OPTIMIZATION OF A THERMOACOUSTIC REFRIGERATOR WITH AN EVOLUTIONARY ALGORITHM APPROACH
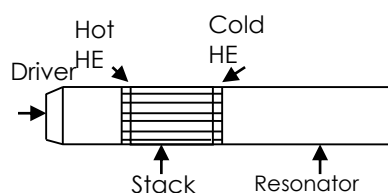
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Graphical Abstract

Abstract

Current non-environmentally friendly refrigerants released into our environment have caused serious concern over reports of the depleting of the ozone layer and global warming. Alternative technologies and efficient energy-related systems are being investigated to perhaps reduce if not stop the environmental degradation. This paper reports the outcomes of an optimization procedure performed on an environmentally friendly standing wave thermoacoustic refrigerator. A typical system to date has a low coefficient of performance (COP) and thus is not attractive to the general public. Optimization is completed using genetic algorithm over four design variables; the stack length and center position within a thermoacoustic resonator, the blockage ratio, and drive ratio. Optimization results show a maximum COP obtainable at 1.64. The outcomes indicate a potential for better thermoacoustic refrigerators in future.

Keywords: Optimization, genetic algorithm, thermoacoustic refrigerator, coefficient of performance

1.0 INTRODUCTION

Thermoacoustic cooling technology involves the interactions of fluid particles over solid boundaries. Acoustical work transfers heat from one end of a solid wall to another which over time generates a temperature gradient over that wall. The magnitude of that temperature gradient is constrained by axial conduction along the wall and within the fluid particles themselves. Generally, even as we speak, the expansion and compression of air particles over any solid boundaries produce a temperature gradient but this is minuscule i.e. of the order of 10^{-5} °C. At high pressure within an enclosure, oscillating fluid particles passing over a system of solid walls (called the stack) could generate a significant temperature difference, as proven by past reports [1]. The absence of any refrigerants deems a thermoacoustic refrigerator environmentally friendly. Unfortunately, the coefficient of performance (COP) of the completed systems so far

is lower than the conventional vapor compression counter parts [2].

Previous optimization work involved either the geometrical or the operating parameters, generally separately. Wetzel and Herman [3], Tijani *et al*. [4] Babaei and Kamran [5], and Zink *et al*. [6] are among those who have investigated the geometry of the thermoacoustic refrigerator. They studied the effects of the stack length and center position, stack plate spacing and the resonator length on the performance of the system. Meanwhile, Minner *et al*. [7], Emmanuel and Azrai [8], Tasnim *et al*. [9] studied the effects of the operating parameters; frequency, working fluid, and the mean temperature and pressure in the resonator.

Despite being an environmentally friendly technology, the standing wave thermoacoustic refrigeration is yet to be considered seriously due to its low performance, particularly for the standing wave type. Optimization of the controlling parameters should be completed to determine the best that can be delivered by this cooling technology [10]. A

literature review completed has shown that hardly any theoretical optimization has been completed on a standing wave thermoacoustic refrigerator [11]. DeltaE, a software specific to thermoacoustic system design which had been generally used in the past, involves vigorous amount of work [8]. Besides, only a local optimum/minimum is obtainable. Genetic algorithm (GA) is based on evolutionary algorithm, a relatively recent optimization scheme with a strong ability in global search for the optimized solution(s) [12]. The algorithm has also been attempted in the optimization of several parameters [13, 14]. This paper presents the outcomes of a GA optimization of the performance of a standing wave thermoacoustic refrigerator. The thermoacoustic system being investigated follows the design of Tijani *et al.* [12].

2.0 METHODOLOGY

2.1 Theory of Thermoacoustic Refrigerator

The simplest thermoacoustic refrigerator consists of four main components, (i) a resonator tube, (ii) a porous medium called the stack, (iii) an acoustic driver at one end of the resonator tube attached to generate the acoustic standing wave inside the tube, and (iv) the working gas. These are shown schematically in Figure 1. For the current analysis, helium is used since the gas has the highest sound velocity and thermal conductivity among the noble gases. Besides, helium can be easily found and cheap. The stack is the core of the thermoacoustic refrigerator where the desired thermoacoustic cooling effects occur. The cooling load is the amount of heat removed from the cold heat exchanger (HE), Q_c, and the work needed is W_n, the acoustical power which is supplied by an acoustic driver. The driver generates and sustains the standing wave against dissipations due to thermal losses and viscous effects. The cooling load and work are described by [12],

$$Q_{cn} = \frac{\delta_{kn} DR^2 \sin 2x_n}{8\gamma(1+\sigma)\Lambda} x \left(\frac{\Delta T_{mn} \tan x_n}{(\gamma-1)BL_{sn}} \frac{1+\sqrt{\sigma}+\sigma}{1+\sqrt{\sigma}} - (1+\sqrt{\sigma}-\sqrt{\sigma}\delta_{kn}) \right)$$ (1)

and

$$W_n = \frac{\delta_{kn} L_{sn} DR^2}{4\gamma} (\gamma 1) B\cos^2 x_n \, x \left(\frac{\Delta T_{mn} \tan x_n}{BL_{sn}(\gamma-1)(1+\sqrt{\sigma})\Lambda} - 1 \right) - \frac{\delta_{kn} L_{sn} DR^2}{4\gamma} \frac{\sqrt{\sigma}\sin^2 x_n}{B\Lambda}$$ (2)

where Λ is defined as

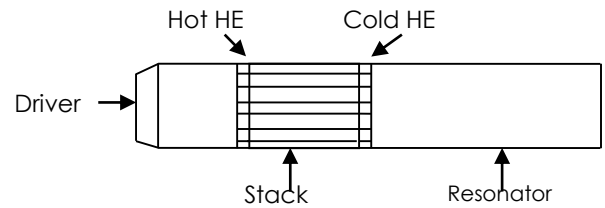$$\Lambda = 1 - \sqrt{\sigma}\delta_{kn} + \frac{1}{2}\sigma\delta_{kn}^2$$ (3)

Figure 1 Schematic of a simple standing wave thermoacoustic refrigerator.

For the stack, the COP is determined from [2],

$$\text{COP} = \frac{Q_{cn}}{W_n}$$ (4)

where an increase in the cooling load will of course increase the work necessary to provide that cooling, they are inter-dependent. Thus, maximization of Equation (4) requires that the controlling parameters in Equation (1) and Equation (2) to be optimized simultaneously. The normalized terms in Equations (1) and (2) are listed in Table 1 with the related parameter values included in Table 2. Past optimizations involved variations of individual variables in Equation (1) and Equation (2) over a selected range with the other selected parameters to be optimized too, being held constant. For this study, if the stack center position, x_{sn}, is to be optimized, the stack length, L_{sn}, which needs to be optimized too, takes on preset values while x_{sn} itself is changed and the outcome of the COP analyzed. Subsequent variation of x_{sn}, with related outcomes are tabulated before similar steps are repeated with L_{sn}. The "optimized" parameters are then selected from the organized results. However, GA optimization scheme performs the optimization simultaneously through a series of selection, cross-over, and mutation of the "chromosomes" that represent the variables, details of which can be found in [15].

Table 1 Operating parameters and properties

Operating parameters	Gas parameters
p_m = 10 bar	a = 935 m/s
T_m = 250 K	σ = 0.68
ΔT_{mn} = 0.3	γ = 1.67
DR = 0.02	B = 0.75
f = 400 Hz	k = 2.68 m^{-1}
	δ_{kn} = 0.66

Table 2 Dimensionless parameters in the thermoacoustic refrigerator system

Operation parameters
Drive ratio: $D = p_0/p_m$
Normalized cooling power: $Q_{cn} = Q_c/p_m aA$
Normalized acoustic power: $Wn = W/p_m aA$
Normalized temperature difference: $\Delta T_{mn} = \Delta T_m/T_m$
Gas parameters
Prandtl number: σ
Normalized thermal penetration depth: $\delta_{kn} = \delta_k/y_0$
Stack geometry parameters
Normalized stack length: $L_{sn} = kL_s$
Normalized stack position: $x_n = kx$
Blockage ratio or porosity: $B = y_0/(y_0 + l)$

In the current study, four parameters appearing in Equations 1 and 2 are simultaneously optimized which will subsequently optimize the COP in Equation 4. These parameters are L_{sn}, , x_{sn}, , B, and DR. These have been identified as the controlling parameters that can be manipulated to achieve the desired COP as high as possible. B is the blockage ratio is defined by

$$B = \frac{y_0}{y_0 + l} \qquad (5)$$

where y_o is the half spacing of the stack and l is the thickness of plate. DR, is the drive ratio defined by

$$DR = \frac{p_0}{p_m} \qquad (6)$$

with p_0 being the dynamic pressure and p_m the mean operating pressure.

2.2 Optimization Using Genetic Algorithm

The thermoacoustic refrigerator designed and fabricated by Tijani *et al.* [12] is used as the basis for the present optimization with GA. The COP of the thermoacoustic refrigerator is selected as the objective function to be maximized. GA optimization scheme in the MATLAB toolbox is used to perform the optimization [16]. Simultaneous optimization of the geometric and operating parameters will produce an optimized COP. The algorithm tries to maximize f(X) through combinations of the variables such that the variables that are finally chosen will be optimized. $f(X)$ is subjected to equality constraints $g_j (x_1, x_2, x_3, ..., x_n) = 0$ and inequality constraints $h_j (x_1, x_2, x_3, ..., x_n) \geq 0$. The objective function to be maximized is written in this form:

maximize $f(X) = COP (X)$ (7)

which is then subjected to the imposed constraints (range) for (X); the stack length, stack center position, blockage ratio, and drive ratio,

$$0 \leq L_{sn} \leq 1$$
$$0.06 \leq x_{sn} \leq 0.42$$
$$0.67 \leq B \leq 0.8$$
$$0.015 \leq DR \leq 0.03$$

Parameters L_{sn}, x_{sn}, B, and DR are the decision variables allowed to vary in the bound. The decision variables are represented in binary strings in order to find the optimum solution which satisfy the constraints and maximise $f(X)$. Figure 2 shows the flowchart of the process involved in a GA application. It begins with a search among the random population of solution sets of solutions.

The flowchart of the Genetic Algorithm is shown in Figure 2 and the operator values used are listed in Table 3. The process of evolution will continue until one of the stopping criteria reached.

The fitness objective function, COP, will have the highest probability to be selected to carry out to become the second generation. The next operator in the evolutionary algorithm, the crossover operator randomly selects one cut-point and exchanges parts of the variables to be optimized to generate different values of the objective function. The crossover fraction will determine the amount of variables that will undergo a crossover; in this case, the crossover fraction is 80%. The mutation operator follows which in this study is taken to be 1% of the whole population solution that will be altered.
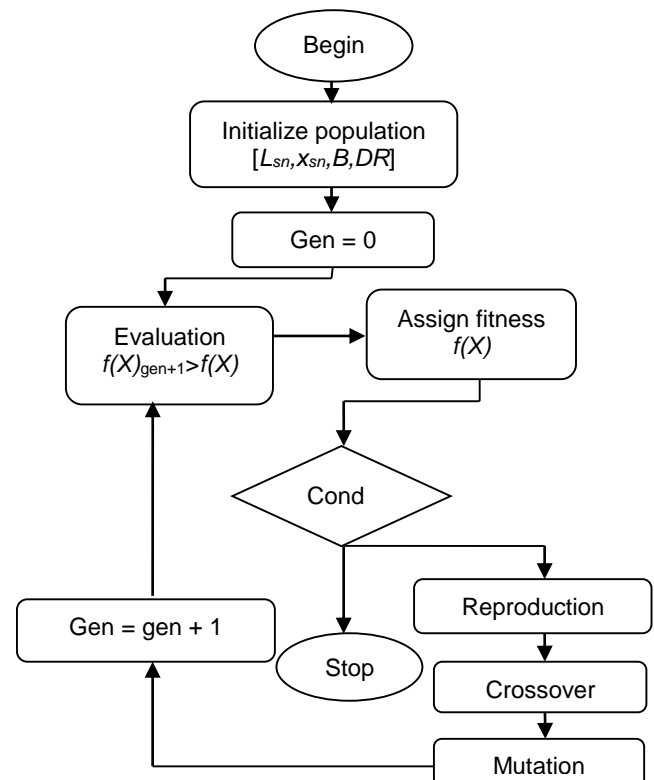

Figure 2 Flowchart of the GA optimization for a thermoacoustic refrigerator

Table 3 Genetic Algorithm operators

Genetic operator	Value
Population size	100
Fitness scaling	Rank
Selection function	Roulette Wheel
Crossover function	Arithmetic
Crossover fraction	0.8
Mutation rate	0.01

3.0 RESULTS AND DISCUSSION

3.1 Two Parameters Optimization

Single objective optimization by using genetic algorithm starts by setting the coefficient of performance (COP) as the objective function to be maximized withthe stack length, L_{sn}, and stack center position,x_{sn}, as the optimized parameters. Figure 3 shows the plot of fitness value versus generations. The solutions converge at the 51st generation and the maximum archive is 1.5582 with L_{sn} = 0.18 or 3.5 cm and x_{sn}= 0.18 or 3.5 cm. Compared with the optimization outcome through a parametric study which necessitates discrete variations within the range of values set, GA has been able to find the combination of parameters to be optimized to achieve the maximum COP through the evolution of solution sets. Since the GA tool was developed to search for a minimum, a maximum value of the function requires a negative sign.
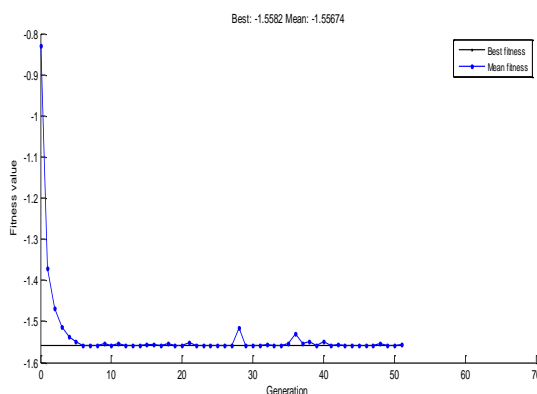

Figure 3 Fitness value versus generation for two parameters optimization

The effect of the stack length and center position is significant in a thermoacoustic refrigerator. The length of the stack determines the surface area prepared for the heat transfer process between the gas particles and the stack plate. An increase in the stack length will increase the power density of the stack. However, increasing the surface area of the stack increases the acoustic impedance and the pressure will drop. This effect will cause the reduction of COP beyond a certain length.

The stack position within the resonator is controlled by two main factors: the acoustic power at the mid-stack position, and the viscous and thermal relaxation losses along the stack channels. The acoustic power is the dot product of the acoustic pressure and the gas parcel velocity. This product is zero at the pressure and velocity nodes (minimum), which occur at the beginning, middle and end of the resonator. The optimum position depends on careful estimation of viscous and thermal relaxation losses. These in turn depend on the Reynolds number, channel width, thermal penetration depth, length and surface roughness in the channels, and all the minor losses in the resonator when the flow cross-sectional area changes. By comparing with Tijani *et al.* [3], the COP here has increased from 1.3 to 1.588 by using Genetic Algorithm optimization, an increment of about 22.15%. This means that a higher COP is possible for the thermoacoustic refrigerator if the optimized parameters are implemented.

3.2 Four Parameters Optimization

Thermoacoustic refrigerator parameters are inter-dependent, thus simultaneous optimization method is important. In this section the study proceeds to include two more parameters and the optimization is repeated. The third parameter is the spacing of the plate, $2y_0$, which has been suggested to be between $2\delta_k$ and $4\delta_k$ [13]. The fourth parameter that is being optimized is the drive ratio, D. Compared to the other parameters, the range of the variables is quite large but for D, the range of this variable is between $0.02 \leq D \leq 0.03$. The range is very limited to make sure the flow in the resonator is less than the Mach number, $M \leq 0.1$.

Referring to Eqn. 5, the blockage ratio consists of the thickness, l and the plate spacing, $2y_0$ of the stack. By fixing the thickness of the plate, blockage ratio will represent the plate spacing parameter. The plate spacing needs to be determined carefully becauseif the walls of the stack are too close to each other, gas parcels cannot pass through the stack efficiently. This is due to the viscous properties of the working fluid preventing the working fluid from oscillating. If the walls are too far apart, effective heat transfer between the gas packets and stack walls cannot occur effectively. As reported by Wheatly *et al.* [17], the desired separation gap between the solid walls has been reported to be between 2 to 4 thermal penetration depths, but the difference is still significant. By using GA, the result shows that the plate spacing produce an optimum performance when $2y_0 = 3\delta_k$.

Figure 4 shows the optimization results for four parameters. By comparing with the outcome of the two parameters optimization, the COP has now increases from 1.55 to 1.58. The improvement of the COP is not significant but for the aspect of cooling power the difference is substantial. For two parameters optimization, Q_c = 3.21 Watt and W = 2.1 Watt. The four parameters optimization producedthe highest COP accomplished that is 1.58. This highest COP achieved is when the acoustic work, W, is 4.33 Watt, providing a

cooling power, Q_c, of 6.84 Watt. The combination of the optimized variables are x_s = 6.7 cm, L_s = 6.7 cm, B = 0.8, D = 0.026. The increment of cooling power is 35%.
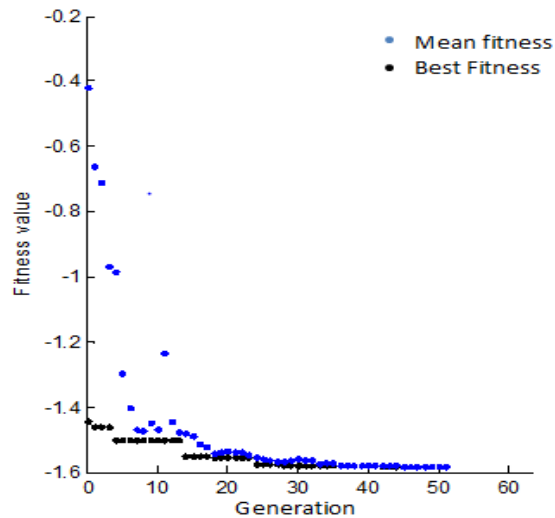

Figure 4 The fitness function for every generation Helium

3.3 Effect of Different Working Fluid

The effect of using a different working fluid has also been investigated. Figure 5 shows the fitness value which is the COP and the generations for the single objective optimization with a mixture of helium and xenon as the working fluid.
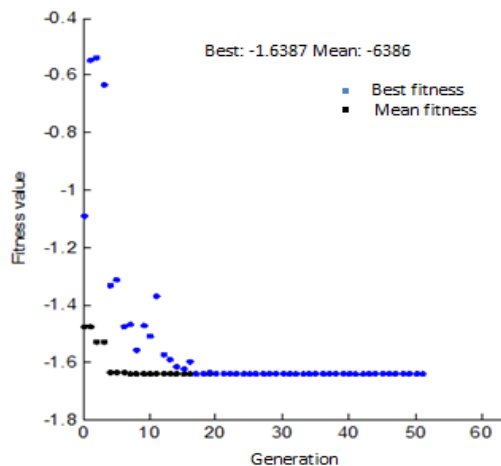

Figure 5 The fitness function for every generation He-xe

For a mixture of helium and xenon (He-Xe), optimization of the COP results in the best COP of 1.64. To achieve this highest COP, the acoustic power needed is 1.18 Watt which provides a cooling power, Q_c = 1.93 Watt. This is obtained from the combination of optimized variables; x_s = 3.57 cm, L_s = 3 cm, B = 0.8, D = 0.026. . There is an increase of 26% in the COP above that obtained by Tijani *et al.* [3]. The

optimization outcome here indicate a possible COP of 1.64 compared to the 1.3 reported.

The maximum cooling power extracted by using helium gas is 6.84 W and by using He-Xe, 1.93 W. This phenomenon was also observed by Tasnim *et al* [9] and Tijani [12] in their works. The results were obtained because the cooling power, Q_c is inversely proportional to the product $p_m a$, where a is the adiabatic speed of sound. The product of $p_m a$ increases when the gas mixture is used thereby reducing the cooling power of the thermoacoustic device. Although helium provides a higher cooling power compared to He-Xe, the COP of the thermoacoustic refrigerator is much higher if He-Xe is the working fluid. The COP of He-Xe is higher because when the Prandtl number is low, the viscous losses are kept at a minimum.

By changing the working fluid from pure to a binary gas mixture, both of which is still from the same group, the transport coefficient which is the Prandtl number changed. The Prandtl number depends on the dynamic viscosity, μ, thermal conductivity, K, isobaric specific heat, c_p, and density, ρ. The viscosity gives the negative effect on the performance of the thermoacoustic refrigerator, reduction of the viscous effects means an increase in the efficiency. The transport coefficient such as the diffusivity, viscosity and thermal conductivity affects the transport of mass, momentum, and energy by means of molecular motion and molecular collision. The low Prandtl number increases the transport coefficient of the gas and improves the performance of the thermoacoustic refrigerator. Unfortunately, the use of a mixture of He-Xe gas lowers the cooling power that can be extracted. This is due to the increase in the working fluid density contributed by the xenon gas which results in in a decrease in the cooling power.

Optimization using Genetic Algorithm is based on the probability to satisfy the objective function, maximization of the COP. Based on the two and four parameters optimization,the   COP has improved. As seen in Figure 6, the optimized stack length is 6.7 cm, which is almost double that of the two parameters optimization results. Thus, more material is needed to built the stack. Due to that, the cooling power extracted from the system is larger because the longer stack has more surface area for heat transfer between the gases and stack plate to take place. The results between the four parameters optimization, pure helium against the He-Xe mixture,shows the shortest stack length possible for the highest COP.The drawback by using a mixture of helium and xenon is that the COP is high at low cooling power.
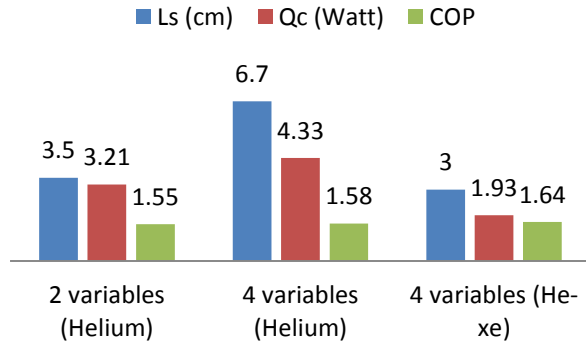
Figure 6 The value of Lsn, Qc, and COP for 2 variables and 4 variables optimization for helium and he-xe

4.0 CONCLUSION

Maximization of the stack coefficient of performance (COP) in a standing wave thermoacoustic refrigerator under four optimized controlling parameters have been completed. Single-objective optimization with genetic algorithm (GA) was performed and an improved COP of the stack was obtainable at 1.58 and 1.64 for helium and helium-zenon respectively, compared to that 1.3 from Tijani *et al* [3]. Results of the optimization of the stack unit means a reduction in the losses of the resonator since this provides a more compact stack but still produces a better performance. This study indicates the potential that GA provides towards the improvement of the performance of a standing wave thermoacoustic refrigerator, in particular the stack component – the core of the system.

Acknowledgement

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