Integer Coefficients Partial Response Signaling in OFDM System

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Abstract - Orthogonal Frequency Division Multiplexing (OFDM) is a successful technique in wireless communication system. However, frequency offset in OFDM system leads to loss of orthogonality among subcarriers, which resulted in intercarrier interference (ICI) to occur. To improve bandwidth efficiency performance in ICI self-cancellation schemes, frequency domain partial response signaling (PRS) has been studied. In this paper, the integer polynomial partial response coefficients are exploited to enhance carrier-to-interference power ratio (CIR) in OFDM system. The CIR is enhanced by about 3.8 dB up to 4.7 dB when the length of PRS polynomial, K is 2, 3, and 4 respectively.

Keywords: CIR; ICI; frequency offset; OFDM; PRS

1. Introduction

In OFDM system, the entire bandwidth is divided into many orthogonal subcarriers, where information symbols are transmitted in parallel over these subcarriers with long symbol duration in order to deal with frequency-selective fading of wireless environments. The overlapping spectrum of the subcarriers in OFDM system requires an accurate frequency recovering system. However, in time variant mobile radio environment, the relative movement between transmitter and receiver resulted in frequency offset due to Doppler frequency shifts, hence the carriers cannot be perfectly synchronized. This imperfection destroys orthogonality among subcarriers and causes intercarrier interference (ICI) to occur in addition to rotation and attenuation. Furthermore, the degradation of BER performance increases rapidly with increasing frequency offset occurrence in OFDM system [1].

Several methods have been proposed to reduce the effect of the ICI. One of the methods is frequency-domain equalization [2]. Time-domain windowing is another way to reduce the effect of frequency offset [3]. The ICI suppression in multiple-input multiple-output (MIMO) OFDM is studied in [4]. A self-ICI-cancellation approach has been proposed, which transmits each symbol over a pair of adjacent or non-adjacent subcarriers with a certain phase shift [5, 6, 7]. This method can suppress the ICI significantly with a reduction in bandwidth efficiency. In single-carrier systems, partial response signaling has been studied to reduce the sensitivity to time offset without sacrificing the bandwidth [8]. In the frequency domain, the partial response with correlative coding was used to

mitigate the ICI caused by carrier frequency offset [9]. The optimum weights for partial response coding that minimize the ICI power were derived [10]. However, by using polynomial coefficients with integer values reduces the hardware implementation of the system. In this paper, we study the partial response signaling OFDM (PRS-OFDM) system with integer polynomial coefficients.

This paper is organised as follows. In Section 2 we describe a PRS-OFDM system. The ICI expressions and analysis is included in Section 3. Then, in Section 4, the numerical results are presented to demonstrate the performance of PRS-OFDM systems with integer polynomial coefficients.

2. OFDM System with Frequency Offset Occurrence

In OFDM system, if a unit signal is modulated onto a single subcarrier k_0th and zero on the rest of subcarriers of an OFDM system with N number of subcarriers, the signals received from all of the subcarriers are defined as the subcarrier frequency offset (SFO) response for the k_0-th subcarrier [5, 9]. Considering only a single path from the transmitter to the receiver, signal on kth subcarrier influences the other subcarrier which are recognised as intercarrier interference (ICI) for $l \neq k$ which is represented as [5, 9],

$$G(l-k) = \frac{\sin(\pi(\varepsilon + l - k))}{N\sin\left(\frac{\pi}{N}(\varepsilon + l - k)\right)} \cdot \exp\left(j\frac{\pi}{N}(N-1)(\varepsilon + (l-k))\right)$$

$$(1)$$

Equation (1) denotes the ICI effect of the lth subcarrier to the kth subcarrier with the occurrence of normalised frequency offset, ε. Some indoor environments can be modeled as this model, because of the negligible channel time delays. If ε is zero in equation (1), it means that no frequency offset exists, hence resulted in no ICI amongst the subcarriers of the OFDM signal.

Consider OFDM system with N subcarriers and M OFDM frames, the demodulated OFDM symbols can be formulated as [5, 7],

$$Y = GX + W \qquad (2)$$

where X is the $N \times M$ input data. Each frame consists of N input bits, and W is the white noise. Noise components will not be discussed and then will be neglected from now

on. Ideally, G is an $N \times N$ identity matrix. However, with the existence of frequency offset, the matrix has nontrivial off-diagonal elements. By assuming that all subcarriers under the influence of the same channel impulse response with ε occurrence, the demodulated symbol at kth subcarrier can be written as

$$Y(k) = X(k)G(0) + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} X(l)G(l-k) \quad (3)$$

The first part in equation (3) consists of desired signals, while the second part is the unwanted ICI components.

3. Partial Response Signaling OFDM (PRS-OFDM) System

In this paper, we consider PRS-OFDM system. The baseband model of PRS-OFDM is shown in Figure 1. At the transmitter, the modulated symbol, $X(k)$ will be encoded by PRS encoder. Let $X(k)$ be the symbols to be transmitted and $c(i)$ be the coefficients for partial response polynomial, the transmitted signal at the kth subcarrier can be expressed as

$$S(k) = \sum_{i=0}^{K-1} c(i)X(k-i) \quad (4)$$

where K is the number of coefficients or length of the polynomial. Without loss of generality, $E|X(k)|^2 = 1$ and $E(X(k)X^*(j)) = 0$ for $k \neq j$ is assumed.
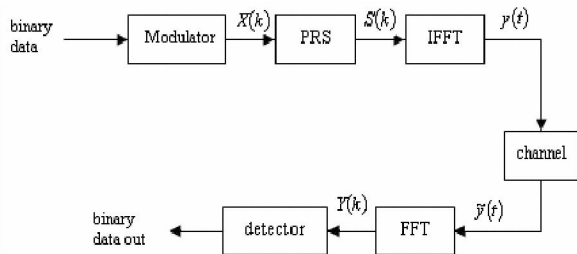

Figure 1. Baseband model of PRS-OFDM system

The transmitted OFDM signal in time domain is

$$y(t) = \sum_{k=0}^{N-1} S(k)e^{j2\pi f_k t}, \qquad 0 \le t < T_s \quad (5)$$

where $f_k = f_0 + k\Delta f$ is the frequency of the kth subchannel, $\Delta f = 1/T_s$ is the subchannel spacing, and T_s is the symbol duration. The coded signals can be recovered by a maximum-likelihood (ML) sequence detector at the receiver.

At the receiver, by performing FFT on the received signal, the demodulated signal can be written as

$$Y(k) = \sum_m \tilde{y}(t)e^{-j2\pi f_m t} \quad (6)$$

By neglecting the additive white Gaussian noise (AWGN) and considering equations (3) and (4), equation (6) can be written as

$$
\begin{aligned}
Y(k) &= \sum_{l=0}^{N-1} S(l)G(l-k) \\
&= S(k)G(0) + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} S(l)G(l-k) \\
&= C(k) + I(k)
\end{aligned} \quad (7)
$$

for $k = 0, \dots, N-1$. The ICI power of PRS-OFDM system can be expressed as

$$P_{ICI} = E\left[|I(k)|^2\right] = E\left[\left|\sum_{\substack{l=0 \\ l \neq k}}^{N-1} S(l)G(l-k)\right|^2\right] \quad (8)$$

By applying equation (4) to (8), the ICI power of K length PRS-OFDM system where $k=0$ also can be written as

$$P^K_{ICI} = E\left[\left|\sum_{l=1}^{N-1}\sum_{i=0}^{K-1} c(i)X(l-i)G(l)\right|^2\right] \quad (9)$$

Using equation (1), (7), and (9), the carrier-to-interference power ratio (CIR) can be formulated as in equation (10).

4. Numerical Result

We are concerned on finding the PRS polynomial coefficients that would result in maximum CIR in OFDM system. By using the CIR in equation (10), appropriate coefficients can be found for each of the respective polynomial length, K. The CIR is a function of PRS polynomial coefficients and the normalised carrier frequency shift, ε.

$$CIR = \frac{\left(\dfrac{\sin \pi\varepsilon}{\pi\varepsilon}\right)^2 \sum_{i=0}^{K-1} c(i)^2}{\left\{\sum_{i=0}^{K-1}\sum_{l=1}^{N-1} c(i)^2 G^2(l) + \left[\sum_{k=1}^{K-1}\sum_{i=0}^{K-1-k} c(i)c(i+k)\sum_{l=k+1}^{N-1} G(l)G^*(l-k) + G(l-k)G^*(l)\right]\right\}} \quad (10)$$

Exhaustive search methodology is used to find the optimum integer coefficients of PRS-OFDM that maximises the carrier-to-interference power ratio (CIR). Some restrictions were applied in order to simplify the findings. Below are restrictions that are applied in the coefficients identification procedure:

Restriction 1: c_0 is set to integer 1
Restriction 2: The maximum value of c_i is set to the maximum coefficient value used in the polynomial of the form $(1-r)^{K-1}$ where K is an integer value and $K>0$. Lastly, with the presence of ε, the value of CIR for each combination of c_i is calculated. At each length K, the appropriate polynomial that gives a maximum CIR is identified.

Table 1 shows the PRS coefficients combination, which gives the maximum CIR for the respective polynomial length, K when N=64.

Table 1: The optimum PRS-OFDM polynomial integer coefficients for the respective K length.

Length, K	PRS integer coefficients
2	1, -1
3	1, -2, 1
4	1, -2, 2, -1

As seen form the Figure 2, the CIR of the integer PRS-OFDM system is significantly better than the normal OFDM system as the normalized frequency offset, ε. The CIR performance when ε is from 0.2 to 0.3 is depicted in Figure 2. K=2 has the lowest range of CIR gain followed by K=3 and K=4 with maximum CIR gain of 3.8 dB, 4.5 dB, and 4.7 dB respectively.
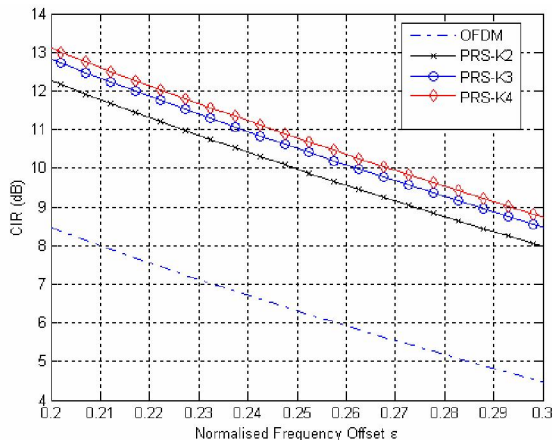

Figure 2. The CIR performance of integer coefficients PRS-OFDM system

5. Summary

In this paper, PRS-OFDM system has been studied. Polynomial coefficients with integer values are used to reduce the complexity of the receiver. ICI is deliberately introduced in a controlled manner through the polynomial functions. The effectiveness of PRS-OFDM system with integer polynomial coefficients on enhancing CIR is investigated. In this chapter, the CIR of PRS-OFDM system has been derived. The integer coefficients of PRS for the respective polynomial length in single OFDM system with maximised CIR are determined through exhaustive search. The numerical and simulation results shows that the longer the polynomial length, K, the higher CIR is obtained. PRS managed to enhance further the CIR of OFDM system by about 3.8 dB up to 4.7 dB when the length of polynomial, K is 2, 3, and 4 respectively. This system is feasible and can be applied in future broadband system development such as MIMO-OFDM system.

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