

# Comparison between the Performance of Spectrogram and Multi-Window Spectrogram in Digital Modulated Communication Signals

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**Abstract**— In this paper, we compare the performance of spectrogram and a new variation of multi-window (MW) spectrogram for various digital modulated signals. The windows used in the MW spectrogram are combination of Slepian sequences instead of using all the sequences. The comparison in the time-frequency representation is made in terms of main-lobe width (MLW) and peak to side-lobe ratio (PSLR) of the signals used. In the presence of noise, performance is compared in terms of variance in the estimated bit duration frequency. Bit duration and frequency are estimated from the instantaneous frequency (IF), which is derived from the time-frequency representation. The bias in the estimated bit duration is also calculated to compare the performance between spectrogram and MW spectrogram. In general, the time-frequency representation of the spectrogram is better than MW spectrogram. However, MW spectrogram is superior to spectrogram in terms of bit duration estimation.

**Index Terms**— digital modulation signals, multi-window spectrogram, Slepian sequence

## I. INTRODUCTION

Spectrogram has been widely used as one of the method for time-varying spectral analysis which is important in many applications such as radar, sonar, speech, geophysics and biological signals [1]. Recently, there has been increasing interest to improve the spectrogram in terms of bias and variance [2] by adapting the multiwindow spectral estimation approach introduced by Thomson in [3]. The concept of multi-window or multi-taper spectral estimation was discussed in [2]-[6]. This is performed by averaging the spectrum of the signal modulated with any set of orthogonal windows [4],

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such as minimum bias windows, sinusoidal windows and the sequence of discrete prolate spheroids or the Slepian sequence [5].

In [2], this method is incorporated into a nonstationary signal analysis context using the spectrogram and is shown to have lower variance and bias as compared to the traditional spectrogram. An improvement to the MW spectrogram has been carried out in [4] where the ambiguity domain filter is extended along the Doppler lag axis. It is shown that the new MW spectrogram has lower variance and bias than the MW spectrogram proposed in [2] at signal-to-noise ratio conditions of -3dB.

In this paper, we compare the performance between spectrogram and MW spectrogram in digital modulated signals such as amplitude shift keying (ASK), frequency shift keying (FSK), multi-frequency shift keying (M-ary FSK) and phase shift keying (PSK) signals. The comparison is made in terms of main-lobe width (MLW), peak-to-side lobe ratio (PSLR) and variance of estimated bit duration and variance of frequency at one particular bit duration in the presence of additive white Gaussian noise (AWGN). Ideally, a good spectrogram should have small MLW and large PSLR. In the presence of noise, the spectrogram should have low variance in both estimated bit duration and frequency to show stability in the estimation. For the MW spectrogram, the Slepian window with the sequence combination of 1 and 6 is used.

## II. SIGNAL MODEL

All the signals used are summarized in Table I. The set of signal chosen such they are similar to signals that are found in digital communication. Signals FSK1, FSK2, ASK, 8FSK, 16FSK and PSK are example of time-varying signals that are normally used in digital communications. 8FSK and 16FSK are examples of M-ary FSK signals. The main reason for choosing these signals is to evaluate the capability of the spectrogram and the MW spectrogram to represent and estimate the signal parameters in the time-frequency plane. The performance of the spectrogram and the MW spectrogram is compared for each of the signals used.

TABLE I  
SIGNALS USED TO TEST TIME-FREQUENCY REPRESENTATION

Signal	Mathematical representation
FSK1	$x(n) = \begin{cases} x_1(n) = A \cos \pi(17/32)n & s = 1 \\ x_0(n) = A \cos \pi(459/800)n & \text{when } s = 0 \end{cases}$ $0 : n : N - 1, \text{ Bit duration} = 80 \text{ samples}$
FSK2	$x(n) = \begin{cases} x_1(n) = A \cos \pi(17/32)n & s = 1 \\ x_0(n) = A \cos \pi(101/160)n & \text{when } s = 0 \end{cases}$ $0 : n : N - 1, \text{ Bit duration} = 80 \text{ samples}$
ASK	$x(n) = \begin{cases} x_1(n) = A \cos \pi(1/2)n & s = 1 \\ x_0(t) = 0 & s = 0 \end{cases}$ $0 : n : N - 1, \text{ Bit duration} = 80 \text{ samples}$
8FSK	$x(n) = \cos 2\pi f_i n \text{ where } 3/40 \leq f_i \leq 1/4,$ $f_i \text{ depends on the combination of 3 bits in a symbol}$ $0 : n : N - 1, \text{ Symbol duration} = 80 \text{ samples, 3 bits/symbol}$
16FSK	$x(n) = \cos 2\pi f_i n \text{ where } 1/20 \leq f_i \leq 17/40,$ $f_i \text{ depends on the combination of 4 bits in a symbol}$ $0 : n : N - 1, \text{ Symbol duration} = 80 \text{ samples, 4 bits/symbol}$
PSK	$x(n) = \begin{cases} x_1(n) = A \cos \pi(1/2)n & s = 1 \\ x_0(t) = A \cos(-\pi(1/2)n) & s = 0 \end{cases}$ $0 : n : N - 1, \text{ Bit duration} = 80 \text{ samples}$

### III. TIME-FREQUENCY ANALYSIS

#### A. Spectrogram

Spectrogram is one of the bilinear time-frequency distributions where the power spectrum of a nonstationary signal is represented in both time and frequency. Spectrogram is the squared magnitude of the short-time Fourier transform (STFT) [7]. It is defined as

$$S_x(n, f) = \left| \sum_{m=0}^{N-1} x(m)w(m-n)\exp(-j2\pi f m T_s) \right|^2 \quad (1)$$

where  $w(n)$  is the observation window. The window length used in this analysis is 128.

#### B. MW Spectrogram

MW Spectrogram is proposed as an improvement to the traditional spectrogram. The window functions used in this paper are the Slepian sequences. Slepian sequences are defined as solutions to the Toeplitz symmetric matrix eigenvalue problem [3]. It is the most common orthogonal set of windows used in the practical spectral analysis. It is defined as

$$\sum_{n=0}^{N-1} \frac{\sin 2\pi W(m-n)}{\pi(m-n)} w_k(n) = \lambda_k(N, W)w_k(m),$$

$$0 \leq m \leq N - 1$$

(2)

where  $N$  is the window length,  $W$  is the bandwidth and  $\{w_k\}$  are orthogonal vectors. The eigenvalues,  $\lambda_k(N, W)$  are arranged in decreasing order and further details are described in [4].

The selection of  $N$  and  $W$  is given in [2] as approximate of  $K = 2NW - 3$ . The choice of  $W$  dictates the number of Slepian sequences,  $K$  used in the analysis. The choice of bandwidth will determine the time and frequency resolution. In this analysis, the bandwidth is set as  $W = 2.5/N$ . This is because in this analysis the number of sequences is set as  $K=2$ . The Slepian sequence can be seen graphically in [2]. The combination of sequences,  $\{w_k\}$ , used in this analysis are sequence 1 and 6. The sequence number represents  $k$  in  $\{w_k\}$ . MW spectrogram is defined as

$$\hat{S}_{MW}(n, f) = \frac{1}{K} \sum_{k=1}^K |S_{MW,k}(n, f)|^2 \quad (3)$$

$$S_{MW,k}(n, f) = \sum_{m=0}^{N-1} x(m)w_k(m-n)\exp(-j2\pi f m T_s) \quad (4)$$

where  $w_k(n)$  is the  $k^{\text{th}}$  Slepian sequence from (2) and  $K$  is the number of Slepian sequences used in the analysis. In this paper, we use the combination of sequence 1 and 6 instead of using all the sequences. Equation (3) is modified as

$$\hat{S}_{MW}(f) = \frac{1}{2} \left[ |S_{MW,1}(n, f)|^2 + |S_{MW,6}(n, f)|^2 \right] \quad (5)$$

#### C. IF Estimation

The IF of a signal indicates the dominant frequency of the signal at a given time [7]. This accounts for the signal spectral variations as a function of time. It can be estimated from the peak of the time-frequency representation

$$f_i(n) = \max_f [S_x(n, f)] \text{ for } 0 \leq n \leq N - 1 \quad (6)$$

where  $S_x(n, f)$  is the time-frequency representation.

From the IF, the bit duration and bit rate can be estimated for ASK and FSK signals. It is not possible to determine the characteristics of PSK signal using the spectrogram. This is because PSK varies in phase and not in frequency. Thus it is not shown in the time-frequency representation.

#### D. Performance Measure

The performance measures for the time-frequency representation that are engaged in this paper are main lobe width (MLW) and peak-to-side lobe ratio (PSLR). The time-frequency representation is first calculated and the power

spectrum which is derived from the frequency marginal is obtained by the following expressions

$$S_x(f) = \sum_{n=0}^{N-1} S_x(n, f) \tag{7}$$

where  $S_x(n, f)$  is the time-frequency representation. The MLW and PSLR are then estimated from the power spectrum as shown in Figure 1.

A good spectrogram should have small MLW and large PSLR. MLW should be as small as possible to enable the representation of signals with two different frequencies, even if the two frequencies are closely together. PSLR should be as large as possible to resolve the problem of representing signals with low SNR.

Next, in the presence of AWGN, the variance in the estimated bit duration and frequency at one particular bit duration is compared between spectrogram and MW spectrogram. The bias in the estimated bit duration is also calculated to compare the accuracy in signal analysis (time-resolution) of each method. A good spectrogram should have low variance in both estimated bit duration and frequency and be able estimate the bit duration as close to the actual as possible. Variance in both estimated bit duration and frequency show stability in the estimation. The bias in the estimated bit duration shows the accuracy in signal analysis in terms of bit duration.

Signals that are chosen for comparison are FSK1 and 16FSK. This is because these signals have similar performance as the other FSK and M-ary FSK signals. For PSK signals, the bit duration cannot be estimated from the spectrogram because both time-frequency representation of spectrogram and MW spectrogram are unable to show the changes in phase. The performance of ASK signal is similar to FSK.

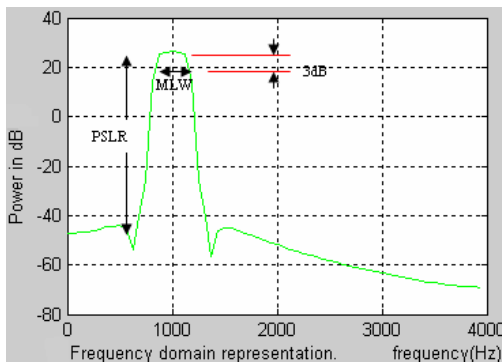


Figure 1. Performance measure used in the analysis

#### IV. RESULTS

The time-frequency plot for FSK1 and 16FSK is shown in Figure 2 and Figure 3 respectively. From these figures, it is shown that the time-frequency representation of MW spectrogram has more side lobes that cause leakages from the main lobe. As a result, the time-frequency representation is wider in both time and frequency.

Table II and III summarized the performance of both the spectrogram and MW spectrogram on various digital communication signals. In general, spectrogram has better performance than the MW spectrogram in terms of MLW and PSLR for all signals used. The spectrogram has smaller MLW and significantly higher PSLR than MW spectrogram. The presence of side lobes in the time-frequency representation results in lower PSLR.

Next, the performance of both methods is evaluated in the presence of noise. IF is derived from the time-frequency representation using equation (6). From the IF, bit duration and frequency are estimated. The performance is compared in terms of the variance of estimated bit duration and frequency and the bias in the estimated bit duration. These performance measures are calculated on 100 realizations of 16FSK and FSK1.

Figure 4-6 shows the variance of the estimated bit duration, the bias in the estimated bit duration and the variance of estimated frequency for the spectrogram and MW spectrogram. At low SNR, the uncertainty to estimate the IF contributes to the bias and variance in bit duration and frequency estimation. In general, the variance in the estimated bit duration increases with SNR. However, for 16FSK, the variance in the estimated bit duration is lower when using MW spectrogram. On the other hand, for FSK1, the variance in the estimated bit duration is lower when using spectrogram. The bias in the estimated bit duration is smaller at SNR above 6dB. In general, the MW spectrogram method gives smaller bias as compared to the spectrogram. For FSK1, the bias is still small at low SNR when using the MW spectrogram. The bias in the estimated frequency for FSK signal using both methods is comparable. However, in general, the bias is lower for MW spectrogram. From Figure 6, it is shown that the variance in the estimated frequency is similar for both methods. Based on the bit duration and frequency estimation, the MW spectrogram performs better than the spectrogram.

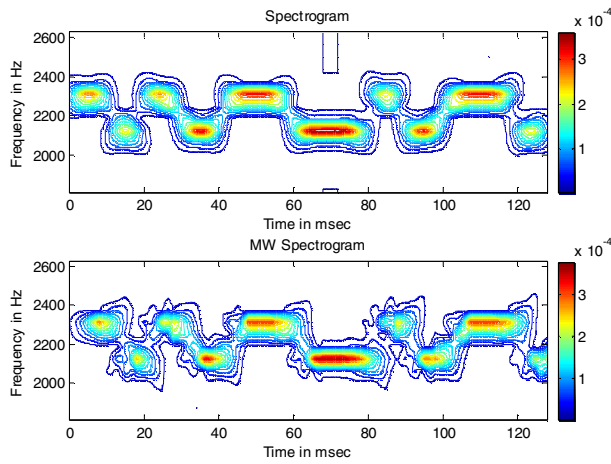


Figure 2. Time- frequency representation of FSK1

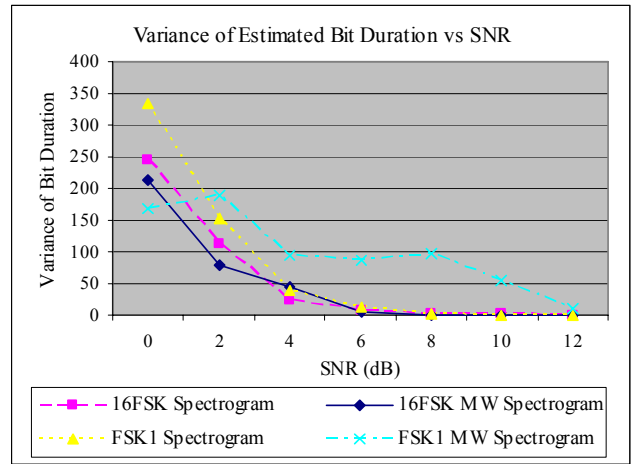


Figure 4. The comparison of spectrogram and MW spectrogram in terms of variance of estimated bit duration

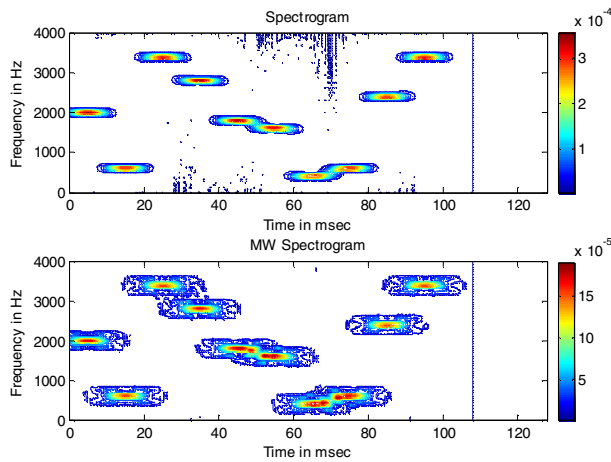


Figure 3. Time- frequency representation of 16FSK

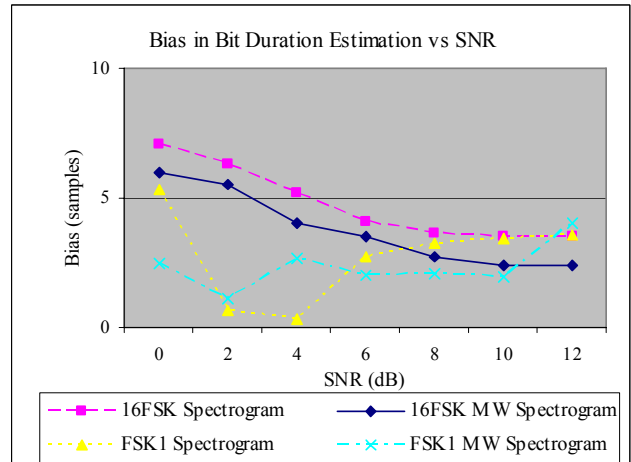


Figure 5. The comparison of spectrogram and MW spectrogram in terms of bias in the estimated bit duration

TABLE II  
PERFORMANCE OF SPECTROGRAM IN VARIOUS DIGITAL COMMUNICATION SIGNALS

Signal	MLW (Hz)	PSLR (dB)
1. ASK	90.000	36.000
2. FSK1	113.00	32.000
3. FSK2	101.00	44.200
4. 8FSK	115.20	32.680
5. 16FSK	120.60	36.260
6. PSK	70.761	32.266

TABLE III  
PERFORMANCE OF MW SPECTROGRAM IN VARIOUS DIGITAL COMMUNICATION SIGNALS

Signal	MLW (Hz)	PSLR (dB)
1. ASK	161.30	3.3750
2. FSK1	280.00	5.2700
3. FSK2	148.50	3.1850
4. 8FSK	190.00	9.4300
5. 16FSK	179.75	6.7400
6. PSK	120.25	3.2020

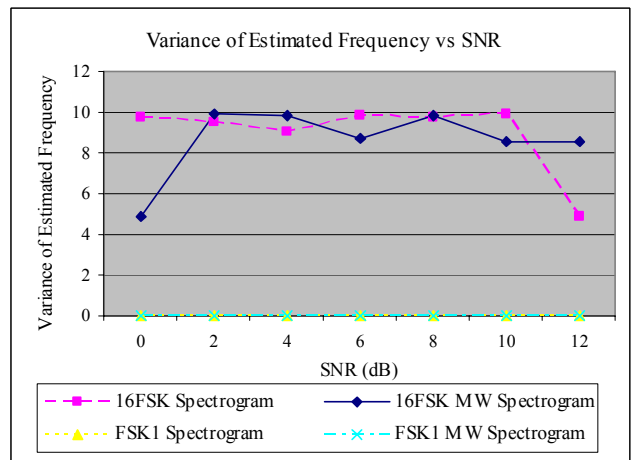


Figure 6. The comparison of spectrogram and MW spectrogram in terms of variance of estimated frequency

## V. CONCLUSION

Spectrogram and MW spectrogram can be used to analyze digital modulated signals such as ASK, FSK and M-ary FSK. Since phase is not represented in the time-frequency representation, the bit duration of PSK signals cannot be estimated using these methods. Spectrogram has superior performance in terms of MLW and PSLR. The variance in the estimated frequency is similar for both spectrogram and MW spectrogram. However, MW spectrogram is more accurate in bit duration estimation. This method has lower bias and lower variance in the estimated bit duration as compared to spectrogram

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## REFERENCES

- [1] L. Cohen, "Time-frequency distributions – A review," *Proc. IEEE*, vol. 77, pp. 941-981, July 1989.
- [2] G. Frazer and B. Boashash, "Multiple window spectrogram and time-frequency distributions", in *Proceedings of ICASSP '94*, vol. 4, pp. 293-296, 19-22 April 1994.
- [3] D. Thomson, "Spectrum estimation and harmonic analysis", *Proceedings of IEEE*, vol. 70, pp. 1055-1096 September 1982.
- [4] N. Stevenson, M. Mesbah and B. Boashash, "An improvement to the multiple window spectrogram using quadratic time-frequency distributions", *Proceedings of the Eighth International Symposium on Signal Processing and Its Applications, 2005*, vol. 2, pp. 651-654, 28-31 August 2005.
- [5] K. S. Riedel and A. Sidorenko, "Minimum bias multiple taper spectral estimation", *IEEE Transactions on Signal Processing*, vol. 43, pp. 188-195, January 1995.
- [6] T. P. Bronez, "On the performance advantage of multi-taper spectral analysis", *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 40, pp. 2941-2946, December 1992.
- [7] B. Boashash, *Time-Frequency Signal Processing: A comprehensive Reference*, Amsterdam: Elsevier, 2003.
- [8] Oppenheim and R. W. Schaffer, *Discrete time signal processing*, edition, Upper Saddle River: Prentice Hall, 1999