Proportional Integral Sliding Mode Control of Hydraulic Robot Manipulators with Chattering Elimination

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Abstract - This paper is concerned with the application of a robust control approach based on Sliding Mode Control (SMC) strategy with proportional integral switching surface in controlling the position trajectory of a hydraulic manipulator. This paper also addresses the suppression technique for the undesirable chattering phenomenon which usually occurs in SMC by replacing the discontinuous controller sign function with a proper continuous function. Chattering is unwanted because it leads to an excessive usage and damages the actuators and therefore the control law may become impractical. In this study, an integrated model of hydraulically actuated robot that considers both the manipulator linkage and actuator dynamics is used to provide a more suitable model for controller synthesis and analysis. The control technique is stable in the large based on Lyapunov theory. Its performance is evaluated and compared with the existing Independent Joint Linear Control (IJC) technique through computer simulation. The results prove that the controller has successfully force the robot manipulator to track the desired position trajectory for all times and has better performance than IJC.

I INTRODUCTION

Hydraulic manipulators find wide applications in heavy duty jobs since they are capable of providing very large torque and fast motion. However, in contrast to electric motor, the modeling and control of hydraulics manipulators are more complex since the torque developed by hydraulic actuators is proportional to the pressure difference or the flow rate towards the cylinder chamber. In other words, the control voltage or current signal controls the speed of the hydraulics' spool position rather than its force or torque directly. Hydraulic actuator also introduces additional nonlinearities to the control problem.

Most of the past researches in the synthesis of control law for manipulators deal with electrically actuated manipulators. Comparatively less work has been done for hydraulically actuated robot [1]. Previous researches have studied the control aspect of hydraulic robots with no manipulator dynamics considered in the model such as in [2] and [3]. Adaptive Control Technique was proposed in [2] to control hydraulic cylinders with the application to robot manipulators, but the control law is synthesized from the actuator model alone. However, the dynamics of manipulator should not be ignored, since it contains the arm dynamic forces such as inertia forces and gravity effects that the controller needs to compensate [1]. Therefore, in this study, the control law is devised based on an integrated model of hydraulically actuated robot that considers both the manipulator linkage and the actuator dynamics. This model represents a closer dynamic behaviour of the real system and thus provides a more suitable dynamic equation for the purpose of controller synthesis and analysis.

The majority of current industrial approaches to the robot control arm design treat each joint of the manipulator as a simple linear servomechanism with proportional plus integral plus derivative (PID) or Computed Torque (CTC) controllers [4]. The problem with PID controllers is that they are not adequate for the cases when the robot moves at high speed and in situations requiring a precise trajectory tracking. Therefore it is less suitable to control hydraulic manipulators. On the other hand, the problem with CTC is that it is based on exact robot arm dynamic model, where the explicit use of an incorrect robot model will deteriorate the control performance. Hence, a robust controller is proposed to drive such a system.

Proportional Integral Sliding Mode Control (PISMC) is an extended version of conventional SMC technique. It has been successfully designed for electrically driven robot manipulator as presented in [5]. It provides the advantages of zero steady error due to the integral term and robustness offered by the Sliding Mode Control (SMC). It is suitable for complex systems and is insensitive to parameters variations and uncertainties. Different from conventional SMC, the proposed technique overcomes the problem of reduced order dynamics by avoiding the need of original plant transformation into canonical form or reduced form. In [5], a three DOF revolute electric robot is used in the simulations. It is verified that the proposed control law is effective in providing the necessary tracking control. Therefore, this paper extends the work of [5] to provide trajectory tracking control of a hydraulically actuated robot manipulator. The approach is stable based on Lyapunov theory. The discontinuous sign function which normally exists in SMC is replaced by a continuous function to eliminate the chattering effects, therefore prevents the actuators from damage and wear. A 3 DOF hydraulic robot is used in the simulation study.

This paper is organized as follows: The system dynamics, including mechanical linkage and hydraulic dynamics are presented in Section II. In Section III, the adopted control approach is described. Chattering effect elimination is discussed in Section IV and simulation results are presented in Section V. Finally, conclusions are drawn in Section VI.

II DYNAMIC MODELING OF THE ROBOT MANIPULATOR

A Manipulator Mechanical Linkage Dynamics

The dynamic equation of mechanical linkage of N DOF robot manipulator with rigid links is governed by [6]:

$$M(\theta(t),\xi)\ddot{\theta}(t) + \hat{D}(\theta(t),\dot{\theta}(t),\xi)\dot{\theta}(t) +$$

$$\hat{G}(\theta(t),\xi)\theta(t) = T(t)$$
(1)

where;

$M(\theta(t),\xi)$: 1	$N \times N$ inertia matrix				
$\hat{D}(\theta(t), \dot{\theta}(t), \xi)\dot{\theta}(t)$: 1	$V \times 1$ vector of coriolis and				
с	entrifugal forces				
$\hat{G}(\theta(t),\xi)\theta(t)$: 1	V×1 vector of gravitational				
f	orces				
T(t) : 1	V×1 vector of driving torques				
a	pplied by the actuators				
$\theta(t), \dot{\theta}(t), \ddot{\theta}(t) = : \Lambda$	1 ×1 vector of generalized joint				
α ξ :υ	isplacements, velocities and ccelerations respectively ncertain parameters of the nechanism (payload mass)				
1	(payroad mass)				

B. Electrohydraulic Actuator Dynamics

With the assumptions that the hydraulic actuator's piston is centered, the valve is an ideal critical center valve with matched and symmetrical orifices; and the return line pressure is zero, the augmented dynamic equation of the NN actuators can be written in the following compact form [4]:

$$\dot{X}(t) = A_n X(t) + BU(t) + FT(t) + W\dot{T}(t)$$
(2)

where;

$$\begin{split} X(t) &= \left[X_1^T(t), X_2^T(t), \dots, X_N^T(t) \right]^T, \\ U(t) &= \left[U_1(t), U_2(t), \dots, U_N(t) \right]^T, \\ T(t) &= \left[T_1(t), T_2(t), \dots, T_N(t) \right]^T, \\ \dot{T}(t) &= \left[\dot{T}_1(t), \dot{T}_2(t), \dots, \dot{T}_N(t) \right]^T, \end{split}$$

$$\begin{split} A_n &= \text{diag}[A_{n1}, A_{n2}, \dots, A_{nN}] \\ B &= \text{diag}[B_1, B_2, \dots, B_N], \\ F &= \text{diag}[F_1, F_2, \dots, F_N], \\ W &= \text{diag}[W_1, W_2, \dots, W_N], \end{split}$$

A_{mi} :

and X(t) is a $3N \times 1$ vector, U(t) is an $N \times 1$ input vector, T(t) is the $N \times 1$ mechanical link torque, and $\dot{T}(t)$ is its time derivative. The state variables, $X_i(t)$ are the actuator displacement velocity and acceleration:

$$X_{i}(t) = \begin{bmatrix} \theta_{mi}(t) & \dot{\theta}_{mi}(t) & \ddot{\theta}_{mi}(t) \end{bmatrix}^{T}, \quad i = 1, 2, \dots, N$$
(3)

and

$$=A_i + N_i , \qquad (4)$$

$$\begin{split} A_{i} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{4\beta_{e}K_{t}G}{V_{t}J_{m}} & -\left[\frac{4\beta_{e}K_{t}B_{m}}{V_{t}J_{m}} + \frac{G}{J_{m}} + \frac{4\beta_{e}D_{m}^{2}}{V_{t}J_{m}}\right] & -\left[\frac{4\beta_{e}K_{t}}{V_{t}} + \frac{B_{m}}{J_{m}}\right] \end{bmatrix}, \\ N_{i} &= \begin{bmatrix} \frac{4\beta_{e}K_{i}G_{n}Gn_{\varepsilon}^{2}}{V_{t}J_{m}}\theta_{i}^{2} + \frac{3G_{n}Gn_{\varepsilon}^{2}}{J_{m}}\theta_{i}\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ B_{i} &= \begin{bmatrix} 0 \\ \frac{4\beta_{e}D_{m}K_{q}K_{v}}{V_{t}J_{m}}\theta_{\varepsilon} \end{bmatrix}, \quad F_{i} &= \begin{bmatrix} 0 \\ 0 \\ -\frac{4\beta_{e}K_{t}}{V_{t}J_{m}}\eta_{\varepsilon}^{2} \end{bmatrix}, \quad W_{i} &= \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J_{m}}\eta_{\varepsilon}^{2} \end{bmatrix}, \\ K_{q} &= C_{d}w\sqrt{(P_{s}-P_{L}\operatorname{sgn}(X_{v}))/\rho}, \\ X_{v} &= K_{v}U(t). \end{split}$$
(5)

 X_v is the displacement of the spool in the servo valve, P_L is the load pressure, K_q is the flow gain which varies at different operating points, C_d is the discharge coefficient, W is the area gradient, ρ is the fluid mass density, P_s is the supply pressure, D_m is the volumetric displacement, θ_{mi} is the angular displacement of the *i*th motor shaft, $\dot{\theta}_{mi}$ is the angular velocity of the *i*th motor shaft, $\ddot{\theta}_{mi}$ is the total leakage coefficient of the hydraulic system, V_t is the total compressed volume, is the effective bulk modulus of the oil, J_m is the motor inertia, B_m is the viscous damping coefficient, T is the load torque due to the joint of the *i*th motor, \dot{T} is the derivative of the load torque due to the joint of the *i*th motor, G is the torsional spring constant, $G_n \theta_{mi}^3$ is the

nonlinear stiffness of the spring, and n_g is the inverse of the gear ratio.

C. Integrated Electrohydraulic Manipulator Dynamics

The integrated model of electrohydraulic manipulator can be described as:

$$\dot{X}(t) = A_n(X,\xi,t)X(t) + B(X,\xi,t)U(t),$$
(6)

where;

$$A_{n}(X,\xi,t) = \begin{bmatrix} I_{3N} - WM(X_{B},\xi,t)Z_{B} \end{bmatrix}^{-1} \begin{cases} A_{n} + \begin{bmatrix} FM(X,\xi,t) \\ +W\widetilde{C}(X,\xi,t) \end{bmatrix}_{B} \\ + \begin{bmatrix} FB \widehat{D}(X,\xi,t) \\ +W\widetilde{D}(X,\xi,t) \end{bmatrix}_{B} \\ + F\widehat{O}(X,\xi,t)Z_{B} \end{bmatrix} \begin{pmatrix} (7) \\ +W\widetilde{D}(X,\xi,t)Z_{B} \\ +F\widehat{O}(X,\xi,t)Z_{B} \end{bmatrix} \end{cases}$$

$$B(X,\xi,t) = \begin{bmatrix} I_{3N} - WM(X,\xi,t)Z_{B} \end{bmatrix}^{-1}B \qquad (8)$$

(A STELATER)

 Z_{B}, Z_{B1} and Z_{B2} are the transformation matrices and;

 \widetilde{C} and \widetilde{D} are the matrices associated with the derivative of the manipulator torques in (1), which can be described as follows:

$$\begin{split} \dot{T}(t) &= M(\theta(t),\xi)\ddot{\theta}(t) + \widetilde{C}(\theta(t),\dot{\theta}(t),\xi)\ddot{\theta}(t) \\ &+ \widetilde{D}(\theta(t),\dot{\theta}(t),\xi)\dot{\theta}(t) \end{split} \tag{9}$$

The system matrix, $A_n(X,\xi,t)$ and the input matrix, $B(X,\xi,t)$ are of dimensions $3N \times 3N$ and $3N \times N$ respectively. From (6), (7) and (8), it is clear that the resulting dynamics description of the robotic system is analytically complex. Each nonzero element of the system and input matrices is a function of the instantaneous position, velocity and payload mass of the manipulator. The equations are time varying, highly nonlinear, contains uncertainty due to the mass of the pay load that the manipulator has to carry; and coupled due to the mechanical linkage as well as the hydraulic dynamics. Therefore, a more robust controller that is capable of catering these plant characteristics as presented in next section is required.

III PROPORTIONAL INTEGRAL SLIDING MODE CONTROLLER

The control strategy is to apply the robust controller based on Proportional Integral Sliding Mode control (PISMC) to force the robot manipulator to track a predefined desired trajectory as closely as possible for all times in spite of the nonlinearity, parameter variations, uncertainties and coupling effect present in the system.

Define the state vector of the system as

$$X(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \quad . \tag{10}$$

Let a continuous function $X_d(t) \in \mathbb{R}^n$ be the desired state trajectory, where $X_d(t)$ is defined as:

$$X_{d}(t) = \left[x_{d1}(t), x_{d2}(t), \dots, x_{dn}(t) \right]^{t} \quad . \tag{11}$$

In this study, the following assumptions are made:

- a. The state vector X(t) can be fully observed;
- b. There exist continuous functions H(t) and E(t) such that for all $X(t) \in \mathbb{R}^n$ and all t:

$$\Delta A(t) = \overline{B}H(t) ; \qquad \left\| H(t) \right\| \le \alpha \tag{12}$$

$$\Delta B(t) = \overline{B}E(t) \ ; \qquad \left\| E(t) \right\| \le \beta \tag{13}$$

where ΔA and ΔB are the uncertainties matrices of the system and input matrices of (6).

c. There exist a Lebesgue function $\Omega(t) \in R$, which is integrals on bounded interval such that

$$\dot{X}_d(t) = \overline{A}X_d(t) + \overline{B}\Omega(t) \tag{14}$$

d. The pair $(\overline{A}, \overline{B})$ is controllable.

Equations (12) and (13) in Assumption (b) ensures that the uncertainties $\Delta A(X,\xi,t)$ and $\Delta B(X,\xi,t)$ lie in the range space of the nominal input matrix \overline{B} so that the control signal, U(t) which enters the system through the input matrix, $B(X,\xi,t)$ can compensate the parameter variations and uncertainties present in the system.

The Proportional-Integral (PI) sliding surface is defined as [5]:

$$\sigma(t) = CZ(t) - \int_{0}^{t} [C\overline{A} + C\overline{B}K]Z(\tau)d\tau, \qquad (15)$$

where Z(t) is the tracking error[5]:

$$Z(t) = X(t) - X_{d}(t) , \qquad (16)$$

 \overline{A} and \overline{B} are the nominal matrices of the system and input matrices of (6) and the structure of the matrix *C* is as follows [5]:

$$C = diag[c_1 \quad c_2 \quad \cdots \quad c_{n_i}], \qquad (17)$$

where n_i is the *n*th state variable associated to the *i*th input of the system. The matrix K is designed to satisfy [5]:

$$\lambda_{\max}(\overline{A} + \overline{B}K) < 0 \quad , \tag{18}$$

Equation (14) guarantees that the system is stable by placing the desired poles in the left half plane. The elements of matrix K can be determined by pole placement technique with pre-specified poles locations.

If the following hitting condition is held, the manifold of equation (15) is asymptotically stable in the large [8]:

$$\left(\sigma^{\mathrm{T}}(t) / \left\| \sigma(t) \right\| \right) \sigma(t) < 0 \tag{19}$$

Theorem: The hitting condition (19) is satisfied if the control u(t) [5]:

$$u(t) = -(C\overline{B})^{-1}[\gamma_1 || Z(t) || + \gamma_2 || X_d(t) || + \gamma_3 || \Omega(t) ||] SGN(\sigma(t)) + \Omega(t),$$
(20)

where

$$\gamma_1 > (\alpha |CB| + |CBK|)/(1+\beta), \qquad (21)$$

$$\gamma_2 > (\alpha ||C\overline{B}||)/(1+\beta), \qquad (22)$$

$$\gamma_3 > (\beta \| C\overline{B} \|)/(1+\beta), \tag{23}$$

The proof of this theorem is given in [5].

If both equations (12) and (13) are satisfied, the system's error dynamics during sliding mode can be described as [5]:

$$\dot{Z}(t) = \left[\overline{A} + \overline{B}K\right]Z(t).$$
⁽²⁴⁾

Equation (24) shows that the system is no longer sensitive to the plant variations and uncertainties during sliding mode. Therefore the system error during sliding only depends on the plant nominal values and can be adjusted through a proper adjustment of the value of K, which can be obtained through a proper selection of the desired closed loop poles locations. The proof of this is given in the Appendix.

IV CHATTERING ELIMINATION

The discontinuous sign function $SGN(\sigma(t))$ in equation (20) leads to an undesirable phenomenon known as chattering. Chattering is unwanted because it leads to a high number of oscillations of the system trajectory around the sliding surface, and causes an excessive use of the actuators. This will eventually damage and wear the motors and therefore the control law may become impractical [5]. To overcome this problem, each element of the discontinuous sign function $SGN(\sigma(t))$ for the *i*th link is replaced by a proper continuous function as [3]:

$$S_{\delta i}(t) = \frac{\sigma_i}{\left|\sigma_i\right| + \delta_{0i} + \delta_{1i} \left|X_j - X_{dj}\right|},$$
(25)

where δ_{0i} and δ_{1i} are positive constants. With an appropriate selection of these two values, the inevitable chattering phenomenon may be eliminated.

V RESULTS AND DISCUSSIONS

A three DOF revolute hydraulic robot manipulator is used as a test bed in evaluating the performance of the proposed approach. It is assumed that identical hydraulic motors are used for all the three joints and; the end effector and the variable load are lumped together as a single mass.

The limits of the manipulator are known and tabulated as in Table 1. The robot manipulator is simulated to carry a variation of mass load in between 0 kg to 10 kg.

TABLE 1	
Manipulator Limit	

Physical	Link Number, <i>i</i>								
Quantity	1		2		3				
	Min	Max	Mi	Max	Mi	Max			
			n		n				
Position, θ_i	-160	160	-225	45	-46	225			
(degree)									
Velocity, $\dot{\theta}_i$	0	120	0	110	0	110			
(degree/s)									

In the simulation, each of the manipulators joint is required to track the desired position trajectory described as in Figure 1.

A linear control approach based on Independent Linear Joint Controller (IJC) technique is used as a comparison purpose. IJC is normally used in most industrial robot and is designed with the dynamics of the mechanical linkage completely ignored. Each joint of the robot arm is treated as an independent servomechanism problem. The linear state feedback controller employed in each of the *i*th joint can be described as:

$$U_i(t) = K_i Z_i(t) + \Omega_i(t), \qquad (26)$$

where, K_i is the linear state feedback gain, $\Omega_i(t)$ is the control component to eliminate the steady state error and

$$Z_{i}(t) = X_{i}(t) - X_{di}(t)$$
(27)

Fig. 2 illustrates the individual joint tracking error of the robot utilizing PISMC and IJC with the robot operating at minimum payload mass or at its lower bound (0 kg). From the figure, it can be seen that PISMC has successfully force the manipulator to track the desired trajectory with negligible error at all times. In comparison, IJC fails to control the manipulator, in which it exhibits larger tracking error. This is because the linear controller is unable to compensate the manipulator's nonlinearities, parameter variations and coupled effects, where as the PISMC is capable of catering these problems.

In order to evaluate the robustness of the control system, the mass of the payload that the robot has to carry is varied to 10 kg (maximum load). Fig. 3 shows the resulting joints tracking errors utilizing PISMC while operating at maximum load (10 kg). It verifies the robustness of PISMC strategy against load variation (uncertainty), in which the controller has efficiently control the robot arm to follow the specified trajectory although the mass of the payload is varied. Therefore, under PISMC control, the system is also insensitive against uncertainty.

Fig. 4 (a) shows the original control input function, U and Fig. 4 (b) shows the control input function after chattering elimination has taken place. Fig. 5 shows that by using the modified proper continuous function with an appropriate choice of δ_{0i} and δ_{1i} , the chattering phenomena as illustrated in Fig. 3 can be suppressed. Moreover, the continuous control inputs still maintain an accurate tracking result for all the joints as demonstrated in Fig. 5. The simulation is conducted with the robot carrying maximum payload mass.

VI CONCLUSION

A robust control technique based on Proportional Integral Sliding Mode Control (PISMC) for hydraulically driven robot manipulator is presented in this paper. The controller is devised based on an integrated model of the plant that does not only represent a closer dynamic nature of the real system, but also is formulated such that the matching condition that is required by sliding mode control is satisfied. In comparison to conventional SMC, the of the adopted technique avoids the need of the original plant transformation into reduced form by including an integral term in the sliding surface. Simulation results show that the proposed approach successfully compensate the manipulator's inertia, coriolis forces, centrifugal forces, gravitational forces, varying payload mass and nonlinearities present in the robotic arm, and renders the robot arm to effectively tracks the pre-specified desired joints position trajectory with negligible error at all times. PISMC is by far has better performance than the normally used IJC, The proposed control strategy is also more practical since the chattering phenomenon associated with sliding mode control may successfully be reduced by using a proper continuous function.

VII REFRENCES

- Sirouspour M. R. and Saleudesn S. E., "Nonlinear Control o Hydraulie Robots", in *IEEE Transactions on Robotics and Automation*, Vol. 17, No. 2, pp.173-182, April 2001.
- [2] Zhu W. H., Piedbouf J.C., "Adaptive Output Force Tracking Control of Hydraulic Cylinders with Applications to Robot Manipulators" in ASME Journal of Dynamic Systems, Measurement and Control, Vol. 127, pp. 206-217, June 2005.
- [3] Chern T. L. and Wu Y. C., "Design of Integral Variable Structure Controller and Application to Electrohydraulic Velocity Servosystems" in *IEE Proceedings*, Vol. 138, No.5, pp. 439-444, 1991.
- [4] Osman J. H. S., "Decentralized And Hierarchical Control Of Robot Manipulators", PhD Thesis, City University London, 1991.
- [5] Ahmad M.N., "Modeling And Control Of Direct Drive Robot Manipulator," Faculty of Electrical Engineering, PhD Thesis, Universiti Teknologi Malaysia, Malaysia, 2003.
- [6] E. H. Merrit, H. E, Hydraulic Control Systems, John Wiley and Sons, New York, 1967.
- [7] Fung,R.F., Wang Y.C., Yang R.T. and Huang H.H., "A variable Structure Control with Proportional And Integral Compensation For Electrohydraulic Position Servo Control System", *Mechatronics*, 1997. Vol.7, No.1, pp.67-81, 1997.

[8] Yan, J.J, Tsai, J.S.H., and Kung, F.C. (1997). Robust Decentralized Stabilization of Large Scale Delay System via SMC. *Journal of Dynamic Systems, Measurement and Control*, 119:307-312.

APPENDIX

The proof of the system dynamics insensitivity to plant parameter variations and nonlinearities during sliding mode is briefly presented in the following [5].

In view of equations (12), (13) and (16)the an error dynamic system can be written as:

$$\dot{Z}(t) = [\overline{A} + \overline{B}H(t)]Z(t) + \overline{B}H(t)X_d(t)$$

$$-\overline{B}\Omega(t) + [\overline{B} + \overline{B}E(t)]u(t)$$
(28)

Differentiating equation (15) gives:

$$\dot{\sigma}(t) = C\dot{Z}(t) - [C\overline{A} + C\overline{B}K]Z(t)$$
(29)

Substituting equation (28) into equation (29) gives: $\dot{\sigma}(t) = C\overline{B}H(t)Z(t) + C\overline{B}H(t)X_{*}(t)$

$$-C\overline{B}\Omega(t) + [C\overline{B} + C\overline{B}E(t)]u(t) - C\overline{B}KZ(t)$$
30)

Equating equation (30) to zero gives the equivalent control, $U_{eq}(t)$:

$$U_{eq}(t) = [C\overline{B} + C\overline{B}E(t)]^{-1} \{C\overline{B}KZ(t) + C\overline{B}\Omega(t) - C\overline{B}H(t)Z(t) - C\overline{B}H(t)X_{d}(t)\}$$
(31)

Noting that $[C\overline{B} + C\overline{B}E(t)]^{-1} = [I_3 + E(t)]^{-1}(C\overline{B})^{-1}$, then the equivalent control of equation (31) can be written as

$$U_{eq}(t) = -[I_3 + E(t)]^{-1} \{ (H(t) - K)Z(t) - \Omega(t) + H(t)X_{-1}(t) \}$$
(32)

The system dynamics during sliding mode can be found by substituting the equivalent control (31) into the system error dynamics (27). After simplification, it can be shown that:

$$\dot{Z}(t) = \left[\overline{A} + \overline{B}K\right]Z(t) \tag{33}$$

Therefore, if the matching condition is satisfied (equations (12) and (13) hold), the system's error dynamics during sliding mode as described by equation (33) is independent of the system uncertainties and couplings between the inputs, and, insensitive to the parameter variations, and can be determined through a proper adjustment of the value of K, which can be obtained through a proper selection of the desired closed loop poles locations.

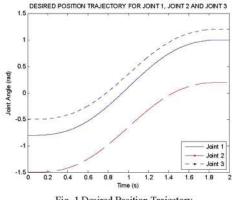


Fig. 1 Desired Position Trajectory

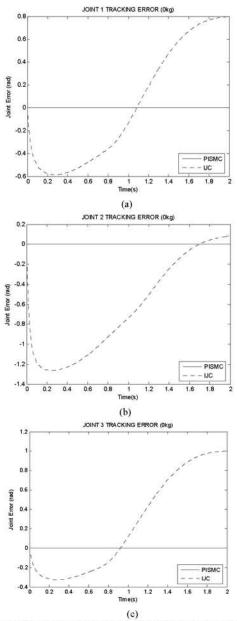


Fig. 2 Tracking Errors of (a) Joint 1 (b) Joint 2 (c) Joint 3 by PISMC and IJC with the Manipulator Handling No Load

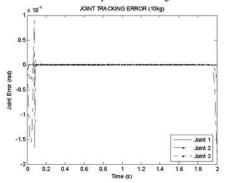
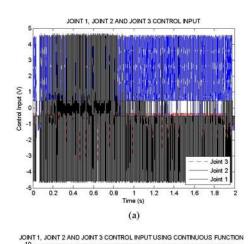
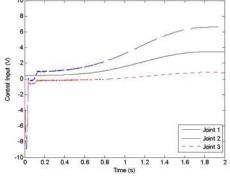


Fig. 3 Joints Tracking Error with Manipulator Handling 10 kg Load





(b) Fig. 4 Joints Control input using (a) Discontinuous Function (b) **Continuous** Function

