transport fluids.

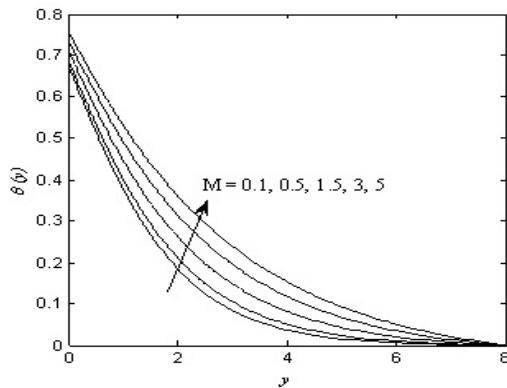
**Table-1.** Comparison on the rate of heat transfer coefficient.

Pr	0.5	1.0	2.0
$\delta = 0.5$	0.490388*	0.715436*	1.039175*
	0.490387**	0.715435**	1.039158**
$\delta = 1$	0.564194*	0.797897*	1.128415*
	0.564189**	0.797884**	1.128379**
$\delta = 2$	0.708790*	0.974867*	1.339203*
	0.704507**	0.969264**	1.332401**
$\delta = 5$	1.025110*	1.379050*	1.847427*
	1.025068**	1.378941**	1.847152**

\*Present results      \*\*Turkyilmazoglu and Pop [11]

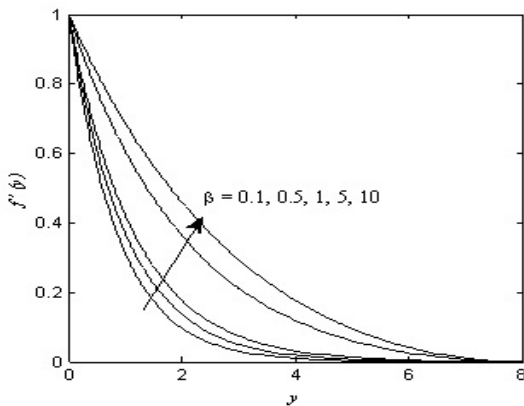


**Figure-1.** Velocity profile at different values of magnetic parameter, M at K = 0.5,  $\beta = 0.5$ , Pr=1, and  $\delta = 1$ .

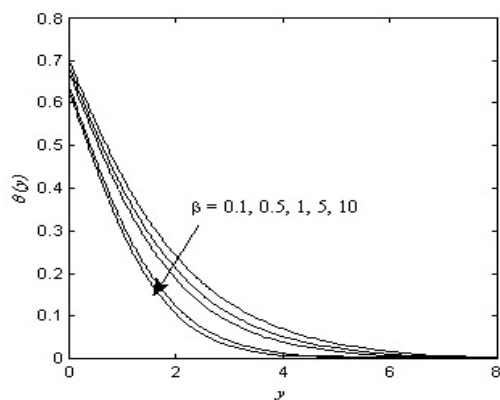


**Figure-2.** Temperature profile at different values of magnetic parameter,  $M$  at  $K = 0.5$ ,  $\beta = 0.5$ ,  $Pr = 1$ , and  $\delta = 1$ .

Figure-3 and Figure-4 shows the significances of the various values of Deborah number on the velocity and temperature profiles respectively. The larger of  $\beta$ , enhance the fluid velocity and retard the fluid temperature.

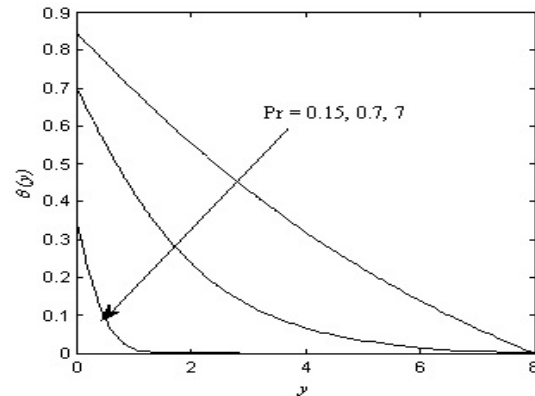


**Figure-3.** Velocity profile at different values of  $\beta$ , at  $K = 0.5$ ,  $M = 0.5$ ,  $Pr = 1$ , and  $\delta = 1$ .



**Figure-4.** Temperature profile at different values of magnetic parameter,  $M$  at  $K = 0.5$ ,  $\beta = 0.5$ ,  $Pr = 1$ , and  $\delta = 1$ .

Figure-5, display the results on the influences of Prandtl number,  $Pr$  towards the temperature distribution of the fluid. As the  $Pr$  increase, the temperature profile is decline. Physically, the increment in the value of  $Pr$ , growth the fluid viscosity and controlled the temperature of fluid.



**Figure-5.** Temperature profile at different values of Prandtl number at values  $K = 0.5$ ,  $\beta = 0.5$ ,  $M = 1$ , and  $\delta = 1$ .

## CONCLUDING REMARK

In this article, the problem on influence of slip velocity on Jeffery fluid towards a stretching sheet with the effect of magnetohydrodynamic (MHD) embedded with convective boundary conditions is discussed. The main observations are summarized in the following point.

- The distribution of velocity is decreased while the temperature is increased in the increment of the value of magnetic parameter,  $M$  but the opposite consequences is observed in the increment of the values of Deborah number
- The larger value of Prandtl number will retard the temperature distribution of the fluid.

## ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial supports received from Universiti Malaysia Pahang, UMP under Vot RDU150101 and Universiti Teknologi Malaysia, UTM.

## REFERENCES

- [1] Cortell R. 2007. Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non uniform heat source, viscous dissipation and thermal radiation. *Int J Heat Mass Transf.* 50, 3152–3162.
- [2] Hayat T., Shafiq A., Mustafa M. and Alsaedi A. 2015. Boundary-Layer Flow of Walters' B Fluid with Newtonian Heating. *Zeitschrift für Naturforschung A.* 70(5): 333–341.



- [3] Kasim A. R. M., Admon M. A. and Sharidan S. 2011. Free convection boundary layer flow of a viscoelastic fluid in the presence of heat generation. *World Academy of Sci. Engng. and Tech.* 75: 492–499.
- [4] Kasim A. R. M., Mohammad N. F., Aurangzaib A. and Sharidan S. 2012. Natural convection boundary layer flow of a viscoelastic fluid on solid sphere with Newtonian heating. *World Academy of Science, Engineering and Technology.* 64: 628–633.
- [5] Kasim A. R. M., Jiann L. Y., Shafie S. and Ali A. 2014. The effects of heat generation or absorption on MHD stagnation point of Jeffrey fluid. *AIP Conf. Proc.* 1605: 404.
- [6] Kasim A. R. M., Mohammad N. F., Shafie S. and Pop I. 2013. Constant heat flux solution for mixed convection boundary layer viscoelastic fluid. *Heat Mass Transf.* 49: 163–171.
- [7] Merkin J. H. 1994. Natural convection boundary layer flow on a vertical surface with Newtonian heating. *Int J Heat Fluid Flow.* 15: 392–39.
- [8] Kasim A. R. M., Mohammad N. F., Aurangzaib A. and Sharidan S. 2013. Mixed convection flow of viscoelastic fluid over a sphere with constant heat flux. *AIP Conf. Proc.* 1522: 453.
- [9] RamReddy Ch., Pradeepa T. and Srinivasacharya D. 2015. Similarity Solution for Free Convection Flow of a Micropolar Fluid under Convective Boundary Condition via Lie Scaling Group Transformations. *Advances in High Energy Physics.* 2015(2015).
- [10] Shehzad S. A., Hayat T., Alsaedi A. and Ahmad B. 2015. Effects of thermophoresis and thermal radiation in mixed convection three-dimensional flow of Jeffrey fluid. *Applied Mathematics and Mechanics.* 36(5): 655-668.
- [11] Turkyilmazoglu M. and Pop I. 2013. Exact analytical solutions for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid. *International Journal of Heat and Mass Transfer.* 57(1): 82–88.