OPTIMIZED DESIGN OF HOME REFUELING APPLIANCE (HRA) WITH NOVEL MULTI-STAGE SYMMETRICAL WOBBLE-PLATE COMPRESSOR

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Abstract

A new multistage symmetrical wobble-plate compressor has been developed with a working pressure from 3 to 250 bars for compressing natural gas for vehicles (NGV). This paper describes the procedures for obtaining the optimum number of stages for this compressor functioning as a Home Refueling Appliance (HRA), refilling the vehicle at a slow rate of 1 Nm^3/hr. Several parameters such as the desired pressure, an appropriate torque and work, pressure ratio, and compact compressor size were taken into consideration. In this HRA design, the number of stages considered were from three to six. Finally, the number of stages for optimum design at a pressure ratio of 2.913 was found to be four-stage arrangement.

Keywords: Symmetrical Wobble-Plate Compressor, High Pressure, Natural Gas

1. Introduction

In general the overall size of a multi-stage wobble-plate compressor depends very much on the flow-rate and the working pressure desired. The flow-rate affects the cylinder diameter and the piston displacement. The pressure governs the choice of the number of stages required. Furthermore a limitation on the overall dimensions of the compressor requires an optimization of these parameters.

Given a specified number of stages and piston clearance, the relationship between the pressure ratio, the discharge temperature and efficiency could be determined. If the calculated discharge temperature is too high, the numbers of stages need to be increased. During preliminary sizing calculations, the isentropic discharge temperature could be used, but if a certain number of stages create a marginal situation, the discharge temperature should be estimated more accurately. As a first estimate, it can be assumed that equal pressure ratios are used for all stages. In practice it is often better to take a higher pressure ratio in the low pressure stages and unload the more critical higher-pressure stages a little [1].

As the number of stages increase, the pressure losses in the valve and piping would also increase, thus reducing the efficiency. Also, the cost of the compressor would increase as the number of stages increases due to the need for additional compressor cylinders, coolers and piping [2,3].

In this particular case, a symmetrical wobble-plate compressor was to be used to compress natural gas at an operating suction pressure of 50 psi (3 bars), a discharge pressure of 3600 psi (250 bars), and with a capacity of 1 Nm^3/hr [4,5]. There are several parameters involved in getting the optimum number of stages i.e. the desired pressure, an appropriate torque and work, pressure ratio, and compact compressor size.

In this design process the number of design stages considered were from three to six, and the optimum number of stages could be obtained at a specified pressure ratio [6, 7].

2. Pressure Ratio

If the compression ratio is increased, the final compression temperature rises and the volumetric efficiency of the compressor decreases. A high compression temperature affects the operation of the delivery valve, diminishes the lubrication properties of the oil. The maximum compression ratio for small single-stage compressors is normally taken to be eight, and for large machines, five [8]. In the case of multi-stage compression, gas leaving the first stage is again cooled in an intercooler before being passed to the second stages, and this process is repeated until the required pressure is reached. By cooling the gas between stages, the compression process approaches the isothermal conditions. Thus the final temperature is appreciably lowered and the work required for the compression is reduced [9].

The total work done is the sum of the low-pressure and the high-pressure work [2,9].

$$-W' = \dot{m}RT_1 \frac{n}{n-1}\left[\left(\frac{P_a}{P_s}\right)^{\frac{n-1}{n}} - 1\right] + \dot{m}RT_1 \frac{n}{n-1}\left[\left(\frac{P_d}{P_a}\right)^{\frac{n-1}{n}} - 1\right] \quad \ldots(1)$$

or

$$-W' = \dot{m}RT_1 \frac{n}{n-1}\left[\left(\frac{P_a}{P_s}\right)^{\frac{n-1}{n}} + \left(\frac{P_d}{P_a}\right)^{\frac{n-1}{n}} - 2\right] \quad \ldots(2)$$

A minimum total work could be obtained by differentiating the expression in the brackets in Eqn. (1) with respect to P_a and setting to zero. After substituting $z = \left(\frac{n-1}{n}\right)$, we obtain:

$$\frac{zP_a^{z-1}}{P_1^z} - \frac{zP_d^z}{P_a^{z+1}} = 0, \quad \ldots(3)$$

$$P_a^{2z} = P_s^z P_d^z \quad \ldots(4)$$

Extracting the root leads to:

$$P_a = \sqrt{P_s P_d} \text{ or } \frac{P_a}{P_s} = \frac{P_2}{P_a} \quad \ldots(5)$$

The pressure ratio at all stages should be the same in order to maintain a lower total power input. Thus, Eqn. (2) becomes:

$$-W' = 2\frac{n}{n-1}\dot{m}RT_1\left[\left(\frac{P_a}{P_s}\right)^{\frac{n-1}{n}} - 1\right] \quad \ldots(6)$$

If each stage of the two-stage compressor is sub-divided into two further stages, then a four-stage compressor is obtained. The total work of the new first and second stages will be minimum if the compression ratios are the same in both stages. This also applies to the new third and fourth stages. Since the compression ratio in both stages of the original two-stage compression was the same, the compression ratios of all four stages will also be equal too. Let us now illustrate the adiabatic four-stage compression on the T-s diagram, Fig. 2. Assuming perfect inter-coolers, then the final compression temperatures for all stages are equal. Hence the entropy changes in all stages are also equal i.e. $(s_0 - s_1) = (s_1 - s_2) = (s_2 - s_3) = (s_3 - s_4)$.

In general for a compressor with n stages,

$$\frac{P_1}{P_0} = \frac{P_2}{P_1} = \ldots = \frac{P_n}{P_{n-1}} = \sqrt[n]{\frac{P_n}{P_0}} \quad \ldots(7)$$

$$P_n = P_{n-1} \times (\text{Pressure Ratio}) \quad \ldots(8)$$

Eqn. 7 will then be used to determine the level of compressor pressure in each stage [3].

Figs. 1 and 2 show the increase of temperature in the compressor from T_s to T_a due to the compression. If there is a cooling system (inter-cooler) after stage 1 the temperature T_1 will be reduced to T_1'. This is then the intake temperature before stage 2, which will increase to become T_2 after stage 2 compression processes . This would repeat accordingly for the next stages. Generally, the discharge temperature after each stage could be determined by using the equation below:

$$T_n = T_{n-1}\left(\frac{P_n}{P_{n-1}}\right)^{\frac{n-1}{n}} \quad \ldots(9)$$

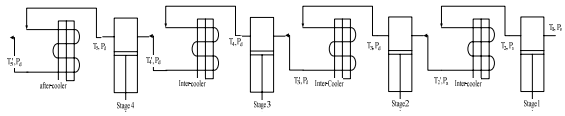

Fig. 1 Schematic diagram showing inter-cooling between stages, and after-cooling at the end

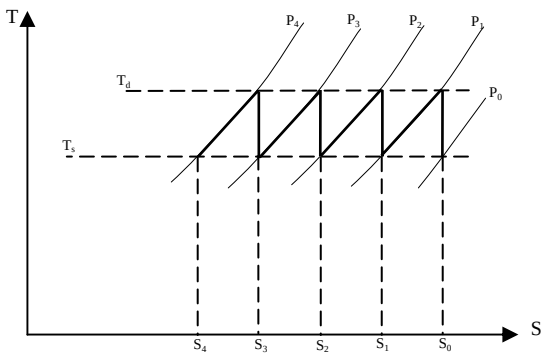

Fig. 2 T-s diagram of the adiabatic four-stage compression

3. Piston Displacement of the Symmetrical Wobble-Plate Compressor

The governing equations for piston, connecting rod and wobble-plate motions could be determined by using standard coordinate transformation method derived for the anti-rotation mechanism. These equations are written and used to demonstrate the effects of anti-rotation mechanism and variation in stroke length, Eqn. 10.

Wobble-plate compressors exhibit complex motion compared to crankshaft reciprocating compressors and swash-plate compressors. In swash-plate compressors, piston movements are sinusoidal and they are affected by the plate movement in shaft axis direction (z-axis) only. However, in a wobble-plate compressor the wobble-plate movements in all the x-, y- and z-axes will affect the movements of the piston due to the usage of ball-joints on the connecting rods to link the plate and the pistons.

Wobble-plate kinematics is obtained from the movement of the connecting rods. Connecting rod ball-joint at the wobble-plate side will show the movement of the plate while the connecting rod ball-joint at the piston side will show the movement of the piston. The plate is constrained from rotating with the rotor and shaft by the anti-rotating mechanism. This constraint is needed to prevent the

connecting rods from being tangled and tied together.

The piston stroke is given by:

$$l = z_{tmax} - z_{tmin} \quad \ldots(10)$$

where:

$$z_t = [t_w\cos\gamma + R_w\sin\theta_n\sin\gamma]\cos\alpha + R_w\cos\theta_n\sin\alpha + l_{cr}\cos\phi_n \quad \ldots(11)$$

At all piston locations, all the values of R_w, R_p, t_w, l_{cr}, θ_n, and $\tilde{\alpha}$ are the same. The only difference is in the value of piston angular location, θ_n. This difference cause each piston set to have a different value of stroke [10].

4. Cylinder Sizing

4.1 Wobble-plate Motion

There are several factors which influence the determination of cylinder size, such as the radius of wobble-plate, tilting angle, stroke of compressor, speed of compressor, and space between cylinders. All of these factors need to be determined simultaneously as they affect each other intrinsically.

The determination of the symmetrical wobble-plate cylinder dimensions was very unique. One of them is the availability of space for cylinders arrangement as described in Fig. 3.

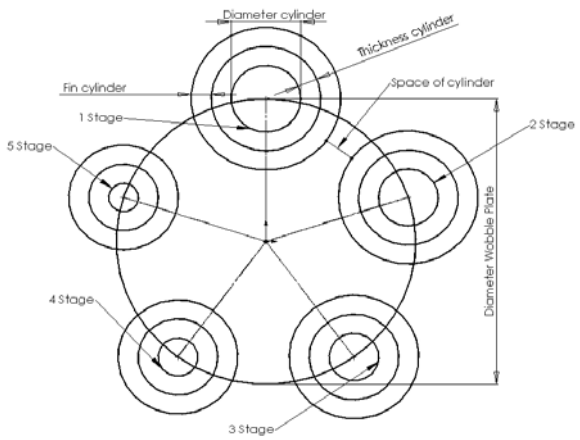

Fig. 3 Cylinder configuration

Based on Fig. 3 above, it could be seen that the inner diameter of the cylinder alone above is not enough to determine the arrangement of the cylinders. The thickness of the cylinder and the diameter of the fin would also have to be considered.

The next step in to determine the optimum wobble-plate tilting angle. This process will repeat the same calculations that were required to determine the number of stages.

To start off the inner diameter of the first stage is determined initially based on the compressor capacity, which is given by Eqn. 12.

$$\dot{V} = D \cdot l \cdot N \quad \ldots(12)$$

l is the stroke of the compressor and it could be calculated by using Eqn. 10,

D is the cylinder diameter, and

N is the compressor speed, in RPM

To obtain V_n, the cylinder volume of stage n,

$$V_1 = \frac{\pi}{4} D_1^2 \cdot l \quad \ldots(13)$$

The diameters of the piston for the higher stages are given by the equation below:

$$V_n = V_{n-1}\left(\frac{P_n}{P_{n-1}}\right)^{\frac{1}{\gamma}} \quad \ldots(14)$$

By obtaining the cylinder volumes using the Eqns. 13 and 14, the diameter of the second stage cylinder may then be determined [8,9]. After the cylinder dimensions are obtained the availability of space to put that cylinder can then be checked.

4.2 Force Acting on the Piston

The design of the wobble-plate mechanism of the compressor could be started with the forces acting on the pistons. One of the causes of the force on the piston is the pressure of the gas. The gas pressure inside the cylinder varies with the angle of rotation of the shaft, corresponding to the suction, compression and discharge processes.

For the case of the multistage compressor the forces produced in each piston have variations appropriate with the pressure and the cylinder width, since the cylinders are of different geometries and also pressures. This is unlike the case of single stage compressors which have the same the force on the pistons due to the same geometry and pressure in every cylinder.

Considering one wobble-plate position at a time, it can be seen that the pressure in every cylinder will be different because there is a different process in each cylinder. For example the first cylinder may be in the suction condition while the next cylinder (the second) in compression and discharge process.

By knowing the pressure and the dimensions of the cylinder, the force on the piston can be determined using Eqn. 15:

$$F = P.A \quad \ldots(15)$$

Compressors for wide ranges of applications tend to run at about the same piston speed. Thus, compressors with a long stroke tend to run slower than those with a short stroke. Further, short stroke compressors tend to be of lighter construction with lower allowable loads.

For the best efficiency and reliability at the expense of increased cost, a piston speed at the low end of the normal range will be used. The compressor speed and the stroke will then be determined by the horsepower requirement.

A low horsepower application will require a light, low-stroke, high-speed compressor. A high horse power application will require a heavy, long-stroke, low-speed compressor. If possible, large compressors are directly coupled to the driving shaft. Thus the speed range of available motors may influence the selection of the compressor.

4.3 Torque in Compressor

There are several reasons that require for the torque in the compressor to be determined. These include to optimize the use of energy while the compressor is operating. Also, torque is required for selection of the appropriate motor to be used for running the compressor. An optimum torque will make the compressor perform better.

Torque in a compressor can be determined after knowing the force in each piston and the distribution of force at each angular position around the rotating shaft. The process of determining the torque on the symmetrical wobble-plate is very different from that for other kinds of compressors. This process can be described using force and torque diagrams as follows.

Fig. 4 shows a sectioned view of a single piston within the cylinder block as it operates within the compressor. In this view, the reaction force between the wobble-plate and the *n*th slot is shown by the symbol R_n. It can be seen that the component of this force in the downward direction is given by [11]:

$$F_n = R_n \sin\alpha \qquad \ldots(16)$$

where α is the wobble-plate angle.

Eqn. 16 represents the downward force exerted on the shaft by the *n*th piston. Summing these forces for all pistons within the compressor yields the total force exerted on the shaft in the downward direction as follows:

$$F = \sum_{n=1}^{k} R_n \sin\alpha \qquad \ldots(17)$$

where k is the total number of pistons within the compressor.

The torque on the shaft is generated by the downward component of the reaction force between the *n*th piston and the wobble-plate, multiplied by the distance of the *n*th piston away from the *z*-axis. From Fig. 5, it can be seen that the piston is located a distance away from the *z*-axis by the expression $r \cdot \cos\theta_n$, where *r* is the piston pitch radius and θ_n is the circular position of the *n*th piston. Multiplying this distance by the right-hand side of Eqn. 16 yields the following result for the torque exerted on the shaft by the *n*th piston:

$$T_n = R_n \sin\alpha \cdot r\cos\theta_n \qquad \ldots(18)$$

Summing this torques for all pistons within the compressor yields the following result for the total torque exerted on the shaft:

$$T = \sum_{n=1}^{k} R_n \sin\alpha \cdot r\cos\theta_n \qquad \ldots(19)$$

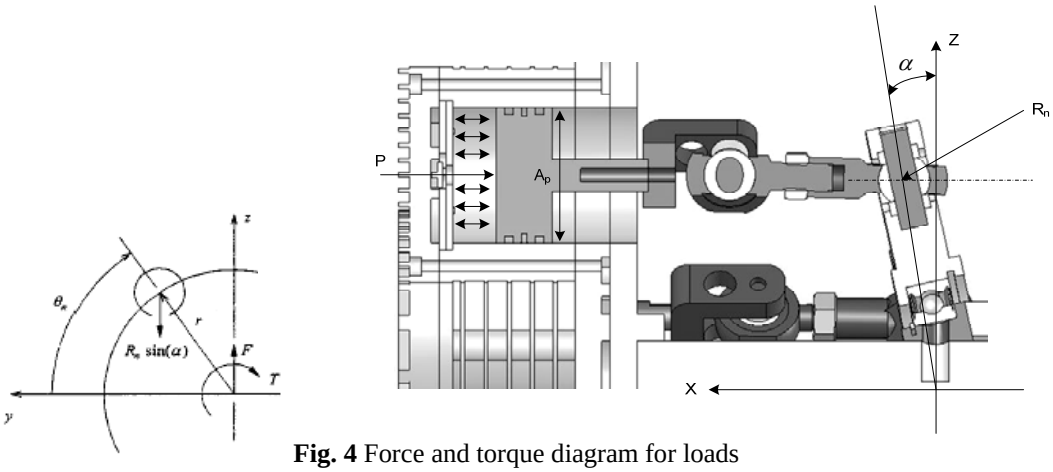

Fig. 4 Force and torque diagram for loads exerted on the shaft

Fig. 5 serves to graphically illustrate the total downward force given by Eqn. 17 while the total of torque given by Eqn. 19.

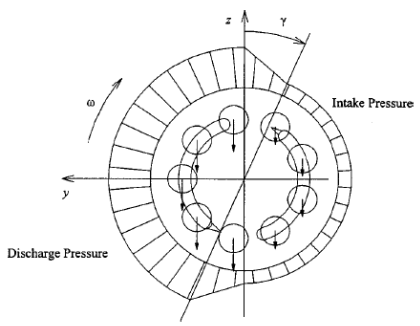

Fig. 5 Piston pressure profile

The reaction between the *n*th piston and the wobble-plate may be determined by summing the forces which are acting on the piston in the *x*-direction and setting them equal to the piston's time rate-of-change of linear momentum. Writing this equation, and rearranging terms, yields the following result for the reaction force between the *n*th piston and the wobble-plate [11]:

$$R_n = \frac{M_p.\ddot{x}_n + P_n.A_p}{\cos\alpha} \quad \ldots(20)$$

where M_p is the mass of a single piston, $\ddot{x}_n$ is the piston acceleration in *x*-direction, P_n is the fluid pressure within the *n*th piston chamber, and A_p is the pressurized area of a single piston.

The general expressions for the downward force and torque exerted on the shaft are given in Eqns. 15 and 17. These equations describe the instantaneous loads that are exerted on the shaft, which tends to oscillate at certain dominant frequencies depending upon the number of pistons within the compressor and the rotational speed of the shaft. If the compressor is designed with a sufficiently large number of pistons, the amplitude of the oscillations can be reduced and the frequency of the oscillations can be increased.

Using Eqns. 21 and 22, the average quantities of force and torque may be computed using the integral-averaging technique. This technique yields the following general forms:

$$\overline{F} = \frac{N}{2\pi}\int_0^{2\pi} R_n.\sin\alpha.d\theta_n \quad \ldots(21)$$

$$\overline{T} = \frac{N}{2\pi}\int_0^{2\pi} R_n.\sin\alpha.r\cos\theta_n.d\theta_n \quad \ldots(22)$$

The results of Eqn. 21, and performing the discontinuous integration of the pressure terms, yield the following for the average force and torque exerted on the shaft:

$$\overline{F} = \frac{NA_p(P_d + P_i)\tan\alpha}{2} \quad \ldots(23)$$

$$\overline{T} = \frac{NA_p(P_d + P_i)r\tan(\alpha)}{\pi} \quad \ldots(24)$$

5. Results and Discussions

The process of choosing the number of stages, and the operations involved in each stage have been described in this paper. For this optimization the numbers of stages considered were from three to six, with inter-cooling between the stages and after-cooling after the last stage. From here the best number of stages was determined for this compressor design.

The pressure ratio and the pressure in each stage for the various cases of numbers of stages chosen are given in Table 1.

In designing the symmetrical wobble-plate compressor, the optimum number of stages will be chosen by considering the overall performance of all stages (hence, cylinders and pistons) together.

Table 2 The maximum force at every cylinder for every stage

Piston	Max force for 3 stages (N)	Max force for 4 stages (N)	Max force for 5 stages (N)	Max force for 6 stages (N)
1	1082.5	1022.2	612.0	883.6
2	1465.7	1283.0	734.1	1028.2
3	1984.5	1610.5	880.5	1196.4
4	-	2021.5	1056.1	1392.2
5	-	-	1266.7	1619.9
6	-	-	-	1884.9

The optimum number of stages is influenced by several factors such as the size of the cylinders. The radius of the centers of the cylinders (from the shaft), pressure ratio, shaft speed (RPM), piston force and torque. However these factors are themselves inter-dependent, for example the piston force is dependent on the cylinder diameter, and the torque is dependent on the radius of the cylinder centers.

The force on each piston is given in Table 2 and shown in Fig. 6. The graph shows that the maximum loading for each piston happens in the three-stage arrangement, and the minimum in the five-stage arrangement.

Fig. 8 shows the total compressor force as the shaft rotates. From here the total torque as shown in Fig. 9 was determined.

However, the sizing of the cylinder is also limited by a minimum diameter, due to practical manufacturing and material strength considerations. This is especially true for the last cylinder, where the diameter (and hence the corresponding material thickness) is smallest but yet the pressure is highest.

Yet another important criterion is the space allowance between cylinders to allow air passage to the fins for effective cooling, and also for ease of installation.

All these considerations needed to be balanced, and the optimum number was then selected subjectively. This is especially important as often two or more criteria would lead to conflicting results. For example, an increase in the number of stages would be preferable in the sense of reducing the pressure ratio and hence the force acting, but this would also cause the radius of the cylinder centers to increase thus increasing the torque, which is undesirable, as a bigger motor would be then needed.

Table 1 Pressure ratio and pressure each stages

Case	No of Stages	Pressure Ratio	P_1 (psi)	P_2 (psi)	P_3 (psi)	P_4 (psi)	P_5 (psi)	P_6 (psi)	P_7 (psi)
1	3	4.160	50	208.0	865.4	3600.0	-	-	-
2	4	2.913	50	145.6	424.3	1235.9	3600.0	-	-
3	5	2.352	50	117.6	276.6	650.7	1530.5	3600.0	-
4	6	2.039	50	101.9	208.0	424.3	865.3	1765.0	3600.0

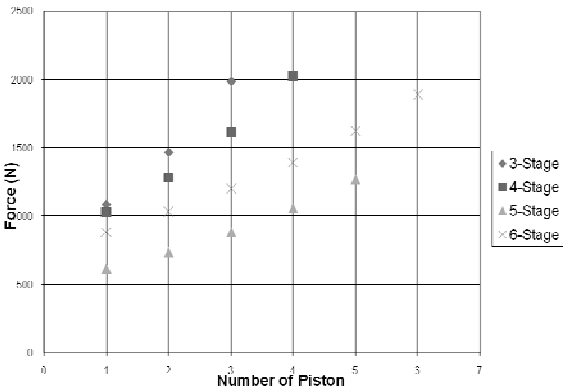

Fig. 6 Load in each piston for each number of stages

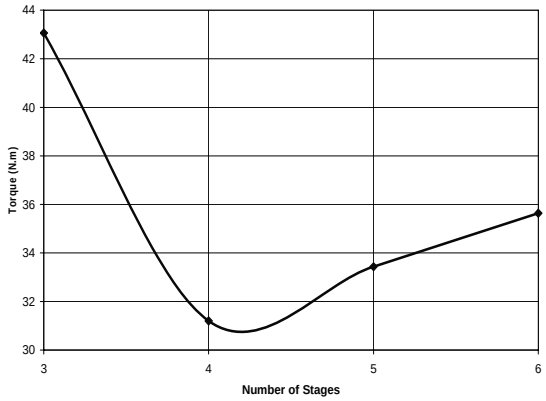

Fig. 7 Maximum torque on the compressor

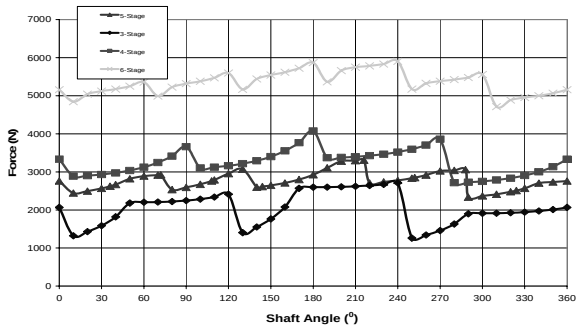

Fig. 8 Compressor total force in each shaft angle rotation with the number stage of compressor

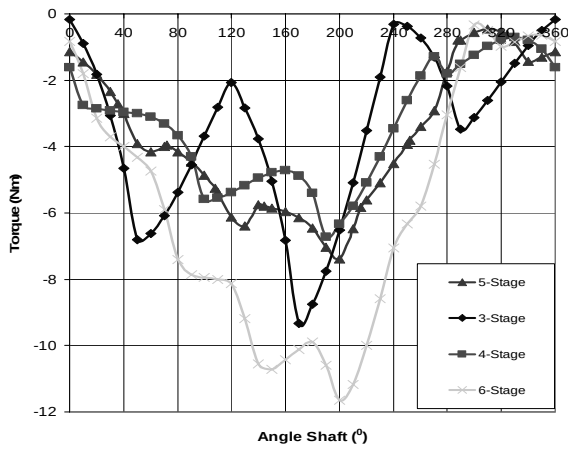

Fig. 9 Total torque at the compressor shaft angle rotation within number stage of compressor

Fig. 9 shows the total torque exerted by the compressor as the shaft rotates for each arrangement. It can see from here that the four-stage arrangement exerted a marginally more desirable torque profile, with a lower overall average torque.

Now, a compressor which requires a smaller torque is considered to be better compared to the one with a bigger torque. This can be related to the energy or the work that done by compressor as shown in Fig. 10. As the torque decreases, the work done by the compressor also decreases, thus decrease the input power. That means, compressor required a less power motor to drive it, hence small and cheaper. A smaller motor would also be advantageous from the size point of view.

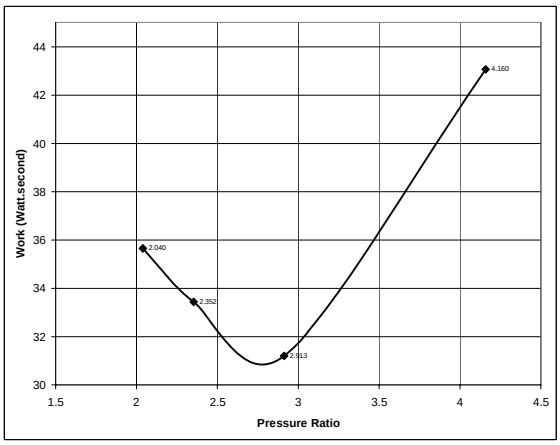

Fig. 10 Work of compressor vs pressure ratio

From here, and looking at Fig. 7, it can be seen that the minimum torque magnitude occurred for the four-stage arrangement. Thus it was decided that the optimum number of stages for this specific requirement was four. This would thus lead to the final specifications for the compressor as shown in Table 3.

Table 3 Optimum specification of symmetrical wobble-plate compressor

Input Data	Calculated Value			
	1 Stage	**2 Stage**	**3 Stage**	**4 Stage**
Cylinder Diameter	36mm	23.6mm	15.5mm	10.2mm
Suction Pressure	50psi	145.7psi	424.3psi	1235.9psi
Discharge Pressure	145.7psi	424.3psi	1235.9psi	3600psi
Stroke	8.7 mm			
Fluid	Natural Gas			
Rotating Speed	1000 rpm			
Tilting Angle	5°			
Pressure Ratio	2.912			
Capacity	1 Nm^3/hr			

5. Conclusions

For a new multistage symmetrical wobble-plate compressor with working pressure from 3 to 250 bars and compressing natural gas at 1 Nm^3/hr, the analysis showed that the number of stages for optimum design at a pressure ratio of 2.913 was four. The optimum design of a symmetrical wobble-plate compressor is actually influenced by several inter-dependent parameters. A crucial determining factor in this particular case was the maximum torque produced. Also, it should be noted that in this paper the effect of temperature, tilting angle, revolution, stroke and efficiency were not considered due design constraints.

5. Acknowledgements

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