PROPORTIONAL-INTEGRAL SLIDING MODE CONTROL OF A QUARTER CAR ACTIVE SUSPENSION

YAHAYA M.SAM[1], JOHARI H.S.OSMAN & RUDDIN A.GHANI[2]
[1]Faculty of Electrical Engineering, University Technology of Malaysia
81310 UTM Skudai. Johor, MALAYSIA
[2]Kolej Universiti Teknikal Kebangsaan Malaysia,
75450 Melaka, MALAYSIA.
yahaya@fke.utm.my

Abstract

The purpose of this paper is to present a new approach in controlling an active suspension system. This approach utilized the proportional integral sliding mode control scheme. Using this type of sliding surface, the asymptotic stability of the system during sliding mode is assured compared to the conventional sliding surface. The proposed control scheme is applied in designing an automotive active suspension system for a quarter-car model and its performance is compared with the existing passive suspension system. A simulation study is performed to prove the effectiveness of this control design.

Key words: Automotive Control, Quarter Car Suspension, Sliding Mode Control.

1. Introduction

In the past millennium, there had been widespread interest in using advanced control techniques to improve the performance of vehicle suspension system. Performance of the suspension system has been greatly increased due to increasing vehicle capabilities. Several performance characteristics have to be considered in order to achieve a good suspension system [1]. These characteristics deal with regulation of body movement, regulation of suspension movement and force distribution. Ideally the suspension should isolate the body from road disturbances and inertial disturbances associated with cornering and braking or acceleration. Furthermore, the suspension must be able to minimize the vertical force transmitted to the passengers for passengers comfort. This objective can be achieved by minimizing the vertical car body acceleration. An excessive wheel travel will result in non-optimum attitude of tyre relative to the road that will cause poor handling and adhesion. Furthermore, to maintain good handling characteristic, the optimum tyre-to-road contact must be maintained on four wheels.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability [2]. Thus the passive suspension systems, which approach optimal characteristics had offered an attractive choice for a vehicle suspension systems and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension system has been proposed by various researchers [3,4,5]. Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, active suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, wheel velocity and wheel or body acceleration. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. Various control laws such as optimal state-feedback, LQG, LQR, H-infinity, sliding mode control, fuzzy control and etc. have been proposed in the past years to control the active suspension system.

In this paper we will consider a control scheme that can improve further the ride comfort and road handling of the active suspension system. The proposed control scheme differ from the previous sliding mode techniques in the sense that the sliding surface is based on the proportional-integral (PI) sliding mode control.

2. Dynamic model of the vehicle

Most of the past active suspension designs were developed based on the quarter-car model as in Figure 1.

0-7803-7490-8/02/$17.00©2002 IEEE.

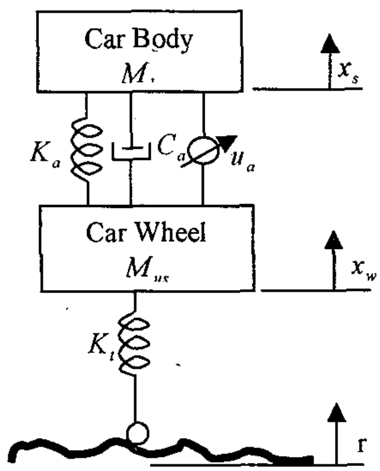

Figure1: A quarter-car model

The following state-space model can be easily obtained from Figure 1.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & -1\\ -K_a/M_s & -C_a/M_s & 0 & C_a/M_s\\ 0 & 0 & 0 & 1\\ K_a/M_{us} & C_a/M_{us} & -K_t/M_{us} & -C_a/M_{us}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 1/M_s\\ 0\\ -1/M_s\end{bmatrix}u_a+\begin{bmatrix}0\\ 0\\ -1\\ 0\end{bmatrix}\dot{r} \quad \ldots (1)$$

where M_s and M_{us} are the masses of car body and wheel , x_s and x_w are the displacements of car body and wheel, K_a and K_t are the spring coefficients, C_a is the damper coefficient , $\dot{r}$ is the road disturbance and u_a is the control force from the hydraulic actuator and assumed as the control input. The state variables are defined as forms: $x_1 = x_s - x_w$ for suspension travel, $x_2 = \dot{x}_s$ for car body velocity, $x_3 = x_w - r$ for wheel deflection and $x_4 = \dot{x}_w$ for wheel velocity. Equation (1) can be written as

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t) \qquad (1a)$$

where $x(t) \in \Re^n$ is the state vector , $u(t) \in \Re^m$ is the control input, and the continuous function $f(t)$ represents the uncertainties with the mismatched condition. The following assumptions are taken as standard:

Assumption i: There exists a known posititve constant such that $\|f(t)\| \le \beta$, where $\|\bullet\|$ denotes the standard Euclidean norm.

Assumption ii: The pair (A, B) is controllable and the input matrix B has full rank.

3. Switching Surface and Controller Design

In this study , we utilized the PI sliding surface define as follows:

$$\sigma(t) = Cx(t) - \int_0^t (CA + CBK)x(\tau)d\tau \qquad (2)$$

where $C \in \Re^{mxn}$ and $K \in \Re^{mxn}$ are constant matrices. The matrix K satisfies $\lambda_{\max}(A+BK) < 0$ and C is chosen so that CB is nonsingular. It is well known that if the system is able to enter the sliding mode, hence $\sigma(t) = 0$. Therefore the equivalent control, $u_{eq}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ [6] i.e,

$$\dot{\sigma}(t) = C\dot{x}(t) - \{CA + CBK\}x(t) = 0 \qquad (3)$$

If the matrix C is chosen such that CB is nonsingular, this yields

$$u_{eq}(t) = Kx(t) - (CB)^{-1}Cf(t) \qquad (4)$$

Substituting equation (4) into system (1a) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{x}(t) = (A + BK)x(t) + \{I_n - B(CB)^{-1}C)\}f(t) \qquad (5)$$

Theorem 1: If $\|\tilde{F}(t)\| \le \beta_1 = \|I_n - B(CB)^{-1}C\|\beta$ the uncertain system in equation (5) is boundedly stable on the sliding surface $\sigma(t) = 0$.

Proof: For simplicity, we let

$$\tilde{A} = (A + BK) \qquad (5a)$$

$$\tilde{F}(t) = \{I_n - B(CB)^{-1}C\}f(t) \qquad (5b)$$

and rewrite (5) as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{F}(t) \qquad (6)$$

Let the Lyapunov function candidate for the system is chosen as

$$V(t) = x^T(t)Px(t) \qquad (7)$$

Taking the derivative of $V(t)$ and substituting equation (5) , gives

$$\dot{V}(t) = x^T(t)[\tilde{A}^T P + P\tilde{A}]x(t) + \tilde{F}^T(t)Px(t) + x^T(t)P\tilde{F}(t) \quad (8)$$
$$= -x^T(t)Qx(t) + \tilde{F}^T(t)Px(t) + x^T(t)P\tilde{F}^T(t)$$

where P is the solution of $\tilde{A}^T P + P\tilde{A} = -Q$ for a given positive definite symmetric matrix Q. It can be shown that equation (8) can be reduced to:

$$\dot{V}(t) = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1\|P\|\|x(t)\| \quad (9)$$

Since $\lambda_{\min}(Q) > 0$, consequently $\dot{V}(t) < 0$ for all t and $x \in B^c(\eta)$, where $B^c(\eta)$ is the complement of the closed ball $B(\eta)$, centered at $x = 0$ with radius $\eta = \dfrac{2\beta_1\|P\|}{\lambda_{\min}(Q)}$. Hence, the system is boundedly stable.□

Remark: For the system with uncertainties satisfy the matching condition, i.e, $rank[B \mid f(t)] = rank[B]$, then equation (5) can be reduced to $\dot{x}(t) = (A + BK)x(t)$ [7]. Thus asymptotic stability of the system during sliding mode is assured.

We now design the control scheme that drives the state trajectories of the system in equation (1a) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter.

For the uncertain system in equation (1a) satisfying assumptions (i) and (ii), the following control law is proposed:

$$u(t) = -(CB)^{-1}[CAx(t) + \phi\sigma(t)] - k(CB)^{-1}\frac{\sigma(t)}{\|\sigma(t)\| + \delta} \quad (10)$$

where $\phi \in \Re^{m \times m}$ is a positive symmetric design matrix, k and δ are the positive constants.

<u>*Theorem 2:*</u> The hitting condition of the sliding surface (2) is satisfied if

$$\|A + BK\|\|x(t)\| \geq \|f(t)\| \quad (11)$$

Proof: In the hitting phase $\sigma^T(t)\sigma(t) > 0$; using the Lyapunov function candidate $V(t) = \frac{1}{2}\sigma^T(t)\sigma(t)$, we obtain

$$\dot{V}(t) = \sigma^T(t)\dot{\sigma}(t)$$
$$= \sigma^T(t)[-(CA + CBK)x(t) - \phi\sigma(t) - \frac{k\sigma(t)}{\|\sigma(t)\| + \delta} + Cf(t)] \quad (12)$$
$$\leq -[\{\|\phi\| + \left\|\frac{k}{\|\sigma(t)\| + \delta}\right\|\}\|\sigma(t)\|^2 + \{\|C\|\|A + BK\|\|x(t)\| - \|C\|\|f(t)\|\}\|\sigma(t)\|]$$

It follows that $\dot{V}(t) < 0$ if condition (11) is satisfied. Thus, the hitting condition is satisfied.□

4. Simulation and Discussion

The mathematical model of the system as defined in equation (1) and the proposed PI sliding mode controller in equation (10) were simulated on computer. Numerical values for the model parameters are taken from [3], and are as follows:

$M_s = 290$ kg, $M_{us} = 59$ kg, $K_a = 16812$ N/m,
$K_t = 190000$ N/m, $C_a = 1000$ N/(m/sec)

Let the set of typical road disturbance be of the form

$$r(t) = \begin{cases} a(1 - \cos(8\pi t))/2 & if\ 0.50 \leq t \leq 0.75 \\ 0 & otherwise \end{cases}$$

where a denotes the bump amplitude (see Fig.2). This type of road disturbance has been used by [4,8] in their studies. Furthermore, the maximum travel distance of suspension travel as suggested by [4] is $\pm 8cm$ has been used. The controller parameters has been chosen as follows for the simulation: *K=[16754 939.14 –1.818e5 172.67], C=[200 50 20 10]*, $\phi = 1000$, *k=1 and* $\delta = 0.001$.

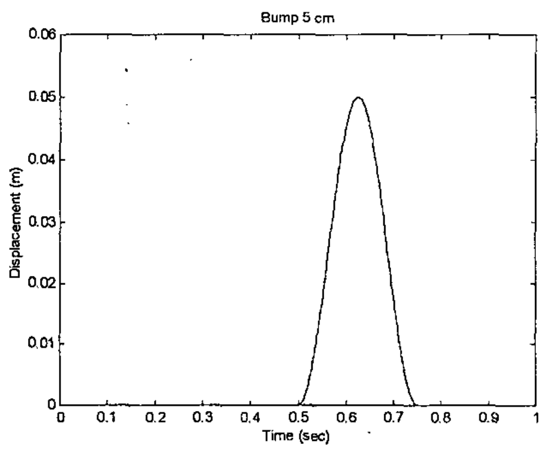

Fig. 2. Typical road disturbance – bump 5cm

In order to fulfill the objective of designing an active suspension system i.e. to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and wheel deflection. Figure 3a shows the suspension travel of both the active and passive suspension systems for comparison purposes. The result shows that the suspension travel within the travel limit . Figure 3b and Figure 3c illustrates clearly how the active suspension can effectively absorb the vehicle vibration in comparison to the passive system. The body acceleration in the active suspension system is reduced significantly, which guarantee better ride comfort. Moreover the wheel deflection is also smaller in the active suspension system.

Therefore it is concluded that the active suspension system improves the ride comfort while retaining the road handling characteristics, compared to the passive suspension system.

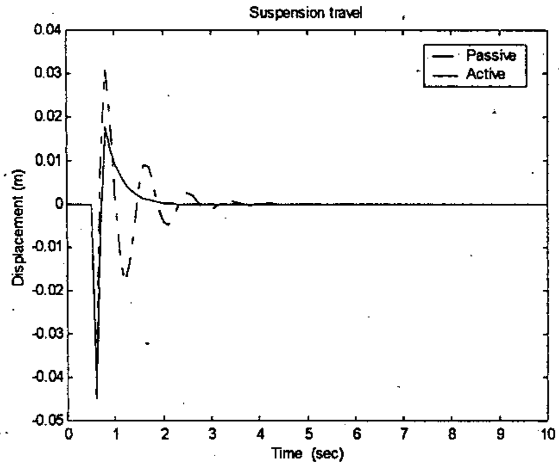

Figure 3(a): Suspension travel – 5 cm bump

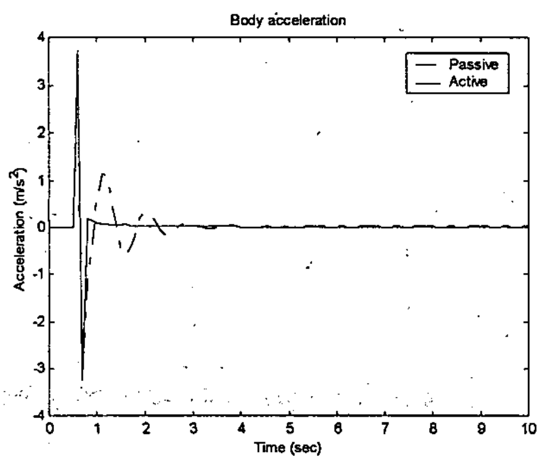

Figure 3(b): Body acceleration – 5 cm bump

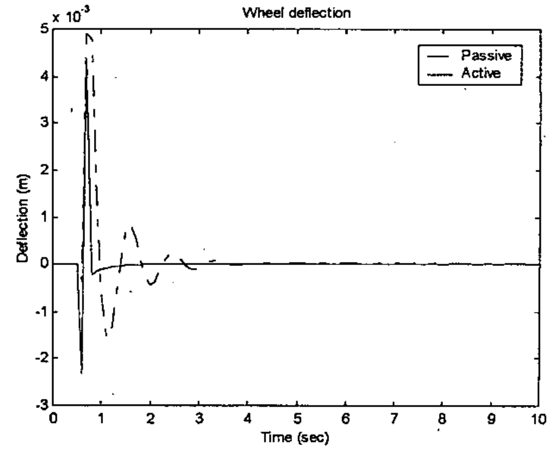

Figure 3(c): Wheel deflection – 5 cm bump

5. Conclusions

In this paper, the PI sliding mode control technique is proposed for controlling an active suspension system. It has been shown mathematically and through computer simulations that the proposed control scheme is capable of improving the ride comfort and road handling characteristics of the active suspension system.

References

[1] M. Appleyard & P.E. Wellstead, Active suspension : some background, *Proc. Control Theory Applications*, Vol. 142, No. 2, 1995, 123-128.

[2] A.G. Thompson, Design of Active Suspension, *Proc. Inst. Mech. Engrs,* Vol. 185, 1971, 553-563.

[3] A. Alleyne & J.K. Hedrick, Nonlinear Adaptive Control of Active Suspensions. *IEEE Trans. Control System Technology*, Vol 3, 1995, 94-101.

[4] J.S. Lin & I. Kanellakopoulos, Nonlinear Design of Active Suspension, *IEEE Control System Megazine*, Vol. 17, 1997, 45-59.

[5] E. Esmailzadeh & H.d. Taghirad, Active Vehicle Suspensions with Optimal State-Feedback Control, *Journal of Mechanical Science*, 1996, 1- 18.

[6] U. Itkis, *Control System of Variable Structure* (Wiley, New York, 1976)

[7] C. Edward & K.S. Sarah, *Sliding Mode control: Theory and Applications* (Taylor and Francis, 2000).

[8] F.J. D'Amato & D.E. Viasallo, Fuzzy Control for Active Suspensions, *Mechatronics*, Vol 10, 2000, 897-920.