A Decentralized Proportional-Integral Sliding Mode Tracking Controller for Robot Manipulators

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Abstract

This paper presents a decentralized Proportional-Integral (PI) sliding mode tracking control for a class of robot manipulators. A robust decentralized sliding mode controller is derived so that in each of the sub-system, the actual trajectory tracks the desired trajectory using the information available only on the local states. The PI sliding mode is chosen to ensure the stability of overall dynamics during the entire period i.e. the reaching phase and the sliding phase. It is shown theoretically and by simulation that manipulator systems controlled by the proposed decentralized control law are practically stable.

1.0 Introduction

Controller designs of robot manipulators that utilize the theory of Variable Structure System (VSS) has became a subject of interest in recent years [1]-[6]. However, most of the works were mainly concentrated on the centralized control concept. The salient feature of the VSS is Sliding Mode Control (SMC). SMC has been widely applied to system with uncertainties and/or input couplings since it is completely robust provided the matching condition hold. The design philosophy behind the SMC is to obtain a high-speed switching control law to drive the nonlinear plant's state trajectory onto a specified and user-chosen surface called the switching surface [7], [8]. When a system is in the sliding mode, its dynamics is strictly determined by the dynamics of the sliding surface and hence insensitive to parameter variations and system disturbances.

Since the dynamics of a robot manipulator basically consists of a set of highly nonlinear and coupled state equations, some researchers treated it as a set of interconnected subsystems with bounded uncertainties and presented decentralized controller for the tracking problem. By treating each of robot joint manipulators as a subsystem, there exist couplings among the control joint torques and as such the controller design objective is to get rid of it beside the uncertainties present in the system. In [9], a class of decentralized continuous nonlinear feedback control law via the deterministic control was introduced but with the couplings among the control joint torques considered negligible. The inclusion of the input coupling was considered in the [10] and [11] where the works were basically based on the Ricatti equation approach. ,

In this paper, a decentralized PI sliding mode control is proposed to control the manipulators. It is shown that the proposed control law guarantees that the hitting condition is always satisfied while the system stability still intact. The bounds of interconnections and the input couplings between the sub-systems are explicitly taken into account while the neighboring states are not required.

2.0 Statement of the Problem

Consider the dynamics of the robot manipulator as an uncertain composite system S defined by an N interconnected sub-systems $S_i, i = 1,2,\ldots, N$. Each sub-system can be described by

$$S_i: \dot{x}_i(t) = [A_i + \Delta A_i(t)]x_i(t) + [B_i + \Delta B_i(t)]u_i(t) + \sum_{j=1, j\neq i}^{N}[A_{ij} + \Delta A_{ij}(t)]x_j(t) + \sum_{j=1, j\neq i}^{N}[B_{ij} + \Delta B_{ij}(t)]u_j(t) \tag{1}$$

where $x_i(t) \in R^{n_i}, u_i(t) \in R^{m_i}$ represent the state and input of sub-system S_i, respectively. A_i, B_i, A_{ij} and B_{ij} are constant nominal matrices. ΔA_i, ΔA_{ij}, ΔB_i and ΔB_{ij} representing uncertainties present in the system, interconnection, input and coupling matrices, respectively.

The following assumptions are introduced:

(A1) Every state vector $x_i(t)$ can be locally observed;

(A2) There exist continuous functions $H_i(t)$, $H_{ij}(t)$, $E_i(t)$ and $E_{ij}(t)$ such that for all $X \in R^N$ and all t:

$$\begin{aligned} \Delta A_i(t) &= B_i H_i(t) \;;\; \|H_i(t)\| \le \alpha_{ii} \\ \Delta A_{ij}(t) &= B_i H_{ij}(t) \;;\; \|H_{ij}(t)\| \le \alpha_{ij} \\ \Delta B_i(t) &= B_i E_i(t) \;;\; \|E_i(t)\| \le \beta_{ii} \\ \Delta B_{ij}(t) &= B_i E_{ij}(t) \;;\; \|E_{ij}(t)\| \le \beta_{ij} \end{aligned} \tag{2}$$

(A3) There exist a Lebesgue function $\Omega_i(t) \in R$:

$$\dot{x}_{di}(t) = A_i x_{di}(t) + B_i \Omega_i(t) \tag{3}$$

where A_i and B_i are the i-th subsystem nominal system and input matrices, respectively;

(A4) The pair (A_i, B_i) is controllable.

The state vector of the composite system S is defined as

$$X(t) = [x_1^T(t), x_2^T(t), \ldots, x_n^T(t)]^T \tag{4}$$

Let $X_d(t) \in R^{3N}$ be the desired state trajectory:

$$X_d(t) = [x_{d1}^T(t), x_{d2}^T(t), \ldots, x_{dn}^T(t)]^T;\; x_{di}(t) \in R^{n_i} \tag{5}$$

Define the tracking error, $z_i(t)$ as

$$z_i(t) = x_i(t) - x_{di}(t) \tag{6}$$

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In view of equations (2), (3) and (6), equation (1) can be written as

$$\begin{aligned}\dot{z}_i(t) &= [A_i + B_i H_i(t)]z_i(t) + BH_i(t)x_{di}(t) \\ &- B_i\Omega_i(t) + [B_i + B_i E_i(t)]u_i(t) \\ &+ \sum_{j=1, j\neq i}^{N} [A_{ij} + B_i H_{ij}(t)]x_j(t) \\ &+ \sum_{j=1, j\neq i}^{N} [B_{ij} + B_i E_{ij}(t)]u_j(t)\end{aligned} \tag{7}$$

Define the local PI sliding surface for S_i as

$$\sigma_i(t) = C_i z_i(t) - \int_0^t [C_i A_i + C_i B_i K_i] z_i(\tau)d\tau \tag{8}$$

where $C_i \in R^{m_i \times n_i}$ and $K_i \in R^{m_i \times n_i}$ are constant matrices. The matrix K_i satisfies

$$\lambda_{\max}(A_i + B_i K_i) < 0 \tag{9}$$

and C_i is chosen such that C_iB_i is nonsingular. For this class of system, the sliding manifold can be described as

$$\sigma(t) = [\sigma_1^T, \sigma_2^T, ..., \sigma_N^T]^T \tag{10}$$

The control problem is to design a decentralized controller for each sub-system using the PI sliding mode (8) such that the system state trajectory $X_i(t)$ tracks the desired state trajectory $X_{di}(t)$ as closely as possible for all t in spite of the uncertainties and non-linearities present in the system. The task can be divided into two parts; a) it must be assured that the system error dynamics is asymptotically stable during the sliding mode, and b) a decentralized controller must be designed in such a way that whatever error the system has during the initial stage, the system is directed towards the sliding surface during the reaching phase.

3.0 Manipulator Dynamics During Sliding Mode

Differentiating equation (8), substitute equation (7) into it, and equating the resulting equation to zero gives the equivalent control, $u_{eqi}(t)$:

$$\begin{aligned}u_{eqi}(t) &= -[I_{n_i} + E_i(t)]^{-1}\{(H_i(t) - K_i)z_i(t) \\ &- \Omega_i(t) + H_i(t)x_{di}(t) \\ &+ \sum_{j=1, j\neq i}^{N} (C_iB_i)^{-1}C_i[A_{ij} + B_i H_{ij}(t)]x_j(t) \\ &+ \sum_{j=1, j\neq i}^{N} (C_iB_i)^{-1}C_i[B_{ij} + B_i E_{ij}(t)]u_j(t)\}\end{aligned} \tag{11}$$

Remark 1: The equivalent control $u_{eqi}(t)$ is only a mathematically derived tool for the analysis of a sliding motion rather than a real control law generated in practical systems.

The system dynamics during sliding mode can be found by substituting the equivalent control (11) into the system error dynamics (7):

$$\begin{aligned}\dot{z}_i(t) &= [A_i + B_i K_i]z_i(t) \\ &+ [I_{ni} - B_i(C_iB_i)^{-1}C_i]\{\sum_{j=1, j\neq i}^{N} [A_{ij} + B_i H_{ij}(t)]x_j(t) \\ &+ \sum_{j=1, j\neq i}^{N} [B_{ij} + B_i E_{ij}(t)]u_j(t)\}\end{aligned} \tag{12}$$

Define $P_{s_i} \triangleq [I_{n_i} - B_i(C_iB_i)^{-1}C_i]$ (13)

where P_{s_i} is a *projection operator* and satisfies the following two equations [12]:

$$C_i P_{s_i} = 0 \quad \text{and} \quad P_{s_i} B_i = 0 \tag{14}$$

In view of assumption (A2), then it follows that by the projection property, equation (14) can be reduced as

$$\dot{z}_i(t) = [A_i + B_i K_i]z_i(t) \tag{15}$$

Hence if the matching condition is satisfied, the system error dynamics during sliding mode are independent of the interconnection between the subsystems and couplings between the inputs, and, insensitive to the parameter variations. Equation (15) shows that the error dynamics can be specified by the designer through appropriate choice of the matrix K_i.

4.0 Decentralized Controller Design

The composite manifold (10) is asymptotically stable in the large, if the following hitting condition is held [13]:

$$\sum_{i=1}^{N} (\sigma_i^T(t)/\|\sigma_i(t)\|)\dot{\sigma}_i(t) < 0 \tag{16}$$

Proof: Let the positive definite Lyapunov function be

$$V(t) = \sum_{i=1}^{N} \|\sigma_i(t)\| \tag{17}$$

Then
$$\dot{V}(t) = \sum_{i=1}^{N} (\sigma_i^T(t)/\|\sigma_i(t)\|)\dot{\sigma}_i(t) \tag{18}$$

Following the Lyapunov stability theory, if equation (16) holds, then the sliding manifold $\sigma(t)$ is asymptotically stable in the large. □

Theorem: The global hitting condition (16) of the composite manifold (10) is satisfied if every local control $u_i(t)$ of system (7) is given by:

$$\begin{aligned}u_i(t) = &-(C_iB_i)^{-1}[\gamma_{i1}\|z_i(t)\| + \gamma_{i2}\|x_i(t)\| + \gamma_{i3}\|x_{di}(t)\| \\ &+ \gamma_{i4}\|\Omega_i(t)\|]SGN(\sigma_i(t)) + \Omega_i(t)\end{aligned} \tag{19}$$

where

$$\gamma_{i1} > \frac{\alpha_{ii}\|C_iB_i\| + \|C_iB_iK_i\|}{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1, j\neq i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}} \tag{20}$$

$$\gamma_{i2} > \frac{\sum_{j=1, j\neq i}^{N}[\|C_jA_{ji}\| + \alpha_{ji}\|C_jB_j\|]}{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1, j\neq i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}} \tag{21}$$

$$\gamma_{i3} > \frac{\alpha_{ii}\|C_iB_i\|}{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1, j\neq i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}} \tag{22}$$

$$\gamma_{i4} > \frac{\beta_{ii}\|C_iB_i\| + \sum_{j=1, j\neq i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]}{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1, j\neq i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}} \tag{23}$$

Proof: In view of equations (7), (8), (18) and (19), it can be shown that the rate of change of the Lyapunov function is

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$$
\begin{aligned}
\dot V(t) \le -\sum_{i=1}^{N} \{\{\{(1+\beta_{ii})\|C_iB_i\| \\
+ \sum_{j=1,j\ne i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}\gamma_{i1} \\
- [\alpha_{ii}\|C_iB_i\| + \|C_iB_iK_i\|]\}\|z_i(t)\| \\
+ \{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1,j\ne i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}\gamma_{i2} \\
- \sum_{j=1,j\ne i}^{N}[\|C_jA_{ji}\| + \alpha_{ji}\|C_jB_j\|]\}\|x_i(t)\| \qquad (24) \\
+ \{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1,j\ne i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}\gamma_{i3} \\
- \alpha_{ii}\|C_iB_i\|\}\|x_{di}(t)\| \\
+ \{\{(1+\beta_{ii})\|C_iB_i\| + \sum_{j=1,j\ne i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]\}(C_iB_i)^{-1}\gamma_{i4} \\
- [\beta_{ii}\|C_iB_i\| + \sum_{j=1,j\ne i}^{N}[\|C_jB_{ji}\| + \beta_{ji}\|C_jB_j\|]]\}\|\Omega_i(t)\|\}
\end{aligned}
$$

Let equations (20)-(23) hold, then the global hitting condition (16) is satisfied. □

Remark 2: The conditions imposed by equations (20)-(23) not only guarantee that the global hitting condition (16) is met, but it also assure that based on the Lyapunov theory, the system dynamics is stable in the large during the reaching phase.

5.0 Simulation of a Two-Link Manipulator

Consider a two-link manipulator (in the horizontal plane) with rigid links of nominally equal length l and mass m as shown in Figure 1. The dynamics of the manipulator can be found in [2].

Define

$$[x_1 \mid x_2]^T \underline{\Delta} [x_{11} \quad x_{12} \mid x_{21} \quad x_{22}]^T = \left[\theta_1 \quad \dot\theta_1 \mid \theta_2 \quad \dot\theta_2\right]^T$$

Suppose that the bounds of the $\theta_i(t)$ and $\dot\theta_i(t)$ are:

$$-150^\circ \le \theta_1 \le 150^\circ \;,\; 0^\circ s^{-1} \le \dot\theta_1 \le 50^\circ s^{-1},$$

$$-35^\circ \le \theta_2 \le 100^\circ \;,\; 0^\circ s^{-1} \le \dot\theta_2 \le 30^\circ s^{-1}$$

With these bounds, the plant can be represented in the form of equation (1) with the nominal value of A_i, A_{ij}, B_i and B_{ij} is calculated as:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1.1684 \end{bmatrix},\; A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -2.4310 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.9724 \end{bmatrix},\; A_{21} = \begin{bmatrix} 0 & 0 \\ 0 & -2.6496 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1.5058 \end{bmatrix},\; B_2 = \begin{bmatrix} 0 \\ 8.4151 \end{bmatrix},$$

$$B_{12} = \begin{bmatrix} 0 \\ -1.9489 \end{bmatrix},\; B_{21} = \begin{bmatrix} 0 \\ -1.9489 \end{bmatrix}$$

Using equation (2), the bounds of $H_i(t)$ and $E_i(t)$ can be computed:

$$\|H_1(t)\| \le \alpha_{11} = 0.6471,\; \|H_2(t)\| \le \alpha_{22} = 0.2889,$$

$$\|H_{12}(t)\| \le \alpha_{12} = 0.6458,\; \|H_{21}(t)\| \le \alpha_{21} = 0.5$$

$$\|E_1(t)\| \le \beta_{11} = 0.1385,\; \|E_2(t)\| \le \beta_{22} = 0.6297,$$

$$\|E_{12}(t)\| \le \beta_{12} = 1.5519,\; \|E_{21}(t)\| \le \beta_{21} = 0.2777$$

It is assumed that each sub-system is required to track a pre-specified cycloidal function of the form:

$$\theta_{di}(t) = \begin{cases} \theta_i(0) + \dfrac{\Delta_i}{2\pi}\left[\dfrac{2\pi t}{\tau} - \sin\left(\dfrac{2\pi t}{\tau}\right)\right], & 0 \le t \le \tau \\ \theta_i(\tau), & \tau \le t \end{cases}$$

where $\Delta_i = \theta_i(\tau) - \theta_i(0)$, $i = 1,2$. In this example, the desired trajectories are set to traverse from 5° to 90° and from 15° to 60° for subsystem 1 and subsystem 2, respectively. The final time, τ is set at 10 s.

In this study, the gains are chosen as follows:

$K_1 = [1.3282 \quad 2.7683]$ so that $\lambda(A_1 + B_1K_1) = \{-1, -2\}$;

$K_2 = [0.7130 \quad 0.3053]$ so that $\lambda(A_2 + B_2K_2) = \{-2, -3\}$;

$C_1 = [3 \quad 1]$ and $C_2 = [4 \quad 1]$.

Therefore, from equations (20)-(23):

$\gamma_{11} > 1.40; \gamma_{12} > 1.72; \gamma_{13} > 0.25; \gamma_{14} > 1.13;$

$\gamma_{21} > 4.19; \gamma_{22} > 0.91; \gamma_{23} > 1.14; \gamma_{24} > 4.48$

For simulation purposes, two sets of controller parameters are chosen:

Set 1: $\begin{cases} \gamma_{11} = 0.5;\; \gamma_{12} = 0.5;\; \gamma_{13} = 0.1;\; \gamma_{14} = 0.5; \\ \gamma_{21} = 4;\; \gamma_{22} = 0.5;\; \gamma_{23} = 0.5;\; \gamma_{24} = 4 \end{cases}$

Set 2: $\begin{cases} \gamma_{11} = 3;\; \gamma_{12} = 3;\; \gamma_{13} = 2;\; \gamma_{14} = 2; \\ \gamma_{21} = 5;\; \gamma_{22} = 2;\; \gamma_{23} = 2;\; \gamma_{24} = 5 \end{cases}$

In Set 1, the controller parameter is selected to study the performance of the system if the gain conditions of equations (20)-(23) are not met; while in Set 2 the controller parameters is selected to represent a situation where the conditions imposed on the controller are met. The output trajectories for subsystem 1 and subsystem 2 for the proposed decentralized controller utilizing the parameter supplied by Set 1 are shown in Figure 2. It can be seen that the tracking performance for both subsystems are unsatisfactory. The simulation was run again but this time with the decentralized controller parameter was supplied from Set 2. The results are shown in Figure 3. As predicted theoretically, the tracking performance is good for both subsystems.

6.0 Conclusion

In this paper, a decentralized PI sliding mode controller is proposed for robot manipulators. It is shown mathematically that the error dynamics during sliding mode is stable and can easily be shaped-up using the conventional pole-placement technique. The controller utilizes only the information available on the local states and the system stability is guaranteed during the reaching phase as well as the reaching phase. Application to a two-link shows that the proposed controller renders the uncertainties system tracks the desired trajectory in spite of the uncertainties, non-linearities and coupling inherent in the system.

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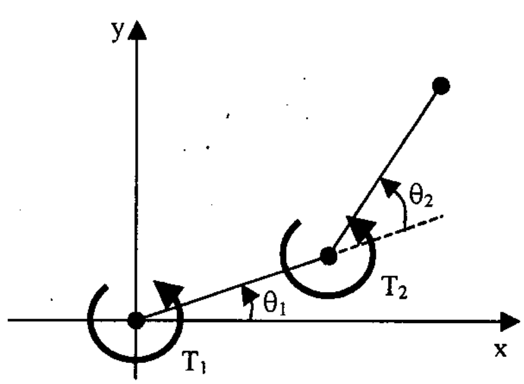
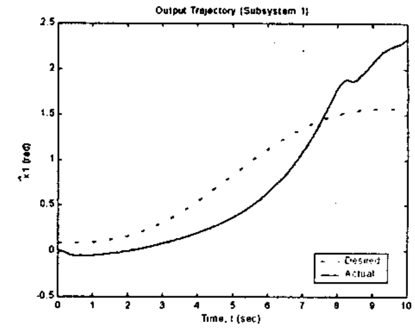
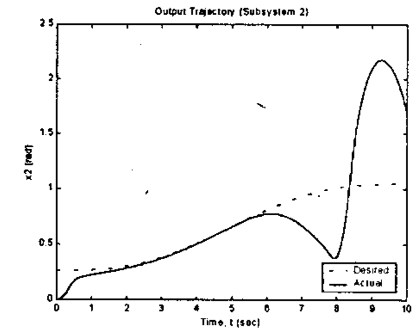


Figure 1: A two-link manipulator.

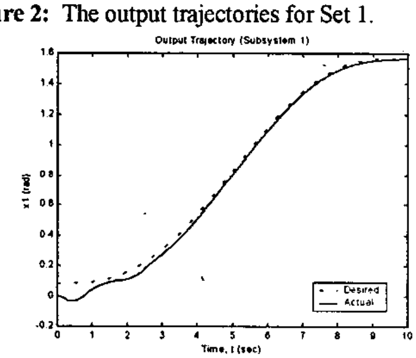


a) Subsystem 1.

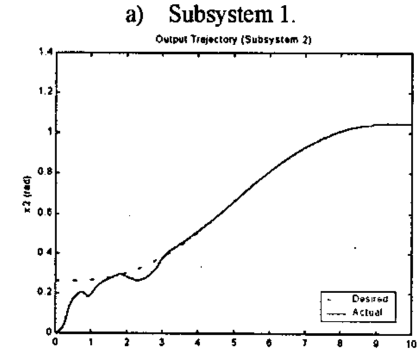


b) Subsystem 2.

Figure 2: The output trajectories for Set 1.



a) Subsystem 1.



b) Subsystem 2.

Figure 3: The output trajectories for Set 2.

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