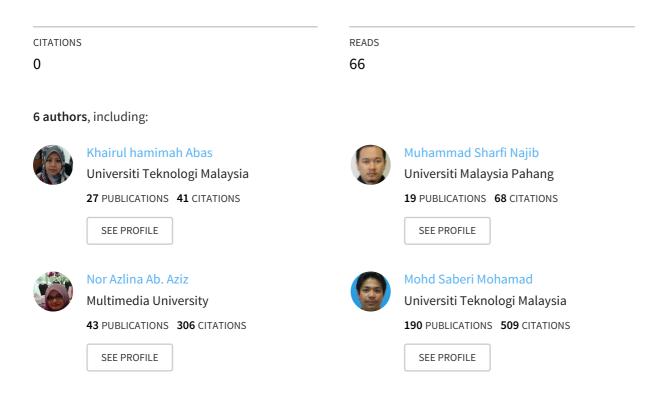
See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/301341671

# Gravitational search algorithm: R is better than R2?

Article in Journal of Engineering and Applied Sciences · April 2016



# Some of the authors of this publication are also working on these related projects:



Neurosignals classification via time sereies analysis View project



An Alternative Mathematical Model of Welding Current to Predict Defects via Small Signal Analysis View project

All content following this page was uploaded by Mohamad Nizam Aliman on 17 April 2016.

©2006-2016 Asian Research Publishing Network (ARPN). All rights reserved.

www.arpnjournals.com

# GRAVITATIONAL SEARCH ALGORITHM: R IS BETTER THAN R<sup>2</sup>?

Mohamad Nizam Aliman<sup>1</sup>, Khairul Hamimah Abas<sup>2</sup>, Muhammad Sharfi Najib<sup>1</sup>, Nor Azlina Ab. Aziz<sup>3</sup>,

Mohd Saberi Mohamad<sup>4</sup> and Zuwairie Ibrahim<sup>1</sup>

<sup>1</sup>Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang, Pekan, Pahang, Malaysia

Faculty of Electrical Engineering, Universiti Teknologi Malaysia, Skudai, Johor, Malaysia

Faculty of Engineering and Technology, Multimedia University, Melaka, Malaysia

<sup>4</sup>Faculty of Computing, Universiti Teknologi Malaysia, Skudai, Johor, Malaysia

E-Mail: MEE13005@stdmail.ump.edu.my

# ABSTRACT

Gravitational Search Algorithm (GSA) is a metaheuristic population-based optimization algorithm inspired by the Newtonian law of gravity and law of motion. Ever since it was introduced in 2009, GSA has been employed to solve various optimization problems. Despite its superior performance, GSA has a fundamental problem. It has been revealed that the force calculation in GSA is not genuinely based on the Newtonian law of gravity. Based on the Newtonian law of gravity, force between two masses in the universe is inversely proportional to the square of the distance between them. However, in the original GSA, *R* is used instead of  $R^2$ . In this paper, the performance of GSA is re-evaluated considering the square of the distance between masses,  $R^2$ . The CEC2014 benchmark functions for real-parameter single objective optimization problems are employed in the evaluation. An important finding is that by considering the square of the distance between masses,  $R^2$ , significant improvement over the original GSA is observed provided a large gravitational constant should be used at the beginning of the optimization process.

Keywords: gravitational search algorithm, newtonian law of gravity, law of motion.

### INTRODUCTION

Gravitational Search Algorithm (GSA) has been firstly introduced by Rashedi *et al.* in 2009 [1]. It is a metaheuristic population-based optimization algorithm which is inspired by the Newtonian law of gravity and law of motion. In GSA, fitness is translated into mass and interaction between agents is simulated based on the Newtonian Law of Gravity and Law of Motion.

However after three years GSA was introduced, Gauci *et al.* [2] has found an inconsistency used of gravitational formulation in GSA. They have proved theoretically that GSA was indeed not genuinely based on Newtonian law of gravity. Specifically, in the calculation of force, distance R is employed instead of  $R^2$ . However, the main reason of this has been explained in the first paper of GSA. The original author stated in [1], original gravitational formulation was not used because of poor experimental result.

Therefore, in this paper, we re-evaluate the performance of standard GSA with distance R, using CEC2014 benchmark dataset. Then, we propose the use of square of distance  $R^2$ , in the calculation of force which we denoted as GSAR2. We also investigate the performance of GSAR2 algorithm by varying the value of initial gravitational constant,  $G_0$ . The performance is then analyzed statistically.

# **GRAVITATIONAL SEARCH ALGORITHM**

In GSA, agents are considered as an object and their performance are expressed by their masses. The position of particle is corresponding to the solution of the problem. Consider a population consisted N quantity of agents, so the position of *i*th agent can be presented by:

$$X_{i} = (x_{i}^{1} \dots x_{i}^{d} \dots x_{i}^{n}) for \ i = 1, 2, \dots, N$$
<sup>(1)</sup>

The mass of *i*th particle at time *t* is derived from Eqn. (2) and Eqn. (3), denoted as  $M_i(t)$ .

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}$$
<sup>(2)</sup>

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^N m_j(t)}$$
(3)

where N is a population size,  $m_i(t)$  is an intermediate variable in agent mass calculation,  $fit_i(t)$  is the fitness value of *i*th agent at time t, best(t) and worst(t) denote the best and the worst fitness value of the population at time t. The best and the worst fitness for the case of minimization problem are defined as follows;

$$best(t) = \min_{j \in \{1,\dots,N\}} fit_j(t)$$
  
$$worst(t) = \max_{j \in \{1,\dots,N\}} fit_j(t)$$
(4)

whereas for maximization problem,

$$best(t) = \max_{j \in \{1, \dots, N\}} fit_j(t)$$
  

$$worst(t) = \min_{j \in \{1, \dots, N\}} fit_j(t)$$
(5)

At specific time "t", the force acting on agent "i" from agent "j" in *d*th dimension can be represented as the following:



www.arpnjournals.com

(6)

$$F_{ij}^{d}(t) = G(t) \frac{M_{pi}(t) \times M_{ij}(t)}{R_{ij}(t) + \varepsilon} \left( x_j^{d}(t) - x_i^{d}(t) \right)$$

where  $M_{pi}(t)$  is the passive gravitational mass of agent "*i*",  $M_{aj}(t)$  is the active gravitational mass of agent "*i*", G(t) is the gravitational constant,  $\varepsilon$  is a small constant, and  $R_{ij}(t)$  is the Euclidian distance between agent "*i*" and "*j*". The distance is calculated using a standard formula as follow;

$$R_{ij}(t) = \|X_i(t), X_j(t)\|_2$$
<sup>(7)</sup>

while gravitational constant, G(t), is defined as a decreasing function of time, which is set to  $G_0$  at the beginning and decreases exponentially towards zero with lapse of time [3].

$$G(t) = G_0 \times e^{-\alpha \frac{t}{t_{max}}}$$
<sup>(8)</sup>

To give a stochastic characteristic to GSA, the total force acted on agent "i" in "d" dimension is a randomly weighted sum of dth components of the forces exerted from other agents;

$$F_i^d(t) = \sum_{j=1, j \neq i}^N rand_j F_{ij}^d(t)$$
<sup>(9)</sup>

where  $rand_i$  is a random number in the interval of [0,1].

According to law of motion, the current velocity of any mass is equal to the sum of the fraction of its previous velocity and the variation in the velocity. Variation or acceleration of any mass is equal to the force acted on the system divided by mass of inertia [3], which is shown in the following formula.

$$a_{i}^{d}(t) = \frac{F_{i}^{d}(t)}{M_{ii}(t)} for M_{ai} = M_{pi} = M_{ii}$$
(10)

Therefore, the new agent velocity and position are calculated using these equations:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t)$$
<sup>(11)</sup>

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1)$$
(12)

Finally, the next iteration is executed until the maximum number of iterations,  $t_{max}$ , is reached. In summary, the principle of standard GSA is shown in Figure-1.

# GSA IS NOT GENUINELY FOLLOWS THE NEWTONIAN GRAVITATIONAL LAW

Newton stated that "Every particle in the universe attract every other particle with a force that is directly proportional to the square of the distance between them" [4]. This definition is known as gravitational force and it is formulated as:

$$F = G \, \frac{M_1 M_2}{R^2} \tag{13}$$

In GSA, the calculation of force is also based on this equation. However, as shown in Eqn. (6), distance R, is used as the denominator instead of  $R^2$ . Let  $\varepsilon = 0$ , then

$$F_{ij}^{d}(t) = G(t) \frac{M_{pi}(t) \times M_{ij}(t)}{R_{ij}(t)} \left( x_{j}^{d}(t) - x_{i}^{d}(t) \right)$$
(14)

since  $R_{ij}(t) = x_j^d(t) - x_i^d(t)$ , therefore,

$$F_{ij}^{d}(t) = G(t) \times M_{pi}(t) \times M_{ij}(t)$$
<sup>(15)</sup>

which clearly shows that the force  $F_{ij}(t)$  is not influenced by the distance between agent *i* and *j*. Thus, the original GSA is not genuinely follows the Newtonian gravitational law.

In this paper, we follow genuinely the Newtonian gravitational law and use the square of distance,  $R^2$ , in the calculation of force. The performance of GSAR2 with different value of initial gravitational constant is investigated as well.

# **EXPERIMENTS**

The parameter setting for all experiments is tabulated in Table-1. Different value of  $G_0$ ,  $G_0 = 10^1$  until  $G_0 = 10^{15}$  were tested in experiments for GSAR2.

In this study, 30 standard benchmark functions from CEC2014 test functions [5] shown in Table-2 were used throughout the experiment. These benchmark functions consist of the shifted, rotated, expanded and combined classical test function. They are categorized into three four groups; unimodal, multimodal, hybrid, and composite function.

# **RESULT AND DISCUSSION**

Convergence curves of the two variations of GSA, which is the original GSA and GSA that employs square of distance between masses (GSAR2), are shown in Figure-2 to Figure-5. For GSAR2,  $G_0=10^9$  is used. These results show that generally better performance can be obtained even though square of distance between masses is used.

Analysis of convergence curves of GSAR2 with different  $G_0$  are shown in Figure-6 to Figure-10. These figures show that solutions can be improved faster and convergence rate is better if smaller  $G_0$  is used. However, there is no guarantee that small  $G_0$  produces better result.

# ARPN Journal of Engineering and Applied Sciences

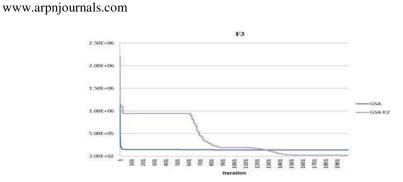
©2006-2016 Asian Research Publishing Network (ARPN). All rights reserved.

# START POPULATION INITIALIZATION V EVALUATE filmess EACH AGENT V UPDATE gravity, best, worst CALCULATE mass, acceleration EACH AGENT UPDATE velocity, position V MEET END CRITERION YES END

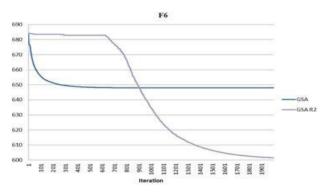
Figure-1. General principle of standard GSA.

Table-1. Parame	ter setting used in	n all experiments.
-----------------	---------------------	--------------------

Parameter	Value
Number of agents, N	100
Number of iterations, t	2000
Number of dimensions, D	50
Number of runs, t <sub>max</sub>	50
Search range	[100,-100]
Alpha, α	20



**Figure-2.** Convergence curve for function 3, in which, G0=102 is used for original GSA and G0=109 is used for GSAR2.



**Figure-3.** Convergence curve for function 6, in which, G0=102 is used for original GSA and G0=109 is used for GSAR2.

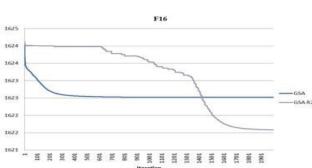
Table-2. CEC 2014 benchmark functions	5.
---------------------------------------	----

Function Type	Function ID	Function Description	Ideal Fitness
	F1	Rotated High Conditioned Elliptic Function	100
Unimodal Function	F2	Rotated Bent Cigar Function	200
Function	F3	Rotated Discus Function	300
	F4	Shifted and Rotated Rosenbrock's Function	400
	F5	Shifted and Rotated Ackley's Function	500
	F6	Shifted and Rotated Weierstrass Function	600
	<b>F</b> 7	Shifted and Rotated Griewank's Function	700
	F8	Shifted Rastrigin's Function	800
<u><u> </u></u>	F9	Shifted and Rotated Rastrigin's Function	900
Simple Multimodal	F10	Shifted Schwefel's Function	1000
Function	F11	Shifted and Rotated Schwefel's Function	1100
Function	F12	Shifted and Rotated Katsuura Function	1200
	F13	Shifted and Rotated HappyCat Function	1300
	F14	Shifted and Rotated HGBat Function	1400
	F15	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	1500
	F16	Shifted and Rotated Expanded Scaffer's F6 Function	1600
	F17	Hybrid Function 1 (N=3)	1700
	F18	Hybrid Function 2 (N=3)	1800
	F19	Hybrid Function 3 (N=4)	1900
Hybrid Function	F20	Hybrid Function 4 (N=4)	2000
	F21	Hybrid Function 5 (N=5)	2100
	F22	Hybrid Function 5 (N=5)	2200
	F23	Composition Function 1 (N=5)	2300
	F24	Composition Function 2 (N=3)	2400
	F25	Composition Function 3 (N=3)	2500
Composite	F26	Composition Function 4 (N=5)	2600
Function	F27	Composition Function 5 (N=5)	2700
	F28	Composition Function 6 (N=5)	2800
	F29	Composition Function 7 (N=3)	2900
	F30	Composition Function 8 (N=3)	3000

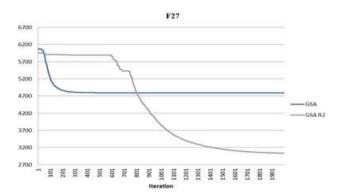
# ARPN Journal of Engineering and Applied Sciences

©2006-2016 Asian Research Publishing Network (ARPN). All rights reserved.

# www.arpnjournals.com



**Figure-4.** Convergence curve for function 16, in which,  $G_0=10^2$  is used for original GSA and  $G_0=10^9$  is used for GSAR2.



**Figure-5.** Convergence curve for function 27, in which, G0=102 is used for original GSA and G0=109 is used for GSAR2.

According to inferential statistic, hypothesis testing can be used to obtain inferences about one or more algorithms from given sample. This can be achieved by defining two types of hypothesis, the null hypothesis  $H_0$  and the alternative hypothesis  $H_1$ . The null hypothesis is a statement of no effect or no difference, whereas the alternative hypothesis represents significant difference between algorithms.

Friedman's test is an omnibus test which can be used to carry out these types of comparison. It allows us to detect differences considering the global set of algorithms. Once Friedman's test rejects the null hypothesis, we can proceed with a post-hoc test in order to find the concrete pairwise comparisons which produce differences.

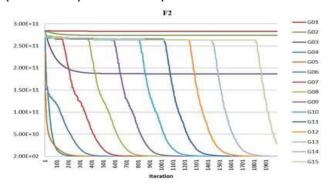


Figure-6. Convergence curves of different G0 values for function 27.

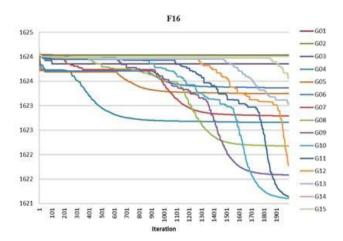


Figure-7. Convergence curves of different G0 values for function 16.

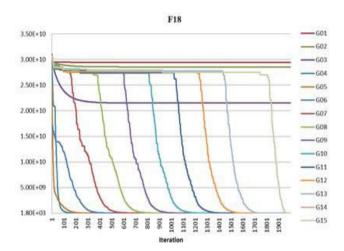


Figure-8. Convergence curves of different G0 values for function 18.

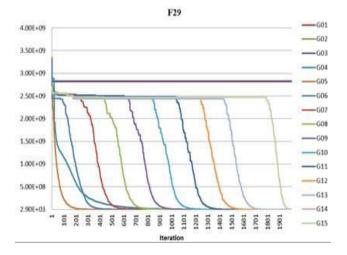


Figure-9. Convergence curves of different G0 values for function 29.

**R** 

©2006-2016 Asian Research Publishing Network (ARPN). All rights reserved.

# www.arpnjournals.com

Table-3. Friedman test result for variant of G0 value.

	-					GSP	with squar	r of the di	GSA with square of the distance between masses (GSA R2)	CONTRACTOR	(GSAR2)			0.000	A COULT	
RT32A6124         SE0044815         Se0057379         S191921         ISRUE         ITSR14         ITSR25         ITSR14         ITSR25         ITSR14         ITSR14         ITSR25         ITSR25 <thitsr25< th=""> <th< th=""><th>FUCUDE</th><th><math>G_{ij} = 10^{1}</math></th><th><math>G_{H} = 10^{2}</math></th><th><math>G_{n} = 10^{3}</math></th><th>5</th><th><math>G_{n}=10^{5}</math></th><th><math>G_{0} = 10^{6}</math></th><th><math>G_{11}=10^{7}</math></th><th><math>G_0 = 10^3</math></th><th>Gn=10<sup>4</sup></th><th><math>G_{0} = 10^{10}</math></th><th></th><th><math>G_0 = 10^{12}</math></th><th></th><th><math>G_{ij} = 10^{14}</math></th><th><math>G_{ij}=10^{15}</math></th></th<></thitsr25<>	FUCUDE	$G_{ij} = 10^{1}$	$G_{H} = 10^{2}$	$G_{n} = 10^{3}$	5	$G_{n}=10^{5}$	$G_{0} = 10^{6}$	$G_{11}=10^{7}$	$G_0 = 10^3$	Gn=10 <sup>4</sup>	$G_{0} = 10^{10}$		$G_0 = 10^{12}$		$G_{ij} = 10^{14}$	$G_{ij}=10^{15}$
2.88273FH         2.78774EH         1.8879EH         1.808796H         710.4445         307.520         370.520         616.700         2.265.74         1.82483           1.80204.355         1.1120883         1.103883         1.103883         1.103883         1.103863         1.103863         1.103863         1.103863         1.103863         1.103863         1.103863         1.11328633         1.11328633         1.1103213164         1.11228         1.1	1	8732661254	8506448153		251 1952.1	1506554	1281282	1132635	1137142	1266821	1781130	2751339	5522629	14421108	5251 1993.8	537606682.7
(882)24.35         (13349.04)         (25036.3         (14058.05         (553.05         (573.05)         (513.43)         (751.43)	2	2.83273E+11	2.73774E+11		-	3440.425	3707.562	3729.365	6416.709	22565.14	182438	1918904	18770454	184440486	1815735743	27739677412
1194885411         1122.08.978         63865.0668         69.82102         547.376         519.9999         520.001         515.002         529.3016           221.456.34         21.4257.27         521.42975         519.9995         519.9995         501.9956         601.6171         602.637           3344.47601         355.070402         2447.3512         700         700.00         70.000         520.016         520.2016           375.12571         21.4257.26         889.9131         963.58         869.533         964.575         91.129         91.129         91.129           1757.1267.1274         1774.10861         1732.010481         1732.010481         260.576         91.23         990.56         949.557         949.557         941.256         949.557           1776.1286         1774.70861         1740.647         1740.57         949.566         949.557         940.571         940.567           17170.88615         1774.70861         1774.708         1774.708         949.566         950.568         950.576         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567         940.567 <td< td=""><td>3</td><td>1689294.335</td><td></td><td>152502831</td><td>11405896</td><td>87550.64</td><td>52177.54</td><td>22883.16</td><td></td><td>175143</td><td>3£.16871</td><td>20236.09</td><td>27013.8</td><td>2826319</td><td>37330.5695</td><td>157738.8777</td></td<>	3	1689294.335		152502831	11405896	87550.64	52177.54	22883.16		175143	3£.16871	20236.09	27013.8	2826319	37330.5695	157738.8777
S21,4554         S21,42572         S21,42975         S19,9997         S19,9997         S19,9997         S19,9997         S19,9997         S10,0572         O10         S20,019         S20,019         S20,019         S20,019         S20,019         S20,019         S20,019         S20,019         S20,019         S20,011         S20,019         S20,010         S20,019         S20,021	4	119488.5411		63365.6885	669 82102	\$47.3768	534.1.977	520.6037	512.5391	515.0092	529.9766	541.9242		623.39436	841.059843	3864.257868
(683 8338385)         (683 8600 746)         (616 971)         (61 53 71)         (61 51 71)         (61 51 71)         (61 51 71)         (61 51 71)         (62 302)           3344 4761         255.07402         2447 3512         700         700         700021         700.022         700.225         700.225           275.12571         27811755         1889 9175         1160.871         98 33.25         93 45.56         89 91.33         93 49.25         141 42.25         300.216         803.34           275.125712571         20811755         1889 9175         160.0371         160.0371         93 305.65         89 91.33         93 956.65         89 93.74           275.12512710         2081784         1776.0387         587 0.02         170.022         170.022         170.022         170.023         170.25         100.023 </td <td>5</td> <td>521.43634</td> <td>521 4257227</td> <td>521.429725</td> <td>519.99</td> <td>519 9999</td> <td>519 9999</td> <td>520.0001</td> <td>520.0019</td> <td>520.0216</td> <td>520 2316</td> <td>521.0816</td> <td>521.4196</td> <td></td> <td>521.422384</td> <td>521.435732</td>	5	521.43634	521 4257227	521.429725	519.99	519 9999	519 9999	520.0001	520.0019	520.0216	520 2316	521.0816	521.4196		521.422384	521.435732
3344 476(1         255.07402         2447 35(1)         700         700         700         700.0004         700.0021         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0025         700.0026         700.0	9	683 8335855	-	_	631 69971	603 8811	602.6572	601.9596	-	1710.100	602 3055	604.1438	607.6323	614.83	631.166609	662.6122999
175:1271         175:1271         176:1271         156:49136         165:1613         861.5879         861.311         844.5941         839.405         857.9425           2073:113791         2048:11756         1849.1756         160.6871         943.525         949.525         941.525         949.525           1774.08845         1774.0864         1732.63068         1677.0463         7588.2786         595.903         4492.371         4491.255         390.216         390.376           1770.658415         1774.0861         1774.0861         1200.694         1200.001         1200.001         1200.001         1200.001         1200.001           1320.658195         120.689483         1207.946         1200.376         1300.526         1300.566         1300.526         1300.526	7	3344.47661	3255.074092		700	700	700	700.0004	700.0021	700.0225	700 2629	700.9354	701.1698	702.59033	71.5.781.77	948.8673236
2073.113791         2048.11756         180.91726         160.8571         949.3205         954.556         931.083         931.333         934.923           17770.8845         1774.7055         180.91726         160.871         1580.2016         3492.397         141.225         390.216         393.341           17770.8845         1774.7055         1774.7055         1774.7055         1774.2055         390.216         349.237         300.216         390.345           132.056.58195         120.0457         130.3992         120.0011         120.001         120.002         120.007         120.025           131.2.265.91         1312.206.581         130.3595         130.395         130.395         130.3595         130.256         390.216         300.316           131.2.265.91         1312.014.571         130.355         130.355         1400.471         1400.452         1400.457         1400.452           11147.9377         182.01266         155.053.47         1400.475         1400.452         1400.457         1400.452           11147.9377         186.0566         55.50.44         140.047         1600.55         1400.452         1400.452           11147.9377         130.0566         155.05674         160.52.71         160.52.71	80	1725.12674	1708.4137	156491326	1057.6139	861.5879	1151.948	844.9941	839.1623	836.8605		838.6608	853.3049	921.39109	110422403	1352.220486
17380.2018         1732.0303         1670.4643         788.2786         569.088         499.397         4141.225         390.216         390.316           17770.8845         1774.70651         1774.70651         1774.70651         1774.70651         120.0001         120.0001         120.0001         120.0001         120.0002         120.0001         120.0002         120.0075         1300.392         1300.392           1312.256504         1312.014347         130.3657         130.30251         130.3026         1300.3021         1300.002         1300.302	6	2073.113791	2048.117556	1804 91726	1160.8371	943.3205	936.0374		931.0835	931.133	934 9526	938.3279	959.2188	1076.904	1341.62882	1516.43828
17770.8845         1774,7081         174,7081         174,7081         1746,7081         1760.342         200.010         120.001         120.001         120.001         120.003         120.001         120.003	10	17380.29708	17325.03038	16770.4643	7588.2786	\$369.088	4959.903	4492.397	4141 226		_	3513.073	3424.28	4325.9472	8920.44787	14887.70658
120.66.88195         120.10.3495         120.001         120.003         120.033         120.043         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045         120.045	н	17770.8845	_			4289.283	3786.806	3642.516	3473845	3496.616	3369.126	3480.942	4013.827		13869.721	17030 29615
1312.266.94         132.014347         1300.3673         1300.3573         1300.3673         1300.3673         1300.3773         1300.3703         1400.4703         1400.5703         1400.5703         1300.3703         1400.4703         1400.5703         1400.5703         1300.4703         1300.5503         1300.4703         1400.5703         1400.5703         1300.4703         1506.5703         1300.4703         1506.5703         1300.4703         1300.5503         1300.4703         1300.5503         1400.4703         1300.5503         1400.4703         1300.5503         1400.4703         1300.5503         1400.4703         1300.5503         1400.4703         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503         1300.5503	12	1206.628195				1200.001	1200.001	1200.002	1200.003	1200.007	1200.023	1200.114	1200.882	1203.5324	120653505	1207.007219
2114.434403         209.30533         186.175842         1400.467         1400.467         1400.5         1400.45         1400.45         1400.45         1400.45         1400.45         1400.45         1400.45         1400.45         1400.45         1400.45         1400.45         150.653         150.653         150.613         150.653         150.643         150.653         150.643         150.653         150.643         150.653         150.643         150.653         150.743         150.653         150.743         150.753         150.410         150.553         150.413         150.553         150.413         150.553         150.713         150.553         150.743         150.653         150.743         150.553         271.443         150.553         271.443         150.553         271.453         275.656         271.453         275.656         271.453         275.656         271.453         275.656         271.453         275.656         271.453	13	1312.296.904	1312.014347	1309.36737		1300.253	1300.256	1300.248	1300255	1300.28	1300.359	1300.484	1300.666		1301.12134	1303.567935
1355126715         1173406341         205478102         150.5936         190.5106         150.6558         150.6558         150.65533         150.6553         150.6553	14	2114.434403	2389.320553	1861.75842	1400.	1400.478	1400.497	1400.5	1400.5	1400.492	1400.467	1400.466	1400.728	1401.0573	1402.71712	1471.34932
IG24.032145         IG24.0382363         IG23.0397         IG23.247         IG23.346         IG23.791         IG23.147         IG23.583         IG23.104           1114799377         1090382363         956772068         23901796         1355403         112017.1         1096529         1625.138         1621.147           29441038773         28556747406         2.1504F10         3489.506         233.825         2199.409         211.236         2255.334         2470.741           6802.013661         6608.619455         5.2555.4008         1950.126         2953.1.20         2064.45         267.43         2254.94         1961.57           19643113.88         16878522.37         4941605         3255.4008         3970.20         2003.31         1969.125         2557.34         2470.741         12087.68           196218.742         733181.9558         55132.556         833.7855         377.50         2556.93         214.588         2476.67         256.673         2556.673         2714.58         2666.75         253.055         2714.58         2714.58         256.673         2714.58         274.56         274.563         2714.58         274.563         2714.58         276.673         2714.58         276.673         2714.58         276.673         2714.58 <td< td=""><td>15</td><td>135512671.5</td><td>117340634.1</td><td></td><td>1510.</td><td>1505.196</td><td>1506.854</td><td>1506.658</td><td>1506.498</td><td>1506237</td><td>1506.523</td><td>1513.366</td><td>1539.534</td><td>1546.4017</td><td>155435725</td><td>2479023983</td></td<>	15	135512671.5	117340634.1		1510.	1505.196	1506.854	1506.658	1506.498	1506237	1506.523	1513.366	1539.534	1546.4017	155435725	2479023983
111479377         1090382363         956772068         23901796         1355403         1120171         1096529         108352.8         279582.1         2956187           29441038883         28556747406         2.15044710         3489.586         2233.825         2199.40         211.236         2733.78         2525.324         2470.741           6802.013661         6008.619455         5255.46006         958.9586         2233.825         1969.123         1965.455         1964.703         1962.814         1961.57           19843113.68         1687852.23         4941606.83         2257.539         3702.06         2233.62         25664.45         2670.43         2566.45         2966.45         2667.43         2665.673         241.657           766218.7422         723181.9558         37418525         372344.1         180931.8         1655.453         3164.844         2923.023         271.647           766218.7422         723181.9558         37141852         2557.483         3765.445         2669.055         2649.05         274.657         271.647           766218.7422         723181.9558         3714.852         377.841         3855.42         2857.58         2857.58         2866.43         2667.45         2667.657         276.80.23         271.647	16	1624.032145	1624.068633		1622.	1623.247	1623.366	1622.791	1622.174	1621 583	1621.104	1621.139	1621.774	1623.0767	162327904	1623.686076
29441038863         28556747406         21.94E+10         389.5086         1929,4708         233.825         2199,409         211.236         2255.324         2470.741           6802.013661         6008.619455         5255.46008         1929,4708         1961.237         1965.455         1964.703         1962.814         1961.57           19843113.88         1687852.337         4941606.83         3257.539         3700.005         20231.62         2664.45         2679.41         1965.455           19843113.88         1687852.337         4941606.83         3257.539         3702.05         20231.62         20231.62         2049.65         25549.41         2106.767           166218.7425         723181.9558         551322.556         3833.7853         357.14         3365.425         3164.847         2053.023         214.557           766218.7425         723181.9558         551322.556         3833.7853         357.542         3164.847         2055.05         257.586           766218.7425         723181         302.1339         5516.482         255.542         2566.032         2648.032         271.4557           766218.7425         723181         302.1338         200.0131         2700.001         2704.24         2720.117         2719.03         2716.51	17	1114799377	1090382363	-	23901	1355403	112017.1	109852.9	106352.8	279582.1	295618.9	324646.7			2761829.76	22716442.25
(802.013661         6608.619455         5255.46068         1929.4708         1961.231         1965.455         1964.703         1962.814         1961.57           198.43113.38         168.7852.37         4941.606.83         32571.39         3702.06         2231.62         2664.45         2799.41         1965.573           198.43113.38         168.7852.37         4941.606.83         32571.539         3702.06         2231.62         2664.45         2549.41         21087.68           766.218.7422         723181.9538         551322.556         3833.785         3773.86         355.744         2064.65         2649.065         2648.02         2714.58           766.218.7422         723181.9538         551322.556         3833.785         3575.79         355.542         365.747         2064.65         2649.05         2648.023         2714.58           714.7434         3286.139418         3022.1338         2001.315         2555.739         2555.746         257.547         2656.673         2714.75         2656.673           311.147434         3286.133         3014.08744         2020.033         2567.136         257.649         2770.11         2719.03         2016.50         2648.023         2671.41         2655.74           3225.6614063         30564.41 <td>18</td> <td>29441038983</td> <td>28556747406</td> <td>2.1504E+10</td> <td>3489.5086</td> <td>2233.825</td> <td>2199.409</td> <td>2211.236</td> <td>Sec. 15.</td> <td>2525324</td> <td>2470.741</td> <td>2519.964</td> <td>2338.366</td> <td>2633.9387</td> <td>2742.06376</td> <td>7931603.248</td>	18	29441038983	28556747406	2.1504E+10	3489.5086	2233.825	2199.409	2211.236	Sec. 15.	2525324	2470.741	2519.964	2338.366	2633.9387	2742.06376	7931603.248
19843113.88         16878522.37         4941606.88         32571.539         3702.05         2233.162         22664.45         26704.9         22549.41           7418338577.7         404233380.4         288429801         374185.25         323.7855         3573.14         3655.425         3164.84         2923.02           766218.7422         723181.9538         551322.556         3833.7855         3553.785         3552.14         3656.45         3164.844         2923.02           6294.418892         6109.699109         4670.55103         2516.482         2565.43         3552.14         365.653         3664.15         293.02           3311.147434         3250.64416         2967.053         2516.482         2500.1315         2556.735         2649.05         2648.02           3311.147434         3250.64416         2967.053         2516.482         2500.133         2557.68         2657.75         2648.02           3311.147434         3250.64416         2967.053         2500.133         2500.133         2656.726         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.75         2657.76         2657.75	19	6802.013661	6608.619455	5255.46068	1929.4	1961.237	1969.123	1965.455	1964.703	1962.814		1966.128	1971.106	1971.5503	1972.67694	2011.130942
418.338.577.1         404.233.380.4         2884.29801         374.185.25         202.344.1         180931.8         16.258.1.4         146788.8         146788.8           766.218.7422         723181.9558         5513.22.556         3833.7855         3557.863         3552.14         365.425         3164.844         2923.023           766.218.7422         723181.9558         5513.22.556         3833.7855         3557.863         3552.14         365.425         3164.844         2923.023           6294.418892         6109.690109         4670.55103         2516.482         2656.43         2552.985         2566.735         2649.065         2648.02           3311.147434         3286.132418         3022.1336         2500.1315         2516.482         2657.35         2657.475         2657.475         2657.475           3311.147434         3286.132418         3022.1336         2500.002         2700.002         2700.013         2666.726         2657.475         2657.475           3329.691419         3305.6416         2982.05317         2700.002         2700.013         2704.24         2712.376         2770.117         2719.05           3329.691419         3305.81818         3014.05714         2907.333         2491.1057         3009.133         2403.309.9667 <t< td=""><td>20</td><td></td><td></td><td>494160683</td><td></td><td>30702.06</td><td></td><td>29664.45</td><td></td><td>22549.41</td><td>21087.68</td><td>23631.41</td><td>25516.52</td><td>36368.637</td><td>47399.2724</td><td>142994.0184</td></t<>	20			494160683		30702.06		29664.45		22549.41	21087.68	23631.41	25516.52	36368.637	47399.2724	142994.0184
766218.7422         723181.958         551322.556         3833.7855         3557.863         3552.14         365.425         3164.844         2923.023           6294.418892         6109.699109         4670.55103         2516.482         265.643         2552.985         259.065         2649.065         2648.023           3311.147434         3286.132418         3022.1338         200.1315         2555.43         2555.93         2555.756         2657.752         269.065         268.0736         2657.475           3276.140837         3290.64416         2982.63517         2700.0002         2700.001         2704.24         2712.316         2730.117         2719.03           3276.140837         3205.64416         2982.63517         2700.002         2700.003         260.133         2657.756         2657.05         2657.05         2657.05           3220.691419         3305.8414         2902.033         2491.1057         3208.013         200.123         2700.123         2796.02           3320.691419         3305.88183         3014.08744         2900.033         2900.133         200.123         2700.123         2796.04           3325.6699.06         618.5917         200.033         2800.133         200.133         200.123         2700.123         200.123 <td>21</td> <td>418358577.7</td> <td>404233380.4</td> <td>268429891</td> <td>37418525</td> <td>202344.1</td> <td>180931.8</td> <td>162851.4</td> <td>146788.8</td> <td>140888.7</td> <td>1856573</td> <td>251913</td> <td>289969.3</td> <td>334712.08</td> <td>614614.432</td> <td>3190329.174</td>	21	418358577.7	404233380.4	268429891	37418525	202344.1	180931.8	162851.4	146788.8	140888.7	1856573	251913	289969.3	334712.08	614614.432	3190329.174
6294,418892         6109,699109         4670,55103         2516,482         2656,43         2552,985         2590,515         2649,065         2648,025           3311,147434         3286,132418         3022,13336         200,1315         2558,279         2557,539         2556,735         2657,475         2657,475           3276,140871         3286,132418         3022,13336         200,1315         2500,001         2700,202         2700,002         2700,002         2700,117         2719,05           3229,691419         3305,818183         3014,08744         2800,053         2600,133         2600,133         2600,133         2656,736         2657,475         2657,475           3229,691419         3305,818183         3014,08744         2800,053         2600,033         2600,133         2704,143         2700,117         2119,05           3229,691416         3305,818183         3014,0874         2800,053         2800,133         290,123         2776,143         2700,112         2770,117         2719,05           3229,69141         3305,6141         2902,5334         2800,053         2800,133         2401,126         3019,961         3029,664         3029,664         3029,664         3029,664         3029,664         3029,664         3029,664         3029,664	22	766218.7422		551322.556	3833.	3757,863	3552.14	3365.425	3164844	2923.023	2714.538	2766.163	2686.664	2799.0476	2945 26427	4923.765685
3311.147434         326.132418         3032.13368         2000.1315         2658.279         265.033         2656.736         2657.475         2657.475           3276.140837         3229.64416         2982.63517         2700.0001         2704.24         2112.376         2657.736         2657.475         2657.475           3276.140837         3229.64416         2982.63517         2700.0002         2700.001         2704.24         2112.376         2730.117         2719.03           3329.691419         309.5818183         3014.08744         2600.0334         2800.033         209.0732         209.0132         2798.024           6155.768633         6179.73393         7044.51333         4341.1057         309.0752         3019.961         3032.667           968.97114         199.47.74393         2403.33842         6618.9917         322.073         3196.162         3188.55         3169.563         302.5667           25334956594         2931.30498         365061933         24012.206         12057.08         3169.162         3039.966         302.5667           2534956594         2931.30498         3653061933         24012.206         12057.08         3169.567         3169.569         309.966         302.5667           2534956594         2931.30498	23	6294.418892	601669.6010	4670 55103	2516	2656.43		2650.515	2649.065			2648.181	2649.523	2650.3922	265621733	2703.859884
3276.140837         3290.64416         2982.63517         2700.000         2700.103         2702.117         2719.03         2715.018         2713.74           3329.601419         3305.818183         3014.08744         2800.0534         2800.033         2800.133         2800.123         2798.024         2798.048         2105.058	24	3311.147434	3286.132418	3032.13398		2658.279	2657.539	2656.993	2656.726		2656.837	2638.303	2661.062	2671.8596	2703.48088	2794.352455
3329.691419         3305.818183         3014.08744         2800.033         2800.133         2798.024         2798.024         2798.024         2798.021         2705.021         2105.021	25			2982.63517	2700.	2700.001	2704.24	2712.376		2719.03				2716.9106	2723.18453	2760.631227
6155768633         6179.73393         7044.51333         4341.1057         3028.074         3019.091         3019.561         3055.574         3105.057           19689.97114         19947.74393         24033.8342         6618.9917         3421.272         3196.162         3188.55         3169.596         3193.567         3105.057           2834996594         29313.0498         353061933         24012.206         12070.83         1388.55         3169.596         3193.583         3193.582           29488599.17         0313586.71         803640558         14839395         12070.83         100780.81         8168.29         6029.955         5577.441         5116.527           39488599.17         0313586.71         803640558         148393955         117847.5         100906.5         79926.06         61653.13         5467.3         516.527         516.527           3948599.17         0313586.71         803640558         148393955         117847.5         100906.5         79926.06         61653.13         5467.3         5826.54         516.527         516.527           3948599.17         0313586.71         803640558         148393955         117847.5         100906.5         79926.06         61653.13         5867.3         5335.54         516.527 <tr< td=""><td>26</td><td>3329.691419</td><td>3305.818183</td><td>3014.08744</td><td>2800.0</td><td>2800.093</td><td>2800.133</td><td>2800.109</td><td>2800.123</td><td>2798.024</td><td>2798.048</td><td>2798.25</td><td>2785.554</td><td></td><td>278581649</td><td>2782.236792</td></tr<>	26	3329.691419	3305.818183	3014.08744	2800.0	2800.093	2800.133	2800.109	2800.123	2798.024	2798.048	2798.25	2785.554		278581649	2782.236792
1968097114         19947.74393         24033.8342         661.8.901         3421.272         3196.162         3188.55         3169.535         3199.596         3193.986         3303.21           2834996594         2939130498         3639061933         24012.206         12057.08         12070.83         10780.81         8168.29         6029.955         5527.441         5116.527           99488599.17         60313586.71         803640558         148339395         1178475         1009065         79926.06         61653.13         534673         5186.23         53355.54           9488599.17         60313586.71         803640558         148333395         1178475         1009065         79926.06         61653.13         534673         53365.54           14.53         13.97         13.30         7.00         5.83         5.67         4.73         4.00         4.13         5.557.441         51.65.27	27	6155.768633	6179.73393	7044 51333	4341.	3028.074	3009.722	3010.903	3019.961	3032.667	3055.574	3105.057	3196.33	3387.0514	3802.90628	4563.212503
2834996594         2939130498         3630061933         24012.206         12070.83         1078081         8168.29         6029.955         5527.441         5116.527           59488559.17         60313586.71         803640658         14839395         1178475         1000065         79926.06         61653.13         534673         5335.54         5335.54           54485         13.97         13.30         7.00         5.83         5.67         4.73         4.13         534573         5355.54	28	19689.97114	19947.74393	24033.8342	6618.5917	3421.272		3188.55	3169,835	3169.596	3193.983	3303.21	3574.232	4661.7652	5790.88651	1006597726
99488599.17         03313586.71         803640658         14839395         117847.5         100006.5         79926.06         61653.13         53467.3         53365.54         99118.85         62242.152           1453         13.97         13.30         7.00         5.83         5.67         4.73         4.13         4.00         4.13         5.53         5.54         99118.85         6.2242.152	29	2834996594	2939130498	363061933			12070.83	1078081	8168.29	6029.955	5527.441	5116.527		6219.6934		3232641.315
1453 13.97 13.30 7.00 5.83 5.67 4.73 4.13 4.00 4.13 5.53 6.73 8.63	30	59488559.17			_	117847.5	100906.5	79926.06	61653.13	534673	\$0826.33	53365.54	9911885	62242.152	102745.936	41 7009.8984
	Avg Rank		13.97	13.30	7.00	5.83	5.67	473	413	400	4.13	553	6.73	8.63	10.03	11.77

©2006-2016 Asian Research Publishing Network (ARPN). All rights reserved.

### www.arpnjournals.com

**Table-4.** G0 variant for post-hoc comparison using Holmprocedure.

 $z = (R_0 - R_i) SE$ i algorithms Holm 105 G01 vs. G09 9.122134 0.000476 0 104 G01 vs. G08 9.006664 0 0.000481 103 G01 vs. G10 9.006664 0 0.000485 102 G02 vs. G09 8 631387 0 0.00049 101 G02 vs. G08 8.515916 0 0.000495 100 G02 vs. G10 8.515916 0 0.0005 0.000505 99 G01 vs. G07 8.487049 0 98 G03 vs. G09 8.054036 0 0.00051 97 G02 vs. G07 7.996301 0 0.000515 96 G03 vs. G08 7.938566 0 0.000521 95 G03 vs. G10 7.938566 0 0.000526 7.794229 0.000532 94 G01 vs. G11 0 93 G01 vs. G06 7,72206 0 0.000538 92 G01 vs. G05 7.534421 0 0.000543 91 G03 vs. G07 7.418951 0 0.000549 90 G02 vs. G11 7.303481 0 0.000556 7.231312 89 G02 vs. G06 0 0.000562 7.043673 88 G02 vs. G05 0 0.000568 87 G01 vs. G12 6.754998 0 0.000575 86 G09 vs. G15 6.726131 0 0.000581 0 85 G03 vs. G11 6.726131 0.000588 84 G03 vs. G06 6.653962 0 0.000595 83 G08 vs. G15 6.610661 0 0.000602 0.00061 82 G10 vs. G15 6.610661 0 81 G01 vs. G04 6.480757 0 0.000617 80 G03 vs. G05 6.466323 0 0.000625 79 G02 vs. G12 6.26425 0 0.000633 0 78 G07 vs. G15 6.091045 0.000641 77 G02 vs. G04 5.990009 0 0.000649 76 G03 vs. G12 5.6869 0 0.000658 75 G03 vs. G04 5.412659 0 0.000667 74 G11 vs. G15 5.398225 0 0.000676 5.326056 0 0.000685 73 G06 vs. G15 72 G09 vs. G14 5.22502 0 0.000694 71 G05 vs. G15 5 138417 0 0.000704 70 G01 vs. G13 5.10955 0 0.000714 69 G08 vs. G14 5.10955 0 0.000725 68 G10 vs. G14 5.10955 0 0.000735 67 4.618802 0.000746 G02 vs. G13 0.000004 66 4.589935 0.000004 0.000758 G07 vs. G14 65 G12 vs. G15 0.000013 0.000769 4.358995 64 G04 vs. G15 4.084753 0.000044 0.000781 63 G03 vs. G13 4.041452 0.000053 0.000794 62 G09 vs. G13 4.012584 0.00006 0.000806 3.897114 0.000097 61 G01 vs. G14 0.00082 60 G11 vs. G14 3.897114 0.000097 0.000833 0.000097 59 G08 vs. G13 3.897114 0.000847 0.000097 0.000862 58 G10 vs. G13 3.897114 3.824946 57 G06 vs. G14 0.000131 0.000877 56 G05 vs. G14 3.637307 0.000276 0.000893 55 3.406367 0.000658 0.000909 G02 vs. G14 54 G07 vs. G13 3.377499 0.000731 0.000926 53 G12 vs. G14 2.857884 0.004265 0.000943 52 G03 vs. G14 2.829016 0.004669 0.000962

Table-3 shows the overall experimental results for Friedman procedure obtained in this study. For the case of GSAR2, value of G0=109 provides the best average ranking among others. These results were subjected to post-hoc test using Holm procedures and the results are shown in Table-4. According to Holm's procedure, hypothesis that have an adjusted p-value less or equal to 0.001887 are rejected.

Table-5. GSA Original vs GSAR2 with G0=109
Wilcoxon test result.

Function	GSA ORI	GSA R2	SIGN	ABS	R	SIGN R
1	14775830.9	1289971.341	1	13485859.58	28	28
2	22443764.2	22565.13575	1	22421199.09	29	29
3	138080.202	17514.3008	1	120565.9014	25	25
4	878.734707	515.0091718	1	363.7255	16	16
5	519.999717	520.0215943	-1	0.0219	2	-2
6	647.955355	601.6171107	1	46.3382	10	10
7	702.097125	700.0225056	1	2.0746	7	7
8	1076.49811	836.8604789	1	239.6376	13	13
9	1250.69963	931.132958	1	319.5667	15	15
10	8193.16657	3900.216001	1	4292.9506	21	21
11	9275.68745	3496.615988	1	5779.0715	22	22
12	1200.00289	1200.007415	-1	0.0045	1	-1
13	1300.47788	1300.280122	1	0.1978	4	4
14	1400.29839	1400.49231	-1	0.1939	3	-3
15	1765.90409	1506.23739	1	259.6667	14	14
16	1622.52317	1621.582546	1	0.9406	5	5
17	2181643.85	279582.1133	1	1902061.733	27	27
18	69338904.1	2525.323747	1	69336378.74	30	30
19	1944.0205	1962.814272	-1	18.7938	8	-8
20	59215.9615	22549.41379	1	36666.5477	23	23
21	1844950.35	140888.6913	1	1704061.661	26	26
22	4133.86178	2923.022917	1	1210.8389	17	17
23	2500	2648.032109	-1	148.0321	12	-12
24	2600.09343	2657.475362	-1	57.3819	11	-11
25	2700	2719.030494	-1	19.0305	9	-9
26	2800.08141	2798.023698	1	2.0577	6	6
27	4789.01228	3032.667359	1	1756.3449	18	18
28	6083.88723	3169.595938	1	2914.2913	19	19
29	3100.15831	6029.954977	-1	2929.7967	20	-20
30	3200.01244	53467.29837	-1	50267.2859	24	-24

In other words, G0=101, G0=102, G0=103, G0=1013, G0=1014, and G0=1015 are significantly different compared to G0=109 which was highlighted in Table-4. The rest of G0 value has no significant difference between each other. However, based on the average ranking, result of G0=109 is chosen for the comparison with the original GSA in pairwise Wilcoxon test. According to the result of the Wilcoxon test shown in Table-5, by using p-value equal to 0.05, the Z-value obtained is -2.931. Based on normal distribution curve it shows p-value for -2.932 is equal to 0.00338 which is smaller than 0.05. So it can be concluding the GSAR2 not only better than the original GSA in terms of performance, but also significant difference exists between these two algorithms.

# CONCLUSIONS

The original GSA algorithm is not genuinely follows the Newtonian gravitational law. In this paper, by correcting the force of calculation in original GSA and investigating various initial gravitational constants G0, ©2006-2016 Asian Research Publishing Network (ARPN). All rights reserved.

### www.arpnjournals.com

GSAR2 has been proposed. It is found that the GSAR2 not only superior to the original GSA, but most importantly, GSAR2 follows more closely to the Newtonian gravitational law.

# ACKNOWLEDGEMENTS

This work is financially supported by the Fundamental Research Grant Scheme (FRGS/1/2015/ICT02/MMU/03/1) and Fundamental Research Grant Scheme (R.J130000.7823.4F615) awarded by the Ministry of Higher Education (MOHE) to Multimedia University (MMU) and Universiti Teknologi Malaysia (UTM), respectively. The first author is thankful to Universiti Malaysia Pahang (UMP) for granting him an opportunity to further his study in postgraduate program in UMP.

# REFERENCES

 E. Rashedi, H. Nezamabadi-pour and S. Saryazdi. 2009. GSA: A Gravitational Search Algorithm. Information Science, 179(13): 2232-2248.

- [2] M. Gauci, T. J. Dodd and R. Grob. 2012. Why "GSA: A gravitational search algorithm" is not genuinely based on the law of gravity. Natural Computing, 11(4): 719-720.
- [3] B. Gu and F. Pan. 2013. Modified gravitational search algorithm with particle memory ability and its application. Int. J. Innov. Comput. Inf. Control, 9(11): 4531-4544.
- [4] R. Robert, J. Walker and D. Halliday 1993, Fundamentals of Physics. 9th Ed. John Wiley & Sons Inc. New York, USA. pp. 330-332.
- [5] J. J. Liang, B. Y. Qu and N. Suganthan. 2014. Problem definitions and evaluation criteria for the CEC2014 special session and competition on single objective real parameter numerical optimization. Technical Report 201311, Computational Intelligence Laboratory, Zhengzhou University, China and Technical Report, Nanyang Technological University, Singapore.