WARPING CONSTANT OF STEEL SECTION

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……. For The God, Creator of the world…..

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ABSTRACT

This project report is aimed to study the warping behaviour of steel section. The main objective of this study is to obtain relationship of warping displacement and warping constant. In BS5950, there is already a basic derivation of warping constant with its relationship with the geometry of cross section. Somehow, this formula is unsuitable to be applied for some section member such as corrugated I-beam. Modifying the formula needs a lot of study and research; it is not economic both in cost and time. To overcome this, there comes a suggestion. There might be a relationship between warping displacements of member with different cross section as they are using the same material. By developing a relationship between warping constant and warping displacement, it is easy to find the warping constant of abnormal section with condition that we know its warping displacement. But, there is no reference available for the relationship governing the warping constant and warping displacement. In order to achieve the objectives, finite element analysis is carried out. Each parameter that affects the warping displacement and warping constant is under the consideration and been studied.

ABSTRAK

Matlamat utama projek sarjana ini adalah untuk mengkaji kelakukan keratan besi. Objektif utama projek sarjana ini adalah untuk mengkaji hubungan antara nilai lendingan dengan kelakukan lendingan. Dalam BS5950, sudah terdapat formula mengenai nilai lendingan. Tetapi, formula ini bukan sesuai digunakan untuk semua keadaan seperti keratan web trapezoid. Jadi, untuk mendapatkan nilai lendingan, pengubahsuaian harus dilakukan. Pengubahsuaian ini adalah memakan masa dan kos. Untuk mengatasi masalah ini, hubungan antara nilai lendingan dan kelakukan lendingan harus dikaji. Dengan adanya hubungan sebegini, ini boleh menyenangkan kerja mencari nilai lendingan sekiranya kelakukan lendingan boleh didapati. Namun, setakat kini, belum dapat sebarang rujukan mengenai hubungan antara nilai lendingan dan kelakukan lendigan. Untuk mencapai objektif projek sarjana ini, finite element analisis digunakan. Semua parameter mempengaruhi nilai lendingan dan kelakukan lendingan telah dikaji.

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NOTATIONS

b	-	Width of flange
D	-	Depth of beam
t_w	-	Thickness of web
t	-	Thickness of flange
H_s	-	Distance between shear center of flanges
E	-	Young's modulus
H	-	Warping constant
J	-	Torsional constant
λ	-	Slenderness of a beam
u	-	Buckling parameter
v	-	Slenderness factor
x	-	Torsional index
n	-	Slenderness of correction factor
λ_{LT}	-	Equivalent slenderness
γ	-	A factor
M_b	-	Buckling resistance moment
p_b	-	Bending strength
θ	-	Angle of twist
p_y	-	Design strength
L	-	Length of section
T	-	Torque
G	-	Shear modulus of material
S_x	-	Plastic section modulus

I_x	-	Second moment of area about the x-axis
I_y	-	Moment of inertia of a section about y-axis
M_{cr}	-	Critical moment
V_f	-	Shear force across the width of flange
$\overline{M}$	-	Equivalent uniform moment
m	-	Equivalent uniform moment's factor
L_E	-	Effective length
r_y	-	Radius of gyration of the minor axis
A	-	Cross-sectional area of a member
T_w	-	Warping restraint torsion
ω	-	Warping displacement
Δx	-	Displacement in the x-direction
ω_s	-	Warping function
$\overline{\omega_s}$	-	Average value of ω_s over the entire cross section
ε_z	-	Axial strain
σ_s	-	Normal stress
τ	-	Shear stress
i	-	Initial point of an element
H_l	-	Warping rigidity
j	-	End point of an element
n	-	Refers to nth element
L_{ij}	-	Length of the element ij
ρ_o	-	Distance from the shear center to an element under consideration
ω_o	-	Unit warping with respect to the shear center
W_n	-	Normalized unit warping of the cross section

CHAPTER I
INTRODUCTION

1.1 Introduction

The use of steel structures has become increasingly popular in different fields of building technology including wall structures, framing systems, trusses and shielding materials.

For years, numerous researches had been conducted to investigate various strength properties of steel sections. One of the studies is on lateral torsional buckling strength of beams. In the study to develop design formulae and related curves, one of the important element is the torsional behaviour of the section. For sections with uniform web throughout the length, the torsional properties i.e, torsional constant and warping constant were defined in Appendix B BS5950: Part 1: 1990.

However, the empirical based design curve in BS5950 is represented by modified form of the Perry equation using a beam slenderness parameter λ_{LT}. The relationship between slenderness, warping constant and their influence in designing a steel member will be discussed in details in this study. The purpose of this study is to examine the warping behaviour of the beams with uniformed web profile i.e, I-beam subjected to pure torsion and lateral torsional buckling moment using finite element modeling.

1.2 Background of Study

Warping constant is an very important parameter in steel designing i.e, in lateral torsional buckling and moment resistance for beams. From BS5950, the buckling resistance moment is given as,

$$M_b = p_b S_x \tag{1.1}$$

where p_b is the bending strength.

S_x is the plastic section modulus.

The bending strength, p_b is related with the equivalent slenderness, λ_{LT}, defined as,

$$\lambda_{LT} = nuv\lambda \tag{1.2}$$

where n is slenderness correction factor,

u is the buckling parameter,

v is the slenderness factor,

λ is the slenderness of the beam, defined as $\lambda = \dfrac{L_E}{r_y}$ (1.3)

L_E is the effective length of beam

r_y is the radius of gyration about minor axis

To acquire the slenderness factor, it is needed to know the torsional index, x. Both buckling parameter, u and torsional index, x are dependant on the geometry of the cross section. They are related to warping constant, H, as,

$$u = \left[\frac{I_y S_x^2 \gamma}{A^2 H} \right]^{0.25} \quad \text{and} \quad x = 1.132 \left[\frac{AH}{I_y J} \right]^{0.5} \tag{1.4}$$

where γ is a factor, defined as

$$\gamma = 1 - \frac{I_y}{I_x} \tag{1.5}$$

I_x is the second moment area about the major axis

I_y is the second moment area about the minor axis

H is the warping constant, defined as

$$H = \frac{h_s^2 t_1 t_2 b_1^3 b_2^3}{12\,(t_1 b_1^3 + t_2 b_2^3)} \quad \text{(Refer Figure 1.1)} \tag{1.6}$$

J is the torsional constant, defined as

$$J = \frac{1}{3}\,(t_1^3 b_1 + t_2^3 b_2 + t_w^3 h_w) \quad \text{(Refer Figure 1.1)} \tag{1.7}$$

t_1, t_2 are the flange thickness

b_1, b_2 are the flange width

t_w is the web thickness

h_w is the web depth

h_s is the distance between shear center of flanges

S_x is the plastic modulus about the major axis

A is the area of cross section

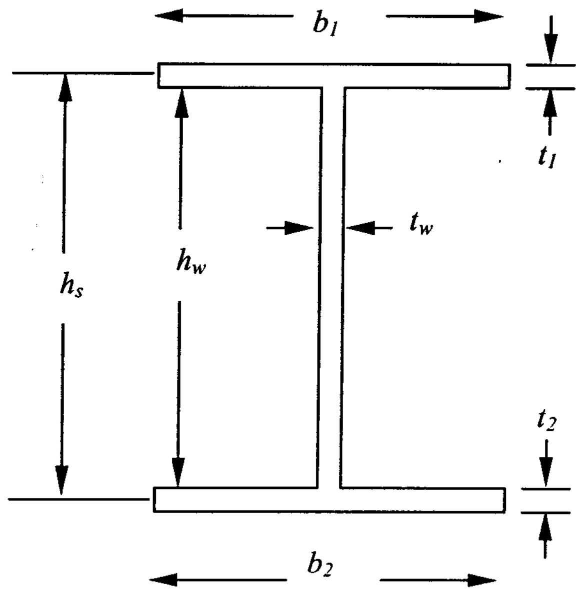

Figure 1.1: Dimension of I-beam.

From these derivations, small changes in warping constant will influence the design of steel structures. By using different cross section, the relationship of equivalent slenderness, λ_{LT} and warping constant, H and influence of warping constant, H in moment resistance during designing will be studied. As an example, take a beam with effective length of 15 m, subjected to a uniformly distributed load

The equivalent slenderness can be obtained from equation 1.2. Subsequently, p_b is obtained from BS5950. Table 1.1 shows the parameters for the calsulation of the lateral torsional buckling strength. The warping constant is shown in the table for comparison.

Serial Size	n	u	x	v	r_y (cm)	λ	λ/x	λ_{LT}	H (dm^6)	p_b (N/mm^2)
254 x 102 x 22	1.0	0.85	36	0.96	2.1	73.171	2.038	59.988	0.018	213.03
254 x 102 x 25	1.0	0.86	31	0.95	2.1	70.093	2.232	57.230	0.023	220.2
254 x 102 x 28	1.0	0.87	28	0.93	2.2	67.568	2.457	54.857	0.028	226.34
254 x 146 x 31	1.0	0.88	29	0.97	3.4	44.776	1.523	38.177	0.066	266
254 x 146 x 37	1.0	0.89	24	0.97	3.5	43.228	1.779	37.084	0.086	268.42
254 x 146 x 43	1.0	0.89	21	0.96	3.5	42.735	2.025	36.472	0.103	269.77

Table 1.1: the warping constant of various sections and the corresponding lateral torsional buckling strength of 15 m strength.

From Table 1.1, it is seen that with the increase in warping constant, equivalent slenderness will decrease as shown in Figure 1.2. And, the influence of warping constant on the lateral torsional buckling resistance is shown in Figure 1.3. Lateral torsional buckling resistance will increase with the increase of warping constant.

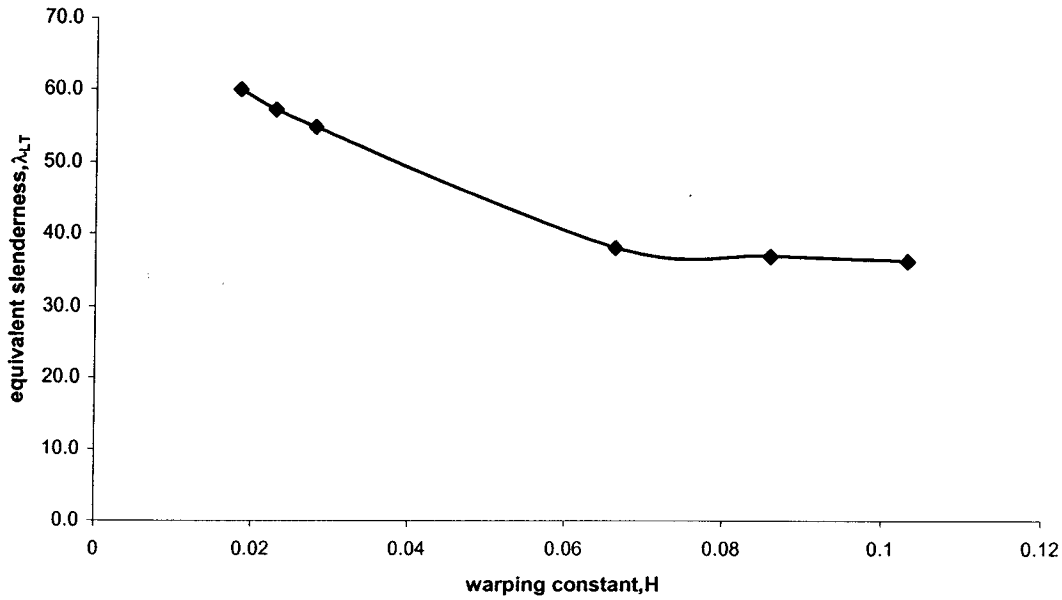

Figure 1.2: Relationship of warping constant with equivalent slenderness.

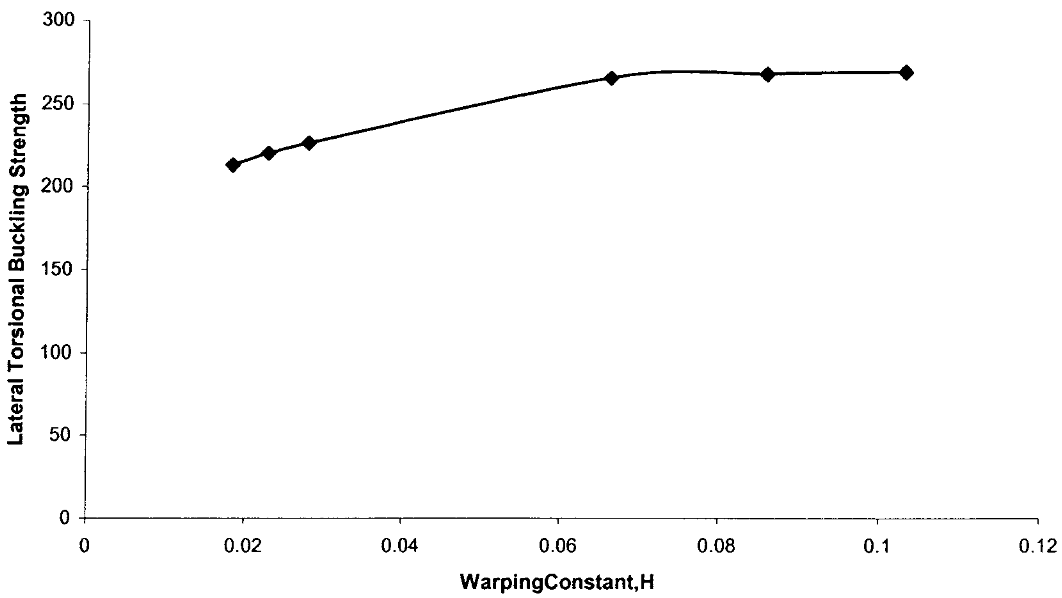

Figure 1.3: Relationship of warping constant with lateral torsional buckling strength.

Thus, it is significant to study in more details about the warping constant. Warping constant, H is important both for safety of structure and to increase efficiency in term of the materials used.

With the increase popularity of the use in steel in construction, various sizes and shapes of members are being used. Taking a short brief of our previous research and development (R&D) of material properties, we find that our R&D is much far behind needs of today's construction field. Most of the time, we found difficulties to derive the warping constant of some non-uniformed cross section as shown in Figure 1.4.

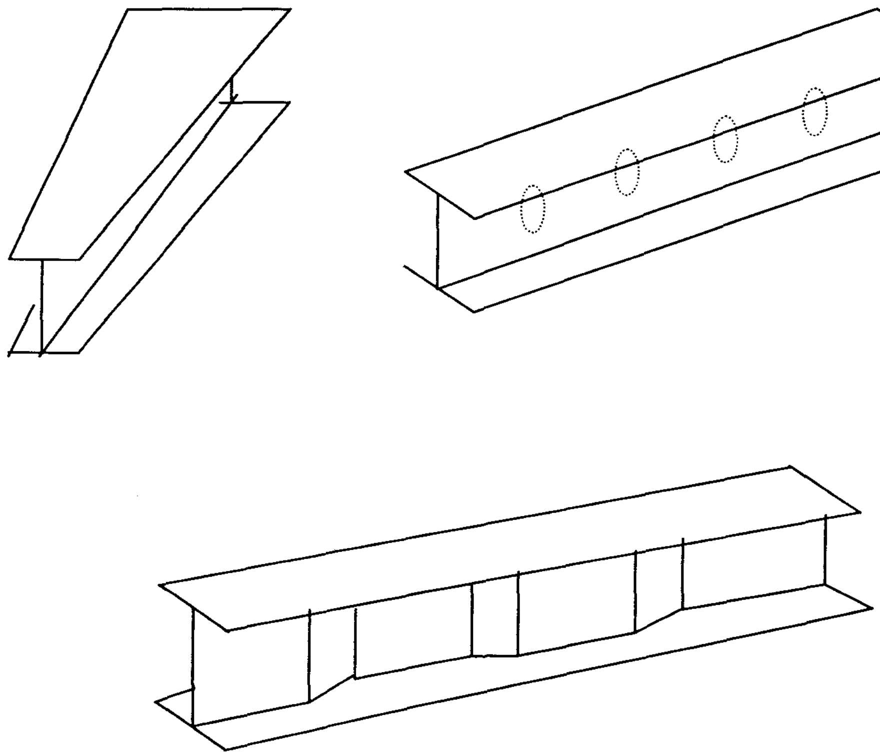

Figure 1.4: Non-uniformed cross section.

In BS5950, there is already a basic derivation of warping constant with its relationship with the geometry of cross section. Somehow, this formula is unsuitable to be applied for some section member such as corrugated I-beam. Modifying the formula needs a lot of study and research; it is not economic both in cost and time. To overcome this, there comes a suggestion. There might be a relationship between warping displacements of member with different cross section as they are using the same material. By developing a relationship between warping constant and warping displacement, it is easy to find the warping constant of abnormal section with condition that we know its warping displacement. But, there is no reference available for the relationship governing the warping constant and warping displacement. Hence, this project report is carried out.

1.3 Objectives of Study

The objectives of this study are:

1. The use of finite element method in the study of warping behaviour.
2. To examine the relationship between warping constant and warping displacement of the beams with uniformed web profile subjected to pure torsion.

1.4 Scope of Study

The scope of study in this project report can be divided as,

1. Basic derivation of warping constant and warping displacement.

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