FUZZY MODELLING FOR REBOILER SYSTEM

Rubiyah Yusof , Mohd Faisal Ibrahim, and Marzuki Khalid
Center for Artificial Intelligence and Robotics (CAIRO)
Faculty of Electrical Engineering,
Universiti Teknologi Malaysia,
Jalan Semarak, 54100 Kuala Lumpur, Malaysia.
e-mail: marzuki@utmkl.utm.my

ABSTRACT

Fuzzy system's ability of providing both heuristic knowledge with quantitative and accurate representation has been exploited for identification of nonlinear and complex system. Takagi-Sugeno (TS) Fuzzy System is one of the most popular method used for fuzzy modelling of multi input multi output (MIMO) dynamical system. In this paper, we propose an automatic tuning methods of fuzzy sets for TS fuzzy models using genetic algorithms. The effectiveness of the approach is illustrated by applying the method to a reboiler of batch distillation column. The results show that the proposed system gives a more accurate model than the conventional TS fuzzy model.

Keywords - Fuzzy model, genetic algorithms, nonlinear identification.

1. INTRODUCTION

Fuzzy modelling techniques have been popular for the past years due to its ability to provide not only heuristic knowledge with quantitative but also accurate representation of complex non-linear system.

One of the most popular fuzzy modelling technique is the Takagi-Sugeno (TS) type [1]. The advantage of TS fuzzy system is that it employs mathematical functions as rule consequent part. This structure gives the system the ability to utilize acquired data in an efficient way. However, as in any other fuzzy system, the fuzzy sets are usually determined using trial and error approach. As the model obtained from the TS method is dependent on the membership functions, the choice of the fuzzy sets will affect the accuracy of the model.

Therefore, one of the challenges of improving the accuracy of the fuzzy model is to tune the fuzzy set such that the mean square error between the model and the actual system is minimized. In this paper, Genetic Algorithms (GA) is used for that purpose.

The proposed technique is applied to a reboiler of a batch distillation column used for oleoraisins extraction.

2. THE REBOILER SYSTEM

2.1. Overview on the plant

The reboiler system used in the batch process distillation column consists of a heating jacket tank and a vessel as shown in Figure 1. Silicon oil inside the heating jacket tank supplies heat to the liquid mixture inside the vessel so that the extraction process can occur at desired boiling point of the final product. The volume of silicon oil in the jacket and the volume of the mixture in the vessel are assumed constant, i.e no material loss occurred.

The inputs are temperature and volumetric flowrate of silicon oil flow into the jacket tank $u=[T_{jin}, F_j]^T$, and the outputs are temperature inside the heating jacket tank and temperature at the vessel $y=[T_j, T_v]^T$.

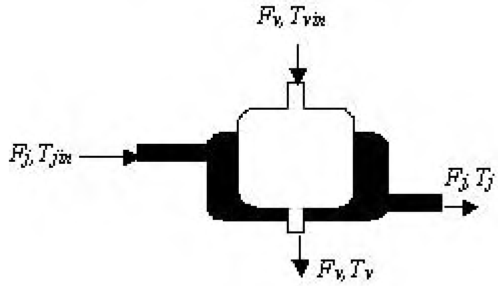

Fig. 1. Reboiler Diagram.

2.2. Mathematical model of the reboiler system

From energy balance derivation, the reboiler system can be described by the following two differential equations [2],[3]:-

$$\frac{dT_j}{dt} = \frac{F_j(T_{jin} - T_j)}{V_j} - \frac{UA(T_j - T_v) + U_a A_a (Tj - Ts)}{V_j \rho_j C_{pj}} \quad (1)$$

$$\frac{dT_v}{dt} = \frac{F_v(T_{vin} - T_v)}{V_v} + \frac{UA(T_j - T_v)}{V_v \rho_v C_{pv}} \quad (2)$$

where the constants are
U : overall heat transfer coefficient, 2000 J/s.m^2.K.
U_a: overall heat transfer coefficient, 200 J/s.m^2.K.
A : heat transfer area (jacket to vessel), 0.43 m^2.

0-7803-8560-8/04/$20.00©2004IEEE

A_a: heat transfer area (jacket to surround), 0.66 m^2.
V_j : jacket volume, 0.053 m^3.
V_v: vessel volume, 0.060 m^3.
ρ_j : silicon oil density, 950 kg/m^3.
Cp_j: silicon oil specific heat capacity, 1800 J/kg.K.
ρ_v: liquid mixture density, 840 kg/m^3.
Cp_v: liquid mixture specific heat capacity, 3000 J/kgK.
F_v : vessel volumetric flowrate, 0.01 m^3/s.
T_s: surrounding temperature, 300 K.
T_{vin}: inlet temperature of vessel, $f(T_v)$.

Simulation is done in LabView in order to obtain input-output data sequences for identification and evaluation of fuzzy model.

3. TAKAGI-SUGENO (TS) FUZZY MODEL

In Takagi-Sugeno (TS) Fuzzy Model, input variables are quantified by the means of linguistic values using membership function as in standard fuzzy system. The major difference of TS fuzzy model is that it uses mathematical function in the consequent part instead of using fuzzy set. The structure can be seen as a combination of linguistic and mathematical regression modelling. The i-th rule of multi input single output (MISO) TS fuzzy model has the following form

$$R_i : \text{IF } x_1 \text{ is } A_1 \text{ AND } x_2 \text{ is } A_2 \text{ AND....AND } x_n \text{ is } A_n \text{ THEN } y_i = f(\mathbf{x}). \tag{3}$$

where,
- $\mathbf{x}$: vector of input variables $(x_1,x_2,...,x_n)$.
- n : total input variables.
- $A_1,A_2,...,A_n$: linguistic terms of input fuzzy set.
- y_i : output of i-th rule.
- $f_i(\mathbf{x})$: mathematical function of i-th rule.

Usually, for simple and practically useful, the function, $f_i(\mathbf{x})$ is in the form of linear equation

$$y_i = \mathbf{a}_i^T\mathbf{x} + b_i \tag{4}$$

where, $\mathbf{a}_i$: vector of parameter of i-th rule.
b_i : scalar offset of i-th rule.

For the fuzzifier, Mamdani MIN operator can be used, and defuzzifiction may be obtained using weighted average method to get the output of the model

$$\text{crisp output, } y = \textstyle\sum_{i=1}^{R}(y_i\mu_i) / \sum_{i=1}^{R}\mu_i \tag{5}$$

where, R is total rules and μ_i is membership value that each y_i holds for the given input. MISO TS fuzzy system can be thought of as a nonlinear interpolator between R-th linear systems [4]. For MIMO model, it can be built by combining j-th MISO model where j depends on number of model output.

In the dynamic modelling, TS fuzzy model can be described as discrete time variant dynamic system. Typically, ARX (auto regression with eXogenous input) model is applied to the function $f_i(\mathbf{x})$.

$$y_i(t+1) = \textstyle\sum_{k=0}^{q}\theta_{a_k}y_i(t-k) + \sum_{k=0}^{p}\theta_{b_k}x_i(t-k) \tag{6}$$

where x(t-k) and y(t-k) are the system input and output regressor. q and p are integers related to the model order.

4. RECURSIVE LEAST SQUARE METHOD

Recursive least square (RLS) is used to obtain the parameters **a** and b in equation 4 and is dependent on the values of the membership functions. Recall MISO TS fuzzy model output in equation 4 and 5. If we define

$$\xi_i = (\mu_i)/(\textstyle\sum_{i=1}^{R}\mu_i) \tag{7}$$

then equation 5 will be

$$y = \textstyle\sum_{i=1}^{R} y_i\xi_i \tag{8}$$

Hence, if we combine equation 4 and 8, then

$$y = \textstyle\sum_{i=1}^{R} (\mathbf{a}_i^T\mathbf{x} + b_i)\xi_i \tag{9}$$

Let

$$\boldsymbol{\xi}(\mathbf{x}) = [x_1\xi_1,...,x_1\xi_R, x_n\xi_1,...,x_n\xi_R, \xi_1,..,\xi_R]^T \tag{10}$$

and

$$\boldsymbol{\theta} = [a_{1,0},...,a_{R,0}, a_{1,n},...,a_{R,n}, b_1,...,b_R]^T \tag{11}$$

then

$$y = \boldsymbol{\theta}^T\boldsymbol{\xi}(\mathbf{x}) \tag{12}$$

Therefore, the RLS algorithm becomes

$$\mathbf{P(s)} = P(s-1) - P(s-1)\,\xi^s(I+(\xi^s)^TP(\xi-1)\,\xi^s)^{-1}(\xi^s)^TP(s-1) \tag{13}$$

$$\boldsymbol{\theta}(\mathbf{s}) = \theta(s-1) + P(s)\,\xi^s(y^s - (\xi^s)^T\theta(s-1)) \tag{14}$$

where s is time index (usually equal to number of data pairs, M) and I is matrix identity. The RLS algorithm is run until the parameter converges or until M times.

5. GENETIC ALGORITHMS (GA)

In this paper, GA is used to tune the parameters of input fuzzy set in antecedent part. GA will search the best configuration of input fuzzy set for each input variable based on the fitness function which in this case is the mean of square error (MSE) between the output of fuzzy model and output of training data. MSE is given by

$$\varepsilon = \frac{1}{M}\textstyle\sum_{i=1}^{M}[Y_i - y_i]^2 \tag{15}$$

where, $\mathbf{Y_i}$ is output from fuzzy model and $\mathbf{y_i}$ is output of training data.

We will limit our discussion here on fuzzy set that is represented by symmetrical Gaussian function. Figure 2 shows an example of 3 Gaussian fuzzy sets.

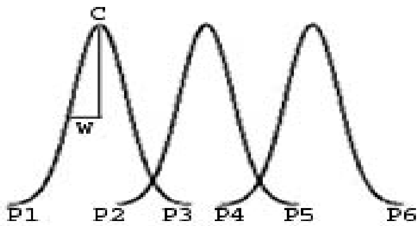

Fig. 2. Gaussian Fuzzy Set.

One step to encode the whole fuzzy set is to directly encode values of P_i as they are in the universe of discourse. But, this would requires constraints for ensuring that P_i is non-decreasing. For instance, if P_2 is bigger than P_3 with a certain gap, it will leave a space where membership value for that space cannot be defined. Therefore, a good alternative is to encode the offset and width between two P_i as shown below:-

P_1	(P_2-P_1)	(P_3-P_2)	(P_4-P_3)	(P_5-P_4)	(P_6-P_5)

Fig. 3. Chromosome.

Once the encoding of the chromosomes has been determined, the algorithm of an automatic tuning of input fuzzy set using GA is as follows:

1. Define string with necessary length to represent parameter of whole fuzzy set.
2. Make random initial population.
3. Evaluate individual fitness for every chromosome.
   i) Use RLS method to define parameter of linear equation with desired fuzzy set configuration.
   ii) Calculate Mean Square Error (MSE) for every chromosome.
4. Perform selection, crossover and mutation operation with appropriate string representation (e.g. binary number) to produce new population.
5. Repeat step 3 and 4 until certain generation (set by user) to get the best chromosome.
6. Using the best chromosome, the best configuration of input fuzzy set function in terms of values c and w will be obtained. The membership functions can then be calculated as follows:

$$\mu(x) = \exp\left(-\frac{1}{2}\left(\frac{x-c}{w}\right)^2\right) \qquad (16)$$

6. APPLICATION TO REBOILER SYSTEM

Figure 4 shows the block diagram of input-output variables for reboiler fuzzy model. From 1000 samples of data, 500 are used for identification of fuzzy parameter and the rest are used for evaluation purpose. Number of generation for GA is 50 and number of population is 20.

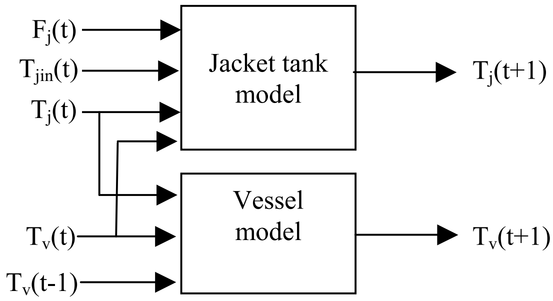

Fig. 4. Block diagram of reboiler fuzzy model.

Since we are only considering MISO fuzzy model, we have divided the system into two MISO system i.e. the jacket tank model and the vessel model. There are 4 and 3 inputs to the jacket tank and vessel models respectively.

The range of values of the inputs and outputs of the fuzzy models are determined. Every input variable for the jacket tank is divided into 2 fuzzy sets: LOW and HIGH. In the case of the vessel, the rate of change of temperature is higher, and therefore we have divided the inputs into 3 fuzzy sets: LOW, MED, HIGH. An example of the fuzzy sets for input $F_j(t)$ is as follows:

$F_j(t)$ i) Fuzzy Set LOW, $c_{low} = 0.0$ $w_{low} = 1.9$
ii) Fuzzy Set HIGH, $c_{high} = 10.0$ $w_{high} = 4.2$
iii) Universe of discourse, 0 – 10 m^3/min

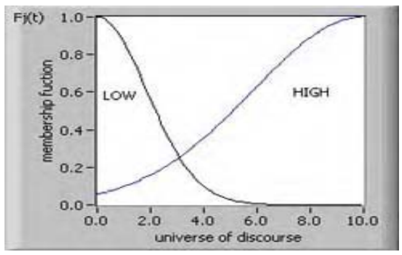

Fig. 5. An example of the fuzzy set for 1 input of jacket tank fuzzy model.

In the consequent part, there are 16 rules and 27 rules for the jacket tank and the vessel respectively. An example of the rule is :-

R_i : IF $\mathbf{F_j(t)}$ **is** $\mathbf{A_1}$ AND $\mathbf{T_{jin}(t)}$ **is** $\mathbf{A_2}$ AND $\mathbf{T_j(t)}$ **is** $\mathbf{A_3}$ AND $\mathbf{T_v(t)}$ **is** $\mathbf{A_4}$ THEN
$\mathbf{T_j(t+1)_i = aF_j(t) + bT_{jin}(t) + cT_j(t) + dT_v(t) + e}$

Using Mamdani MIN operation, the membership value, μ_i for every i-th input will be obtained. Then, using weighted average as defuzzification operation, the output will be obtained. The values will be used by RLS to estimate the best value of the linear equation parameters such as {a,b,c,d,e} for the jacket tank system.

In this paper we have performed several experiments to test the effectiveness of the fuzzy model with automatically tuned fuzzy sets using GA. We compare the result of the experiments with the conventional TS fuzzy model.

For the jacket tank system, we tested the accuracy of the model by varying the inputs $F_j(t)$ and $T_{jin}(t)$ as shown in Figure 6. The corresponding output $T_j(t)$ obtained using the conventional TS fuzzy model and our proposed fuzzy model is shown in Figure 7. The mathematical model represents the simulated system in which the data set have been obtained. Table 1 shows the mean square error (MSE) for each model, meanwhile figure 7 shows the simulated output of TS Fuzzy model and output from mathematical equation.

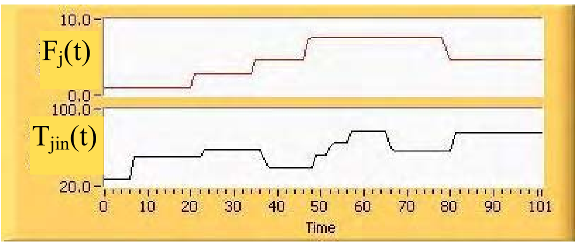

Fig. 6. Input changes for jacket tank model, $F_j(t)$ and $T_{jin}(t)$.

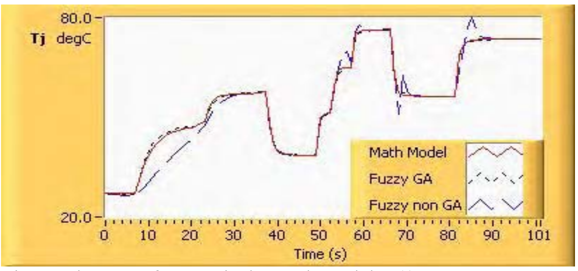

Fig. 7. Changes of output jacket tank model $T_j(t)$.

Model	MSE
with GA	1.55×10^{-2}
without GA	9.72×10^{-2}

Table 1 : Mean square error comparison for jacket tank model.

To test the effectiveness of the vessel model, changes in the input temperature $T_{jin}(t)$ are made as shown in Figure 8. Figure 9 shows the corresponding output $T_v(t)$ changes for both the conventional TS fuzzy model and the fuzzy model with GA.

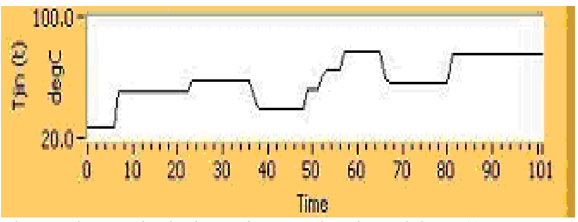

Fig. 8. Changes in the input for vessel tank model, $T_{jin}(t)$.

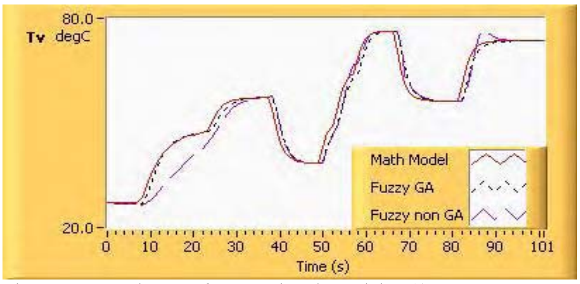

Fig. 9. Output changes for vessel tank model $T_v(t)$.

Model	MSE
with GA	1.93×10^{-3}
without GA	4.59×10^{-3}

Table 2 : Mean square error comparison for vessel tank model.

For both cases, it can be seen that the fuzzy model with GA gives a more accurate model as compared to the conventional TS fuzzy model. From figures 6, 7, 8 and 9, it can be seen that as the inputs changes, the outputs of the proposed fuzzy model follow the output of the mathematical model closely and does not exhibit any significant deviation. For the conventional TS fuzzy model, there are significant deviations from the actual output at the points of input changes and require sometime to closely following mathematical model.

7. CONCLUSION

In this paper, the TS based fuzzy model using GA as an automatic tuner of the fuzzy set is discussed. From the result, it has been shown that the proposed method can minimize the error between fuzzy model output and data output by optimizing the parameters of fuzzy model.

8. ACKNOWLEDGMENTS

This project is supported by Ministry of Science, Technology and Innovation, Malaysia through an Intensified Research Grant Allocation (IRPA). This project is also conducted using facilities at Centre of Lipids Engineering and Applied Research (CLEAR), Universiti Teknologi Malaysia, courtesy of Assoc. Prof. Dr. Noor Azian Morad.

9. REFERENCES

[1] Takagi and M.Sugeno, "Fuzzy Identification of systems and its application to modeling and control", IEEE Transactions on Systems, Man and Cybernetics, 15(1):116-132, 1985.

[2] Willian L. Luyben, "Process Modeling, Simulation and Control for Chemical Engineers", McGraw Hill, 1990.

[3] Christi J. Geankoplis, "Transport Processes and Unit Operations", Prentice Hall Int., 1995.

[4] Kevin M. Passino and Stephen Yurkovich, "Fuzzy Control", Addison Wesley Longman Inc., 1998.

[5] Goldberg, David E (David Edward), "Genetic Algorithms in Search, Optimization & Machine Learning", New York: Addison Wesley, 1989.

[6] Lenhart Ljung, "System Identification Theory for the User", Prentice Hall, 1999.

[7] Hao Ying, "Fuzzy Control and Modeling – Analytical Foundations and Application", IEEE Press, 2000.