# The performance of correlative coding in MIMO-OFDM systems

Sharifah K. Syed-Yusof
Faculty of Electrical Engineering
Universiti Teknologi Malaysia
81310 UTM Skudai, Johor, Malaysia
e-mail: kamilah@fke.utm.my

Norsheila Fisal
Faculty of Electrical Engineering
Universiti Teknologi Malaysia
81310 UTM Skudai, Johor, Malaysia
e-mail: sheila@fke.utm.my

Muladi
Faculty of Electrical Engineering
Universiti Teknologi Malaysia
81310 UTM Skudai, Johor, Malaysia
e-mail: elum_mlg@yahoo.com

***Abstract***— **This paper considers the use of frequency-domain correlative coding in MIMO-OFDM systems to improve the BER caused by channel frequency offset. At BER of $10^{-4}$, the $E_s/N_0$ needed managed to be reduced by 2-4 dB when correlative coding was used in MIMO-OFDM systems with the presence of frequency offset, $\varepsilon$=0.15.**

***Keywords-*** *frequency offset, inter-carrier interference (ICI), multiple-input multiple-output (MIMO), Orthogonal Frequency Division multiplexing (OFDM), partial response coding (PRC), space-time coding (STC).*

## I. Introduction

The main advantage of OFDM transmission comes from the fact that the Fourier basis forms an eigenbasis for time invariant channels [1]. This allows simplification on hardware implementations especially at the receiver. For example, when the channel is time-invariant within one OFDM transmission block, the equalizer is just a single-tap filter in the frequency domain. Combined with multiple-input multiple-output channels (MIMO), MIMO-OFDM has a potential of achieving high data rates. However, the block time-invariance assumption may not be valid at high mobile speeds where impairments such as frequency offset are likely to occur. Such scenario resulted to loss of orthogonality of the OFDM subcarriers that leads to inter-carrier interference (ICI) in addition to signal rotation and attenuation. With the increasing frequency offset occurrence, the degradation of BER performance in OFDM system enhances further [1].

In single-input single output (SISO) OFDM system, several methods have been proposed to reduce the effect of the ICI. One of the methods is frequency-domain equalization [2]. Time-domain windowing is another way to reduce the effect of frequency offset [3]. A self-ICI-cancellation approach has been proposed, which transmits each symbol over a pair of adjacent or non-adjacent subcarriers with a certain phase shift [5, 6, 7]. This method can suppress the ICI significantly with a reduction in bandwidth efficiency. In single-carrier systems, partial response signaling has been studied to reduce the sensitivity to time offset without sacrificing the bandwidth [8]. The partial response with correlation polynomial $F(D) = 1 - D$ was used in the frequency domain to mitigate the ICI caused by carrier frequency offset [9]. In this technique, ICI is deliberately introduced in a controlled manner through the polynomial functions. The ICI suppression in multiple-input multiple-output (MIMO) OFDM is studied in [4] by using time-domain filtering based. In this paper, we investigate the performance of correlative coding used by [8] in reducing BER caused by frequency offset in MIMO-OFDM system. A simple symbol-by-symbol suboptimum detection technique is used in this study.

This paper is organised as follows. In Section II we describe the correlative coding in MIMO-OFDM system. The ICI expressions and analysis is included in Section III. Then, in Section IV, the simulation results are presented.

## II. Correlative coded OFDM System

Let $X_k$ be the symbols to be transmitted and $c_i$ be the coefficients for correlation polynomial, the transmitted signal at the $k$-th subcarrier can be expressed as

$$S_k = \sum_{i=0}^{K-1} c_i X_{k-i} \tag{1}$$

where $K$ is the number of coefficients or length of the polynomial. Without loss of generality, $E|X_k|^2 = 1$ and $E(X_k X_j^*) = 0$ for $k \neq j$ is assumed.

The transmitted SISO-OFDM signal in time domain is

$$y(t) = \sum_k S_k e^{j2\pi f_k t}, \qquad 0 \le t \le T_s \tag{2}$$

where $f_k = f_0 + k\Delta f$ is the frequency of the $k$-th subchannel, $\Delta f = 1/T_s$ is the subchannel spacing, and $T_s$ is the symbol duration.

After passing through a time-varying channel with the impulse response $h(t, \tau)$, the received signal is

$$\tilde{y}(t) = \int h(t,\tau) y(t-\tau) d\tau \tag{3}$$

The channel impulse response for the frequency-selective fading channel can be described by

$$h(t,\tau) = \sum_{l=0}^{v-1} h_l(t)\, \delta(\tau - \tau_l(t)) \tag{4}$$

where $v$ is the total number of non-zeros taps in the channel response, $h_l(t)$ represents the time variant attenuation factor of the $l$-th path and $\tau_l(t)$ is time varying delay of $l$-th path. The channel impulse response can also be represented in terms of Doppler frequency shifts, $f_{D_l}(t)t$ caused by movement of mobile receiver as

$$h(t,\tau)=\sum_{l=0}^{v-1} h_l \exp\left(j\,2\pi f_{D_l}(t)t\right)\delta\left(\tau-\tau_l(t)\right) \qquad (5)$$

where $h_l$ is the amplitude of the $l$-th path.

The received OFDM signal can be written as

$$\tilde{y}(t)=h(t,\tau) * y(t)+z(t) \qquad (6)$$

where $z(t)$ is the additive white Gaussian noise (AWGN).

The output of the DFT at the receiver for a time-block {-($v$-1), ..., $N$-1] where $N$ is the number of carriers can be written as

$$Y(p)=G(p,p)S(p)+\sum_{\substack{q=0\\q\neq p}}^{N-1}G(q,p)S(q)+Z(p) \qquad (7)$$

for $p$=0, ..., $N$-1. The $G(p,p)S(p)$ gives the desired signal value for subcarrier $p$ with an average carrier power of $E[|G(p,p)S(p)|^2]$. While $G(q,p)$ is defined as the subcarrier frequency offset response for the $p$-th subcarrier [5]. It is also the ICI effect of the $q$-th subcarrier to the $p$-th subcarrier with the occurrence of normalised frequency offset, $\varepsilon$. In the case of time-variant the equation $G(q, p)$ becomes

$$G(q,p)=\frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{v-1}h_{l,n}\exp\left(j\theta_{l,n}\right)\exp\left(j\frac{2\pi\pi}{N}(p-q)\right)\exp j\frac{2\pi}{N}n\varepsilon_{l,n} \qquad (8)$$

Therefore, the ICI power on the $m$-th subcarrier can be expressed into

$$P_{ICI}(p)=E\left|\sum_{\substack{q=0\\q\neq p}}^{N-1}G(q,p)S(q)\right|^2 \qquad (9)$$

## III. ICI ANALYSIS OF CORRELATIVE CODED MIMO-OFDM SYSTEM

In the case when we have transmit and receive diversities with $M_r$ receive and $M_t$ transmit antennas, we can modify (7) to

$$\mathbf{Y}(p)=\mathbf{G}(p,p)\mathbf{S}(p)+\sum_{\substack{q=0\\q\neq p}}^{N-1}\mathbf{G}(p,q)\mathbf{S}(q)+\mathbf{Z}(p) \qquad (10)$$

where $[\mathbf{G}(p,q)]_{i,j}$ = $\left[\tilde{Q}H^{(i,j)}Q^H/N\right]_{p,q}$ is the subcarrier frequency offset for the $p$-th subcarrier between the $i$-th receiver and $j$-th transmitter for $(1\le i\le M_r, 1\le j\le M_t, 0\le p\le N-1, 0\le q\le N-1)$. $\tilde{Q}$ is the DFT matrix and $H^{(i,j)}$ is the equivalent channel matrix between i-th receiver and j-th transmitter. Also the received vector for the $p$-th subcarrier frequency is $\mathbf{Y}(p)$= $\left[Y^{(1)}(p),\ldots,Y^{(M_r)}(p)\right]^T$, while the transmitted vector is $\mathbf{S}(p)$= $\left[S^{(1)}(p),\ldots,S^{(M_t)}(p)\right]^T$ and the noise vector is $\mathbf{Z}(p)$= $\left[Z^{(1)}(p),\ldots,Z^{(M_r)}(p)\right]^T$.

By ignoring the additive noise, the ICI power on the $n$-th subcarrier can then be evaluated as

$$P_{ici}(n)=\mathrm{E}\left[\sum_{\substack{p=0\\n\neq m}}^{N-1}\sum_{\substack{n=0\\n\neq m}}^{N-1}\mathbf{G}(m,p)\mathbf{S}(p)\mathbf{S}^{\mathrm{H}}(n)\mathbf{G}^{\mathrm{H}}(m,n)\right] \qquad (11)$$

By substituting eq. (1) into (11) becomes

$$P_{ici}(n)=\mathrm{E}\left[\sum_{\substack{p=0\\n\neq m}}^{N-1}\sum_{\substack{n=0\\n\neq m}}^{N-1}\mathbf{G}(m,p)\sum_{i=0}^{K-1}c_i\mathbf{X}(p-i)\sum_{ii=0}^{K-1}c_{ii}\mathbf{X}^{\mathrm{H}}(n-ii)\mathbf{G}^{\mathrm{H}}(m,n)\right] \qquad (12)$$

## IV. SIMULATION RESULTS

In this section, the performance of correlative coded MIMO-OFDM scheme is compared with the conventional MIMO-OFDM system. The simulation is of limited complexity. OFDM cyclic extension is not considered in this simulation because no delay spread between the transmitter and receiver is assumed. Other parameters considered in the simulation:

- Flat channel frequency response in each OFDM subcarrier
- Slow changing channel (quasi-static during block time)
- Complex path gains are uncorrelated
- Perfect channel state information is known at the receiver
- Perfect symbol timing synchronization is assumed at the receiver

Three different types of MIMO channels are investigated in the simulation:

(a). 1x2 classic MRC technique
(b). 2x1 space-time coding (special class of MIMO-OFDM [10, 11] )
(c). 2x2 MIMO

Fig.1 shows the BER performance of MIMO-OFDM system when correlative coding is used. A frequency offset, $\varepsilon$ of 0.15 is introduced in each path of the MIMO-OFDM channels. As seen from the figure, the classic MRC has the best performance while STBC-OFDM has the worst performance. In fig. 2, 3 and 4, correlative coded MIMO-OFDM systems are compared to conventional MIMO-OFDM system. Correlative coded improves the BER as the $E_s/N_0$ gets higher. It can be seen that correlative coding improves the BER performance of MIMO-OFDM system with the presence of frequency offset in the channel. At probability of $10^{-4}$, The $E_s/N_0$ needed by 2x2 and 2x1 MIMO configuration is reduced by 2 dB compared to the conventional method. The MRC diversity technique managed to reduce up to 4 dB at the same BER performance.

## V. Conclusion

In this paper, MIMO-OFDM systems with correlative coding were studied. ICI was deliberately introduced in a controlled manner through the correlative coding polynomial functions. The effectiveness of correlative coding in improving BER of MIMO-OFDM system with the presence of constant frequency offset was demonstrated. At BER of $10^{-4}$, the $E_s/N_0$ needed managed to be reduced by 2-4 dB when correlative coding was used in MIMO-OFDM with the presence of frequency offset, $\varepsilon$=0.15 in the channels.

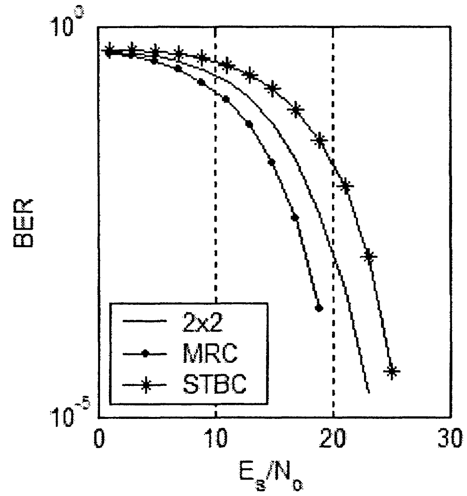


Fig.1. Correlative coded MIMO-OFDM systems

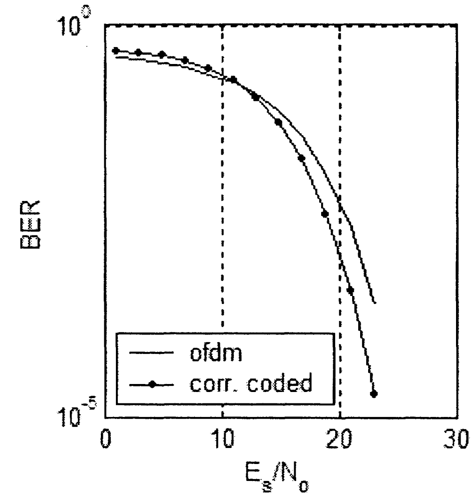


Fig. 2. BER comparison of 2x2 MIMO-OFDM system with respect to $\varepsilon$=0.15

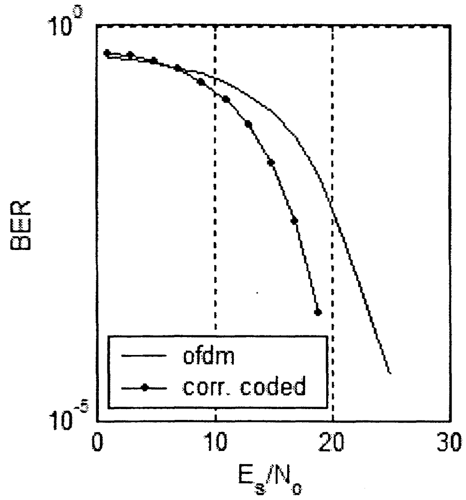


Fig. 3. BER comparison of MRC-OFDM system with respect to $\varepsilon$ = 0.15

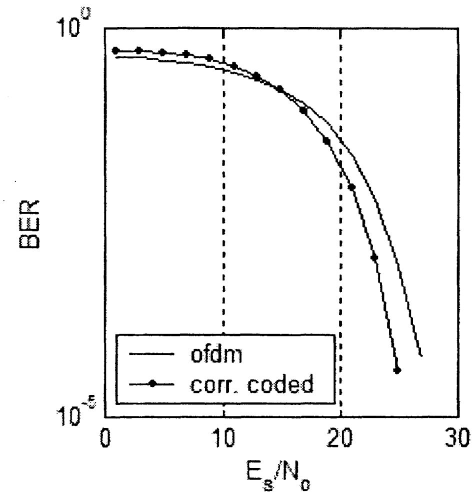


Fig. 4. BER comparison of STBC-OFDM system with respect to $\varepsilon$ = 0.15

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