



PERFORMANCE OF TWO NEW EMPIRICAL EQUATIONS COMPARED TO POLYNOMIAL, EXPONENTIAL, POWER AND LOGARITHMIC FUNCTION FOR MODELLING LOW FLOW AND HIGH FLOW DISCHARGES

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ABSTRACT

Empirical equations to describe flow duration curve (FDC) are mostly in the form of exponential, logarithmic, power or even polynomial functions but none of these fit the dataset of the study site of this research. This paper proposed two new empirical functions, modified from soil water retention equations. The efficiency and prediction accuracy of our new empirical equations were evaluated against each mentioned common function at the study site. Polynomial function was discarded as it failed to fit the dataset. Power function over-predicted nearly every quantile and induced un-acceptable huge difference especially at high flow end of the FDC. Logarithmic was the only function that yields negative predicted low flow and under predicted peak flow by 85%. On the other hand, exponential function almost under predicted peak flows by 100%. New empirical equations have highest Nash-Sutcliffe efficiency with lowest overall RMSE, quantile cumulative RMSE at high flow range and percentage error at the highest peak flow points. A parsimonious form of the new empirical equation was also presented and discussed in this paper.

Keywords: flow duration curve, new empirical equation, soil water retention equation.

INTRODUCTION

A Flow duration curve (FDC) depicts magnitude and frequency of observed flow by defining the proportion of time for which any discharge is equalled or exceeded within a period of study interest (Vogel & Fennessey, 1994). FDC has been used by hydrologist since the end of the 19th century (Sugiyama & Whitaker, 1999). It contains useful information for evaluating flow variability and characterizing hydrological regimes at a particular site. FDC captures full range of flow information thus it becomes a common tool in water management applications (Vogel & Fennessey, 1995; Smakhtin & Masse, 2000). It represents a compact signature of temporal runoff variability which can also be used to diagnose catchment rainfall-runoff responses, including similarities and differences between catchments (Cheng *et al.* 2012).

Studies of FDC allow in depth analysis of flow return period, determine probable maximum peak flow to aid flood prediction and low flow assessment of a region through its lower end of the curve. It is also useful for assessing water supply (McMahon, 1993), water quality (Vogel & Fennessey, 1995), evaluating river habitats (Booker & Dunbar, 2004) and designing hydropower facility (Warnick, 1984; Niadas & Mentzelopoulos, 2007; Heitz & Khosrowpanah, 2010; Baltas, 2012). FDC is also useful to assess flow requirements for different instream uses, to protect aquatic environment and evaluate anthropogenic environmental impacts. Some researchers also used FDC as an indicator of changes in land use (Lane *et al.* 2005; Brown *et al.* 2013). Male & Ogawa (1984) showed how FDCs can be used in the selection of a waste-water treatment plant capacity. Hughes & Smakhtin

(1996) developed a method based on FDC for patching and extending observed time series of daily stream flow. Petheram *et al.* (2008) studied flow characteristics of rivers in northern Australia as an implication of anthropogenic effect. Smakhtin & Masse (2000) proposed a methodology to simulate continuous FDC from observed daily rainfall data in South African while Patil & Stieglitz (2011) analysed FDC to determine the hydrologic similarity among catchments in the USA. In China, Huang & Zhu (2009) used FDC for hydro model calibration in their research. Studies of FDC allow in depth analysis of flow return period, determine probable maximum peak flow to aid flood prediction and low flow assessment of a region through its lower end of the curve. It is also useful for assessing water supply (McMahon, 1993), water quality (Vogel & Fennessey, 1995), evaluating river habitats (Booker & Dunbar, 2004) and designing hydropower facility (Warnick, 1984; Niadas & Mentzelopoulos, 2007; Heitz & Khosrowpanah, 2010; Baltas, 2012). Some researchers also used FDC as an indicator of changes in land use (Lane *et al.* 2005; Brown *et al.* 2013), to assess flow requirements for different instream uses, protect aquatic environment and evaluate anthropogenic environmental impacts. Male & Ogawa (1984) showed how FDCs can be used in the selection of a waste-water treatment plant capacity. Hughes & Smakhtin (1996) developed a method based on FDC for patching and extending observed time series of daily stream flow. Petheram *et al.* (2008) studied flow characteristics of rivers in northern Australia as an implication of anthropogenic effect. Smakhtin & Masse (2000) proposed a methodology to simulate continuous FDC from observed daily rainfall data in South African while Patil & Stieglitz



(2011) analysed FDC to determine the hydrologic similarity among catchments in USA. In China, Huang & Zhu (2009) used FDC for hydro model calibration in their research.

Empirical equations to describe FDC are mostly in the form of exponential, power or logarithmic function. In general, fitting results from these functions are unable to fit extreme values at both high and low flow quantiles. We took a different approach and explored possibilities of using other type of function to describe the curve. This paper presents two new empirical functions, modified from water retention equations developed by van Genuchten (1980) and Fayer and Simmons (1995). Both new empirical functions are aimed at improving FDC extreme flow quantiles fitting ability.

NEW EMPIRICAL EQUATIONS AND OTHER COMMON FITTING FUNCTIONS

van Genuchten (1980) adopted the following general equations from Mualem (1976)

$$Q - Q_{min} = \frac{Q_{max} - Q_{min}}{[1 + (\alpha \times h)^n]^m} \tag{1}$$

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{2}$$

By equating (1) to (2) and re-arrange the equation, the new form of equation becomes:

$$\theta - \theta_r = \frac{\theta_s - \theta_r}{[1 + (\alpha \times h)^n]^m} \tag{3}$$

where Θ is the effective saturation of soil water retention curve which depends on the pressure potential of soil (h). h in above equations is assumed to be positive. α , n , and m are unknown parameters. θ is the soil water content depending on h , θ_r is the residual water content and θ_s is the saturated water content. The graphical interpretation of equation (3) can be illustrated in Figure-1. The graph of the soil-water retention vs. pressure potential of soil closely resembles the shape of FDC thus inspired and initiated the attempt to adopt and modify van Genuchten's equation for fitting FDC in this study. θ_r becomes Q_{min} , θ_s becomes Q_{max} and θ becomes Q . The general form of the modified and proposed first new FDC fitting equation by us is:

$$Q - Q_{min} = \frac{Q_{max} - Q_{min}}{[1 + (\alpha \times h)^n]^m} \tag{4}$$

where α , n , and m are curve fitting parameters. Q_{max} and Q_{min} are the highest and lowest observed

discharge within the period of study interest. When Q_{min} is equal to zero equation (4) simplifies into (5)

$$Q = \frac{Q_{max}}{[1 + (\alpha \times h)^n]^m} \tag{5}$$

More than a decade later, Fayer and Simmons (1995) proposed another modified soil water retention function from the Brooks-Corey and van Genuchten functions which resulted in a better fitting for lower ranges of water content and relative conductivity values compared to van Genuchten and Ross, and Nimmo's functions (see Figure-2).

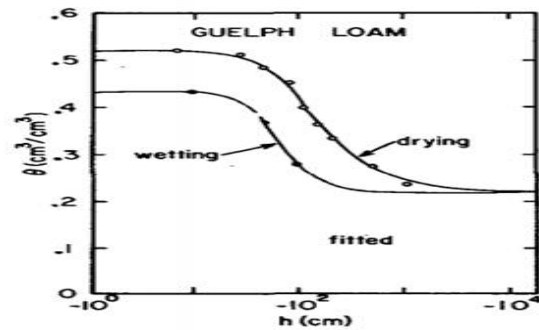


Figure-1. Soil water retention graph of van Genuchten (1980).

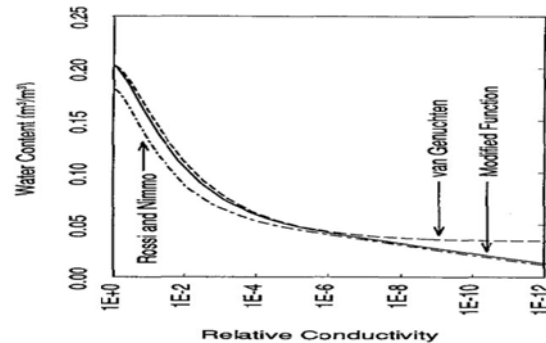


Figure-2. Fayer and Simon's modified function fitting ability comparison.

Fayer and Simmons (1995) adopted the water absorption of soil equation proposed by Campbell and Shiozawa (1992):

$$\theta = \left[1 - \frac{\ln(h)}{\ln(h_m)} \right] \theta_a \tag{6}$$

where θ_a is a curve fitting parameter representing the volumetric water content when $h = 1$ and h_m is a curve fitting parameter representing the matric suction at the oven dried water content. A general form of equation (6) is made possible by multiplying h to a parameter β , where β



=1 and has the inverse of the units of h used to fit θ_a . The bracketed term in equation (6) became (7).

$$x(h) = \left[1 - \frac{\ln(\beta h)}{\ln(\beta h_m)} \right] \quad (7)$$

Fayer and Simmons (1995) proposed to replace θ_r in equation (3) with the combined version of (6) and (7) to become (8). The adopted equation became:

$$\theta = x(h)\theta_a + [\theta_s - x(h)\theta_a [1 + (\alpha h)^n]^{-m}] \quad (8)$$

We were inspired to modify equation (8) into (9) where θ_a becomes Q_{\min} , θ_s becomes Q_{\max} and θ becomes Q . Aimed at improving low flow end fitting ability of equation (4) as Fayer and Simmons (1995) achieved better fitting result than equation (3), the general form of the modified and proposed second new FDC fitting equation is:

$$Q = x(h)Q_{\min} + [Q_{\max} - x(h)Q_{\min} [1 + (\alpha P)^n]^{-m}] \quad (9)$$

$$\text{with } x(h) = \left[1 - \frac{\ln(P)}{\ln(P_{\min})} \right]$$

where α , n , and m are undetermined parameters of curve fitting while P represents the exceedance probability of a distributed frequency discharge. P_{\min} denotes the exceedance probability of the minimum flow at low flow end. Q_{\max} and Q_{\min} are the highest and lowest observed discharge respectively. Numerical analysis approach can be conducted to obtain optimum values for α , n , and m . When $Q_{\min} = 0$, equation (9) can also be presented as (5).

Two parameters logarithmic, power, exponential and polynomial functions were compared against our proposed FDC empirical equation (4) and (9). The general structures of these functions are given by equations (10) to (13) listed below:

$$\text{Logarithmic function: } Q = a \ln(P) + b \quad (10)$$

$$\text{Exponential function: } Q = a e^{bP} \quad (11)$$

$$\text{Power function: } Q = a P^b \quad (12)$$

$$\text{Polynomial function: } Q = aP^2 + bP + c \quad (13)$$

where a , b and c are model fitting parameters, P is the exceedance percentile for which the flow is equalled or exceeded.

Data and Methodology

Daily river discharge data (Figure-3) from 1960 to 2012 at Segamat station (station no 2528414) in Johor state, Malaysia was obtained from the Department of Irrigation and Drainage (DID), Malaysia. The station is

located at the southern part of peninsula Malaysia in Johor state. Johor state is situated at the north-west direction from the neighbouring country Singapore. The maximum river discharge was logged at 1,559 m³/s and mean discharge of 16.18 m³/s at this site. The FDC data was ranked and constructed with exceeded probabilities from Weibull formula.

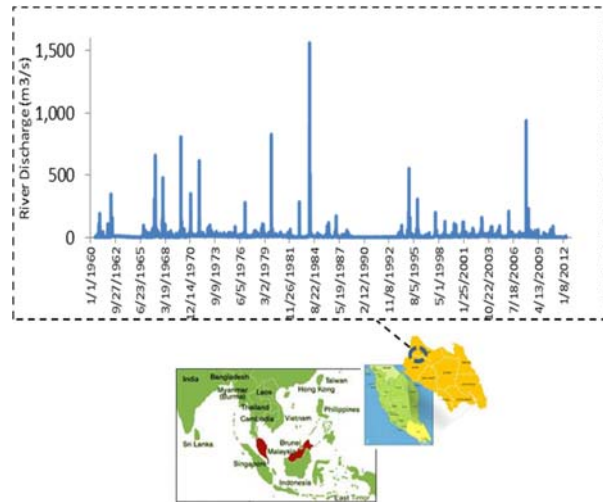


Figure-3. Daily discharge (m³/s) at Segamat station from 1960 to 2012.

The data record of this site actually contains one entry of zero m³/s, under the suspicion of recording errors. Equation (4) and (9) take into consideration of both Q_{\max} and Q_{\min} , when $Q_{\min} = 0$, the general form of the modified FDC fitting equations in (4) and (9) can be simplified as (5). If zero m³/s was excluded, the next non-zero lowest flow (0.2 m³/s) would be used as Q_{\min} . This study analysed the data set both ways. When zero m³/s was included in the analysis, Equation (5) was used, when zero m³/s was excluded, either equation (4) or (9) was used in the analysis where $Q_{\min} = 0.2$ m³/s. It is of interest to explore and compare the predictability of the omission decision using equation (5). Statistical insignificant difference between predicted discharge generated by both choice is required to warrant that the exclusion decision would not impede the overall prediction result or else the zero m³/s cannot be excluded from the analysis.

RESULTS AND DISCUSSIONS

The optimum fitted first empirical equation for this study site is:

$$Q = \frac{1559}{\left[1 + (54.25P)^{1.31} \right]^{0.49}} \quad (14)$$

Equation (14) included zero m³/s as Q_{\min} . The optimum fitted second empirical equation is:



$$Q = 0.2x(h) + [1559 - 0.2x(h)] \left[1 + (54.18P)^{1.31} \right]^{-0.49} \quad (15)$$

$$\text{with } x(h) = \left[1 - \frac{\ln(P)}{4.61} \right]$$

Equation (15) excluded zero m^3/s as Q_{min} and used next lowest flow of 0.2 m^3/s as Q_{min} . Other fitted functions via least square fitting methodology of this site are listed below:

$$\text{Polynomial function: } Q = 0.014P^2 - 1.9P + 64.79 \quad (16)$$

$$\text{Logarithmic function: } Q = -24.91 \ln(P) + 105.9 \quad (17)$$

$$\text{Power function: } Q = 205.31 P^{-0.84} \quad (18)$$

$$\text{Exponential function: } Q = 44.42 e^{-0.03P} \quad (19)$$

Evaluation of Equation (14) and (15)

The predicted discharges from equation (14) and (15) seem to overlap each other as shown in Figure-4. The predicted discharges from both models were plotted against the observed data together with 1:1 line (Figure-4). A magnified view near the origin on the right side of Figure-4 illustrates significant deviation from the 1:1 line. The predicted values from both equations crossed the 1:1 line at observed discharge of 9 m^3/s . This suggests that these equations have overestimated the discharge when the observed discharge is less than 9 m^3/s and vice versa. Graphical predictability pattern of equation (14) and (15) are identical.

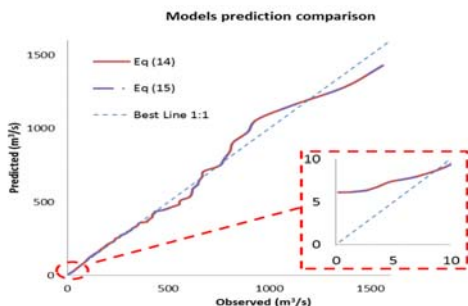


Figure-4. Prediction comparison graph of equation (14) and (15).

Statistical Evaluation of Equation (14) and (15)

The predicted discharges from both equations are almost identical when compared across every quantile range. Figure-4 cannot denote the difference. From Table 1, both prediction results are largely positively skewed while their median values deviated away from their mean significantly thus indicating a non-normal distribution of the data series. RMSE was also calculated to compare the prediction abilities of both equations across different flow quantiles.

Table-1. Statistical summary comparison.

Fitting Equation	Eq (14)	Eq (15)
Mean	16.57	16.56
Median	9.61	9.60
Standard Deviation	35.11	35.11
Variance	1232.5	1232.7
Skweness	17.72	17.72
Kurtosis	476.9	476.8

From Table-2, the overall RMSE and quantile cumulative RMSE of equation (15) are slightly lower than those of (14). The basic statistical properties (Table-1) obtained from equation (14) and (15) are indeed very similar, thus suggesting high degree of agreement between both models.

Table-2. Prediction comparison between empirical equations.

Fitting Equation	Eq (14)	Eq (15)
Nash-Sutcliffe	0.992	0.992
Quantile Cumulative RMSE:		
<1%	23.38	23.38
<5%	10.70	10.71
0-50%	3.74	3.73
51-100%	2.58	2.57
>95%	4.48	4.46
>99%	5.09	5.06
%Error at Peak flow	-8.41%	-8.41%
Max Observed: 1559 m^3/s	1427.83 m^3/s	1427.82 m^3/s
Min Observed: 0.2 m^3/s	6.14 m^3/s	6.11 m^3/s

Because the median of both prediction results deviate significantly from their mean values (Table-1), non-parametric Levene and Krusal-Wallis (KW) test were used to test the null hypothesis that equation (14) and (15) give similar prediction results. Should the p value of any tests is smaller than alpha level of 0.05; the null hypothesis can then be rejected at that alpha level. As p value from non-parametric Levene test is greater than 0.05 ($p=0.23$), the null hypothesis cannot be rejected. Both results indicate that the variances between both prediction models are fairly homogeneous. The assumption of similar distribution applies thus both models were re-examined using Kruskal-Wallis test again.

KW mean rank values are within a close range and statistically similar (equation (14): 18,976.48, equation (15): 18,906.52, $p>0.05$). Once again the null hypothesis cannot be rejected. There is no statistical significant difference between discharge results generated from equation (14) and (15). In the interest of the Law of Parsimony, the simpler empirical equation (14) referred to as new empirical equation was selected for the remaining discussion to benchmark against other fitting functions.

Other Fitting Functions

Many studies adopted polynomial function for fitting FDC; this paper also evaluated its fitting performance. Normal scale graph is unable to show an appropriate curve fitting grand schema view, the fitting



condition at low flow quantiles is unclear, and therefore log normal scale graph was applied. Log normal scale graph of polynomial functions in Figure-5 reveals dip and bounce curvature pattern even into negative flow zone (the solid blue curve is the FDC curve). Due to the mathematical nature of a polynomial function, 6th degree polynomial function (red dash curve) is wavier than 2nd degree polynomial function (black dash curve). Flow discharge data do not fluctuate nor behaves as wavelet function. Therefore, this study discarded polynomial function family because it fails to fit the observed data.

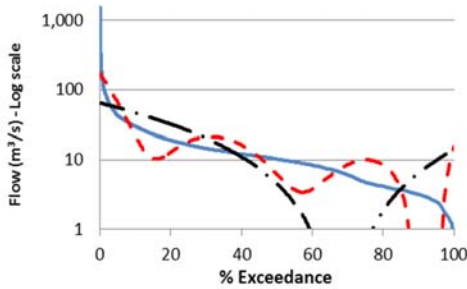


Figure-5. Polynomial curve fitting equations.

The power function, equation (18) over predicted flow values at the highest (exceedance probability <10%) and low quantile ranges (exceedance probability >70%). The maximum predicted flow was unacceptably high, reaching 15,500 m³/s. Due to this over prediction issue (Nash-Sutcliffe efficiency of the power function is less than zero) and to avoid skewing the scale, power function was not included in Figure-6 and 7. Figure-6 shows the overall model prediction fitting pattern on a log normal scale. The prediction pattern of logarithmic function, equation (17) closely resembles the observed FDC until at discharge values less than 10 m³/s where the curve begins to taper off around 70% quantile. However, its Nash-Sutcliffe efficiency is only 0.49. Exponential function, equation (19) falls short at peak flow with maximum predicted value of only 45 m³/s, its Nash-Sutcliffe efficiency is 0.22. On the other hand, both new empirical equations simulated the highest peak flow of 1,428 m³/s which is quite close to the observed peak of 1,559 m³/s.

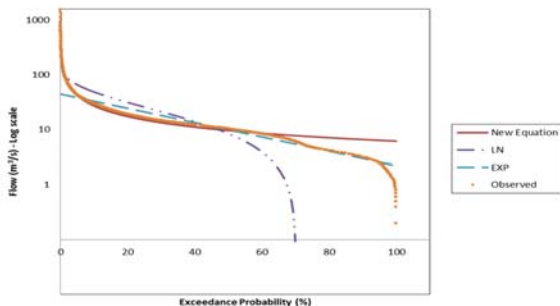


Figure-6. Log normal graph of equation (14), logarithmic and exponential function.

The simulated discharges from each model were plotted against the observed data in Figure-7 together with 1:1 best line. Figure-7 reveals vital information behind Figure-6 where Log normal scale graph is unable to display negative values (because log of any negative number is undefined). Spreadsheet auto scaling graphing feature presented minus scale on Figure-7. A magnified view near the origin (dash circle) revealed that logarithmic function actually generated negative flow after 70% quantile. Figure-7 also shows exponential function with higher prediction accuracy than our new empirical equation at low flow quantile end only.

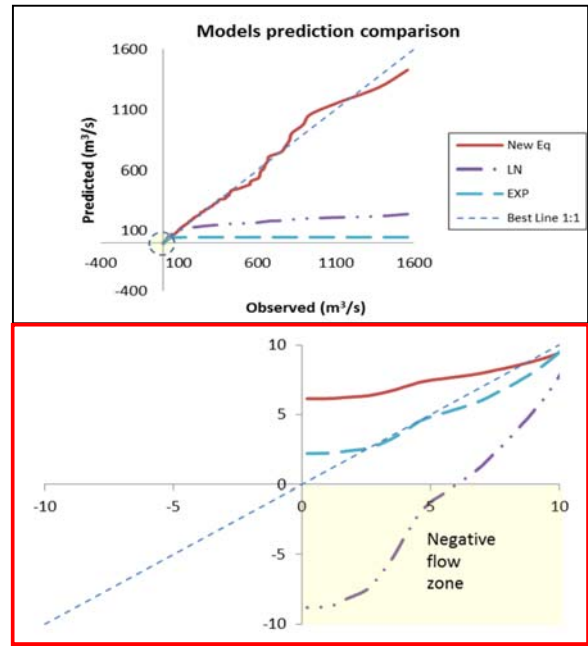


Figure-7. Prediction comparison of equation (14), logarithmic and exponential function and magnified low flow end.

Parameters Convergence Analysis of Equation (14)

Convergence analysis was conducted on all three parameters (α , m and n) of the new empirical equation in order to map their sensitivity and convergence behaviour. This was carried out using the isolation test of each parameter by varying toward positive and negative scales while the other two were kept constant. In Figure 8, the dash line represents optimum fitted result derived from least square fitting. The $\pm 100\%$ lines indicate model fitting conditions when certain parameter(s) was (were) deviated away from the optimum fitted values. For example, in Figure-8a, when α value was increased by 100% while m and n remain constant, new empirical equation (14) would predict below best fitted dash line and vice versa.

Since parameters m and n are the power of a quantitative group, they behave in similar manner and are very sensitive to minute variation. The convergence analysis concludes that setting initial guesses of both parameters toward same scale will converge model fitting



toward optimum values faster; however, setting values toward negative range by decreasing both m and n values will not yield feasible solution. As shown in Figure-8b, by reducing parameters m and n toward 100% scale will generate nearly a flat line model prediction pattern, approaching a threshold where the entire model will collapse with undefined values when m or n reached a certain negative value. On the other hand, α variation does not alter the model fitting shape as dramatically as m and n , it is also the only parameter in this equation with positive constraint and cannot be a negative number. All three parameters share a common converging characteristic where positive scale variation will drop the model fitting shape below the optimum fitted result and vice versa.

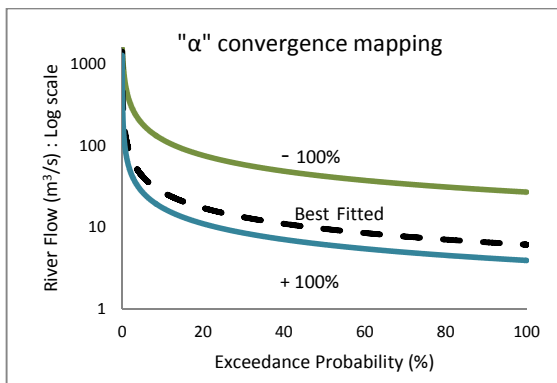


Figure-8a. Convergence and sensitivity test of α .

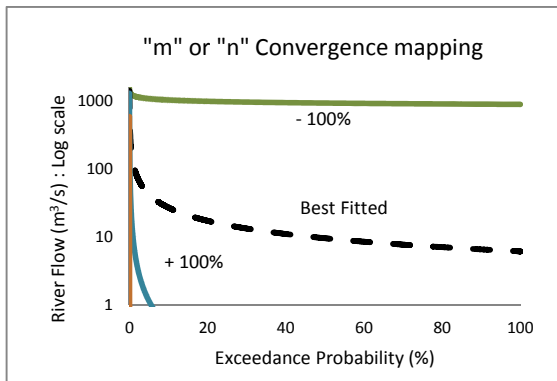


Figure-8b. Convergence and sensitivity test of m and n .

Note: The $\pm 100\%$ lines represent variation scale of each parameters and model fitting conditions.

Statistical Evaluation

Statistical tests were conducted using IBM PASW version 18 to further evaluate differences in the modelling results. The simulated flow is given in Figure-7. The new empirical equation (14) gave the best simulation and closely matched the observed data except at low flow end, followed by logarithmic, exponential and power function. The prediction results from each model

also violated the normality requirement of having skewness within 1 ± 0.50 . Therefore, non-parametric tests are more appropriate for further statistical analyses.

Non-parametric Friedman test is capable to determine overall statistical significant difference of a group referred as omnibus test. If the p value of the omnibus test is less than a pre-determined alpha value, null hypothesis can be rejected at the alpha confidence level. Friedman omnibus test result indicates that there is significant difference ($p < 0.05$) between mean ranks of different groups; however, the omnibus test is unable to further differentiate differences between any chosen pair within this group. This was resolved by using Post-hoc test. Since there are four models within the test group, there are six possible pairing combinations. Wilcoxon signed ranks test is only capable of comparing two groups at a time thus separate tests were repeated for six possible pairings within this group.

In order to avoid committing type I error multiple times during pairing comparisons, post-hoc tests require an adjustment on alpha value under Bonferroni law. For six pairing groups, Bonferroni law requires an adjustment to divide alpha level by 6 ($0.05/6 = 0.0083$) thus Wilcoxon tests' p values < 0.0083 is now required to reject null hypothesis at 95% confidence level. 2-tailed p values of all 6 groups are less than 0.0083 hence there are significant differences up to 98% confidence level between every possible pairing within this group. Each comparing function is therefore unique and different from each other.

CONCLUSIONS

Proposed new empirical equation (15) and its special condition form (14) has a lot higher Nash-Sutcliffe efficiency with low variance in overall RMSE, quantile cumulative RMSE at high flow range and percentage error at highest peak flow point compared to other fitting equations. Both prediction results from equation (14) and (15) demonstrated non normal distribution pattern while several data normalization techniques failed to adjust the skewness and transform the data distribution back to normal distribution. Therefore, non-parametric tests were used to evaluate both new empirical equation (14) and (15). Non-parametric tests concluded that there was no significant difference in the predicted flow produced by using either empirical equation (14) or (15) for this study site. Observed variances are due to chances. Empirical equation (15) also did not improve the fitting ability at low flow range compared to equation (14). In the interest of Parsimony principal, empirical equation (14) was selected for FDC fitting at this site, and therefore included $Q_{\min} = 0 \text{ m}^3/\text{s}$ in this study. The exclusion of zero m^3/s did not statistically improve model predictive accuracy in this study.

Overall, the new empirical equation (14) has outperformed the polynomial, power, logarithmic and exponential function but underperformed at low flow quantiles prediction compared to exponential function only. Both logarithmic and exponential functions underpredicted the stream flow values and fall short of peak



flow prediction with prediction error greater than 85%. Both logarithmic and exponential functions grossly underestimated the high flows with maxima up to 236.5 m³/s and 44.4 m³/s respectively compared to the observed peak flow of 1,559 m³/s. Exponential function was found to be second best fitting function followed by logarithmic and power function as the poorest. Polynomial function was discarded as its wavelet fitting form is not suitable for FDC fitting. In term of return period analysis, new empirical equation (14) has the lowest upper quantile RMSE (at exceedance probability range less than 5%), this indicator signifies equation (14) as best prediction model with least error in return period analysis for this site. General form of the proposed new empirical equations (4) and (9) are suitable for flood forecasting and peak flow related study use. In order to predict peak flow, the absolute cumulative error of optimum prediction model at the high flow exceedance probability end of the curve must be kept at minimum. New empirical equations were able to fit the FDC of this study site very well with high accuracy and emerged as the best choice.

On the other hand, should the study interest be at the low flow end of FDC, exponential function appears to outperform all others with lowest RMSE at low flow quantile range only which will be a better choice for water resources management study. Researchers are cautioned to understand and consider their study need prior to adopt any fitting function blindly.

Curve fitting parameter m and n can be combined into single value to reduce fitting parameters in the empirical equation in order to improve its model efficiency even further under parsimony principal. It is the future intention for author to apply this empirical equation to different sites within a region in order to map a relationship between parameter m and n . It is also possible to utilize these two parameters as an index to represent site specific hydrological characteristic within a region. Such attempt will enable model prediction into ungauged sites within the study region.

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