EFFECT OF MATERIAL PROPERTIES ON DUCTILITY FACTOR OF SINGLY RC BEAM SECTIONS

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ABSTRACT: Ductility may be defined as the ability to undergo deformations without a substantial reduction in the flexural capacity of the member. The ductility of reinforced concrete beams depends mainly on the shape of the moment-curvature relationship of the sections. The constituents of reinforced concrete are very complex due to its mechanical properties. The stress-strain behavior of concrete is considered parabolic and that of the steel is elastic plastic. Concrete and reinforcing steel are represented by separate material models that are combined together to describe the behavior of the reinforced concrete sections. The end displacements of the steel element are assumed to be compatible with the boundary displacements of the concrete element which implied perfect bond between them. The curvature ductility factor of singly reinforced concrete rectangular beams is derived taking into account the possible nonlinear behavior of the unconfined compressed concrete and reinforcing steel. Effects of material properties such as concrete compressive strength, reinforcement ratio and yield strength of reinforcement on the curvature ductility factors are derived analytically. From the analyses it is observed that an increasing steel content decreases the curvature ductility of a singly reinforced concrete section and this pattern is valid for any concrete strength. On the other hand, for the same reinforcement content curvature ductility increases as the concrete strength is increased.

Keywords - moment-curvature relationship; singly RC beams; reinforcing steel; concrete compressive strength; curvature ductility factor.

INTRODUCTION

It is generally well understood that reinforced concrete sections exhibit highly nonlinear load-deformation response due primarily to the nonlinear stress-strain relationship of the constituent materials. The ductility of reinforced concrete beams depends mainly on the shape of the moment-curvature relationship of the sections, since ductility may be defined as the ability to undergo deformations without a substantial reduction in the flexural capacity of the member. The ductility is important in a structure during severe earthquakes.

The ductility of a section is normally expressed as the curvature ductility factor φ_u/φ_y, where φ_y is the curvature when the tension steel first reaches the yield strength and φ_u is the ultimate curvature normally defined for unconfined concrete as when the concrete compressive strain reaches a specified limiting value. Figure 1 presents a typical moment-curvature relationship of a singly reinforced concrete section. It is assumed that plane sections remain plane after bending and that the stress-strain relationship of concrete and steel are known.

For reinforced concrete beams with unconfined concrete the flexural strength and ductility mainly depends on the tension reinforcement ratio, ρ. The compressive strength of the concrete f'_c has a less significant effect. The linear stress-strain relationship of concrete does not hold true when the tension reinforcement reaches yield strength and calculation of the curvature at first yield should be used on the nonlinear behavior of concrete.

The objective of this paper is to calculate the curvature ductility factors for singly reinforced concrete beams with rectangular sections considering the possible nonlinear material behavior of the unconfined compressed concrete. Effect of material properties on the curvature ductility at various stages of loading such as yield and ultimate are also observed.

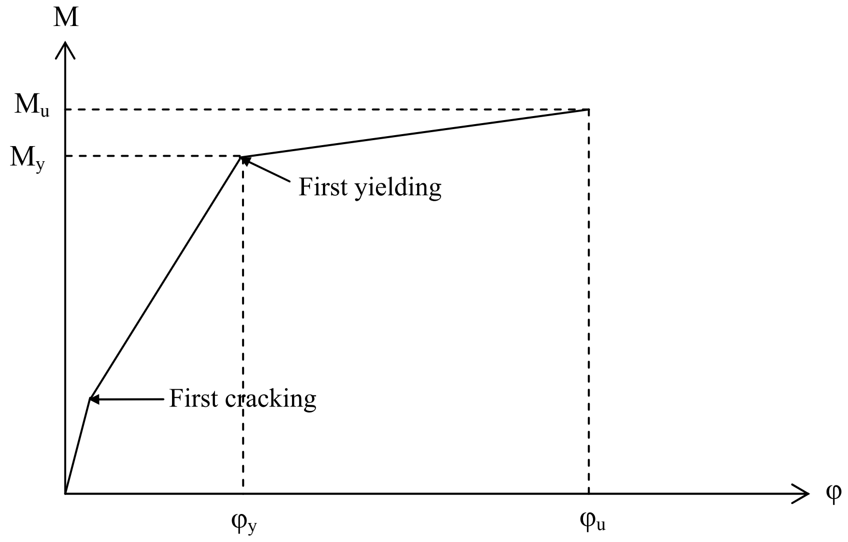

Fig1. Typical moment-curvature curve for a singly reinforced concrete section falling in tension.

STREES-STRAIN RELATIONSHIP OF CONCRETE

There are various mathematical models to idealize the mechanical behavior of concrete and these are summarized by ASCE (1982), Lin and Scordelis (1975) and European Concrete Committee (CEB 1978). In the design of RC structural members, uniaxial compressive strength of concrete obtained from cylinder test is one of the most important design parameter and is widely used. Typical stress-strain curves presented in Nilson et al. (2003), Park, and Paulay (1975) for normal density and lightweight concrete had shown that concrete had a similar nonlinear character even at a normal level of stress in compression. The curves are linear up to about half of the compressive strength. The peak of the curve for high strength concrete is relatively sharp, but for low strength concrete the curve has a flat top. Simply, all the curves consist of an initial relatively straight line portion which then begins to curve to the horizontal, reaching the maximum stress for normal density concrete followed by a falling branch. The stress-strain concrete for unconfined concrete may be assumed to be represented by a second degree parabola for both the ascending and descending branch. The stress-strain curve can be expressed by the following equations as suggested by Hognestad (1951) with some modifications for descending branch as shown in Figure 2. In the present model stress-strain relationship of concrete is parabolic with an ultimate compressive strain of 0.003.

The equation can be written as

$$f_c = f'_c[2\varepsilon/\varepsilon_m-(\varepsilon/\varepsilon_m)^2] \qquad \text{for } \varepsilon \leq 0.003 \qquad (1)$$

where, f_c = concrete compressive stress

ε = longitudinal concrete strain
ε_m = longitudinal strain in concrete corresponding to crushing strength = $2f'_c/E_c$.
f'_c = concrete compressive strength
E_c = elastic modulus of concrete = $57000\sqrt{f'_c}$ psi ($4700\sqrt{f'_c}$ MPa, f'_c in MPa)

The distribution of concrete stress over the compressed part of the section can be found from the strain diagram and the stress-strain relationship of the concrete. For a given concrete strain ε in the extreme compression fiber of a rectangular section of width b and neutral axis depth kd the concrete compressive force is given by

$$C_c = \alpha f'_c bkd \quad (2)$$

where,
C_c = compressive force
k = uncracked depth ratio of the section
b = width
d = effective depth of the section

The mean stress factor α can be determined for beams with rectangular sections from the stress-strain relationship as follows

$$\alpha = \frac{\int_0^{\varepsilon_m} f_c d\varepsilon}{f'_c \varepsilon_m} \quad (3)$$

Substituting Eq. 1 into Eq. 2 and rearranging results in the following equation for α

$$\alpha = \frac{\varepsilon}{\varepsilon_m}[1 - \frac{\varepsilon}{3\varepsilon_m}] \quad (4)$$

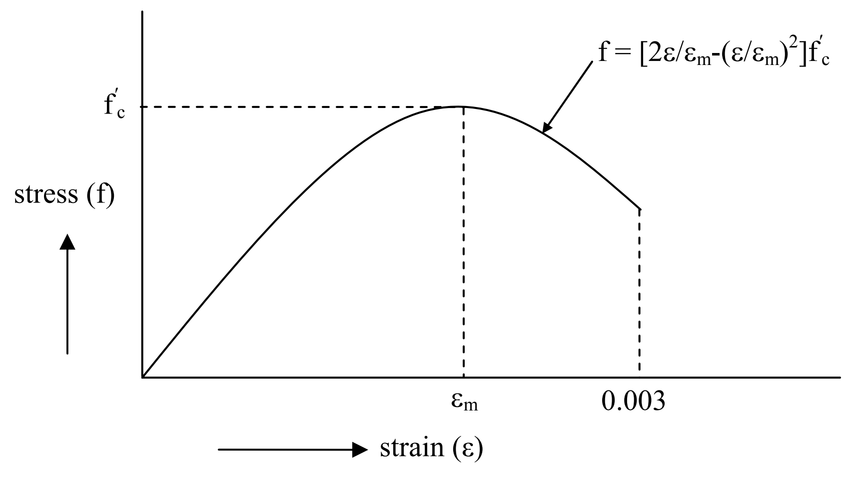

Fig 2. Stress-strain relationship of concrete used in the analysis

For flexural strength calculation ACI Building Code assumes ultimate concrete strain 0.003 and a concrete stress of $0.85f'_c$ uniformly distributed over a depth of a, where a/c = β_1 =0.85 for $f'_c \leq 4000$ psi (27.6 MPa) and for strength above 4000 psi (27.6 MPa), β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi (6.89 MPa) of strength in excess of 4000 psi (27.6 MPa), but β_1 shall not be taken less than 0.65.

Table 1 compares the α values predicted by Eq. (4) and the rectangular stress block of ACI code when $\varepsilon_u = 0.003$. For the range 2500 psi (17.2 MPa) $\leq f'_c \leq$ 5500 psi (37.9 MPa), the agreement is within 6%. The range of concrete strength is considered the commonly used as normal strength concrete and the stress-strain relationship of this range is assumed to be parabolic.

Table 1: Values for the mean stress factor α when extreme concrete fiber strain is 0.003

f'_c, psi (MPa)	2500(17.2)	3000(20.7)	4000(27.6)	5000(34.5)	5500(37.9)
α, from Eq. (4)	0.74	0.75	0.74	0.72	0.70
α, according to ACI Code	0.72	0.72	0.72	0.68	0.66

STRESS-STRAIN RELATIONSHIP OF REINFORCING STEEL

Typical stress-strain curves for reinforcing steel bars used in structure are obtained from the test of bars loaded in tension. For all practical purposes steel exhibits the same stress-strain curve in compression as in tension. The stress-strain relationship of steel exhibits an initial linear elastic portion, a yield plateau, a strain hardening range. The extent of the yield plateau is a function of the tensile strength of steel. The stress-strain relationship may be idealized elastic perfectly plastic as shown in Figure 3.

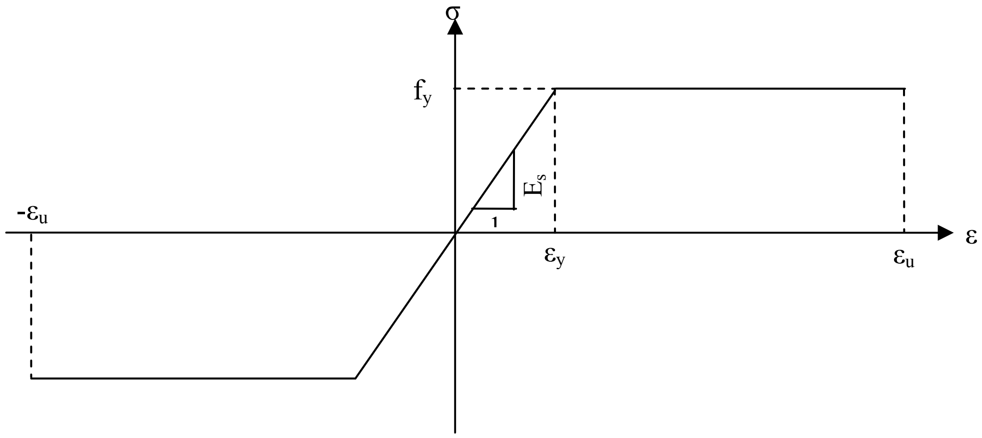

Fig 3. Stress-strain relationship of steel reinforcement

The curvature at first yield φ_y, defined as the curvature when the longitudinal tension reinforcement first reaches the yield strain f_y/E_s, where f_y = yield strength of steel and E_s = modulus of elasticity of steel.

The curvature at yield can be written as

$$\varphi_y = \frac{f_y / E_s}{d(1-k)} \tag{5}$$

For a chosen value of concrete compressive strain in the top fiber ε_{cu}, the neutral axis depth factor k at first yield of the tension reinforcement can be written as

$$k = \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}} \tag{6}$$

Alternatively, the neutral axis depth may be related to the section dimensions, steel ratios, and steel and concrete strengths by considering the strain diagram and the requirements of internal forces. For equilibrium of internal longitudinal concrete and steel forces it can be expressed as

$$k = \frac{f_y A_s}{\alpha f_c^{'} bd} \tag{7}$$

The curvature at ultimate φ_u, defined as that curvature when the concrete compressive strain reaches a limiting value ε_{cu}, can be written as

$$\varphi_u = \frac{\varepsilon_{cu}}{c} \tag{8}$$

where, c= neutral axis depth from top of the section

From equilibrium of the internal forces of concrete and steel in a section it can be expressed as

$$c = \frac{f_y A_s}{\alpha f_c^{'} b} \tag{9}$$

CURVATURE DUCTILITY FACTORS

The curvature ductility factors for a range of concrete compressive strength are computed using Eq. 1 through 9 and are presented in Figures 4 and 5 with various percentages of reinforcement and strengths of steel. Figure 4 presents the effect of reinforcement ratio on the curvature ductility for yield strength of 40 ksi (276 MPa) and Figure 5 that of the 60 ksi (414 MPa) steel reinforcement. The ultimate concrete compressive strain is considered to be 0.003 for computed values using the equations.

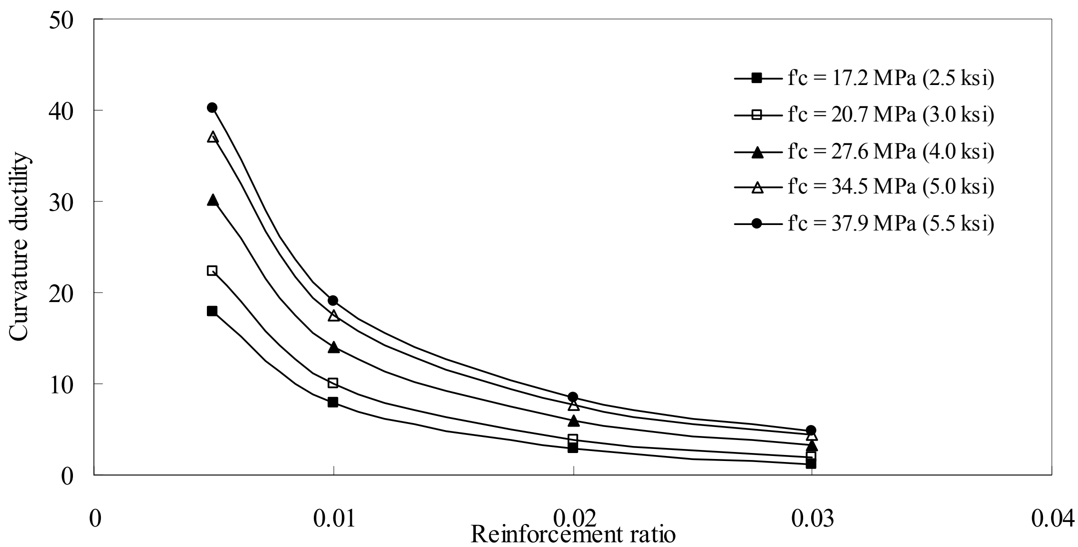

Fig 4. Effect of reinforcement ratio on curvature ductility of a singly RC section, f_y = 276 MPa (40 ksi)

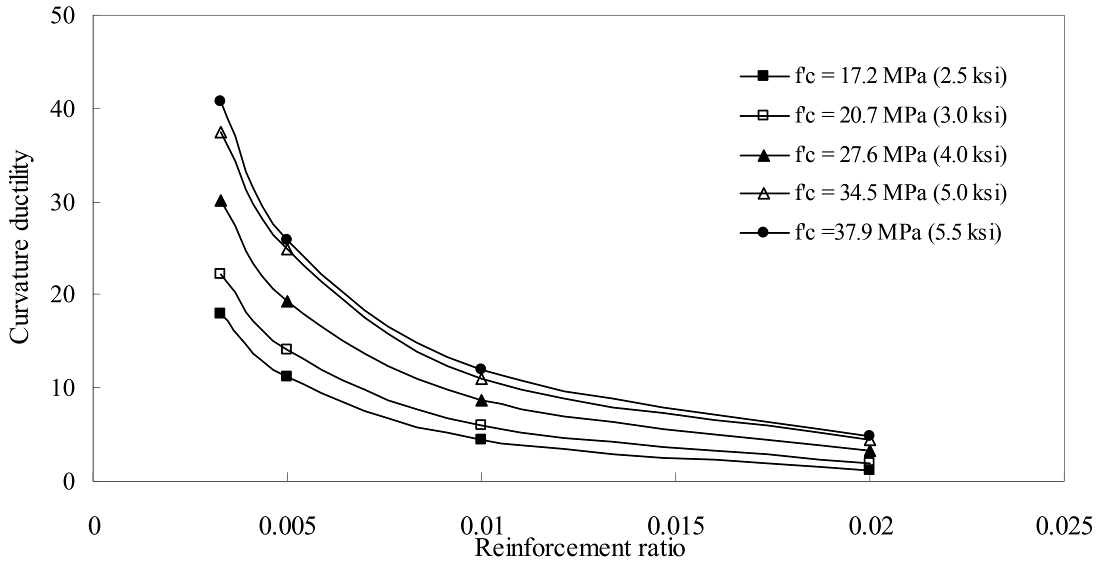

Fig 5. Effect of reinforcement ratio on curvature ductility of a singly RC section, f_y = 414 MPa (60 ksi)

To observe the effect of concrete compressive strength on curvature ductility factors for the same strength of steel, f′c = 17.2 MPa to 37.9 MPa is considered for analysis. Figures 6 and 7 present the curvature ductility for the range of strength considered. The ultimate concrete compressive strain is considered to be 0.003 for computed values using the equations. From these figures it is observed that for the same reinforcement ratio increasing concrete strength increases the curvature ductility and this pattern is the same for different yield strengths of steel.

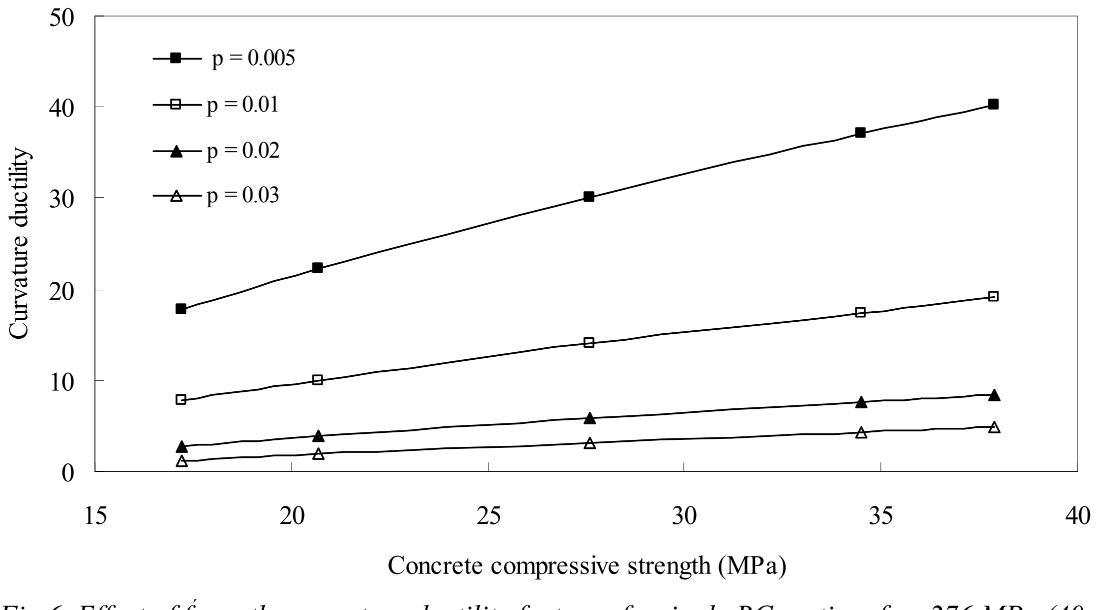

Fig 6. Effect of f'_c on the curvature ductility factors of a singly RC section, f_y = 276 MPa (40 ksi)

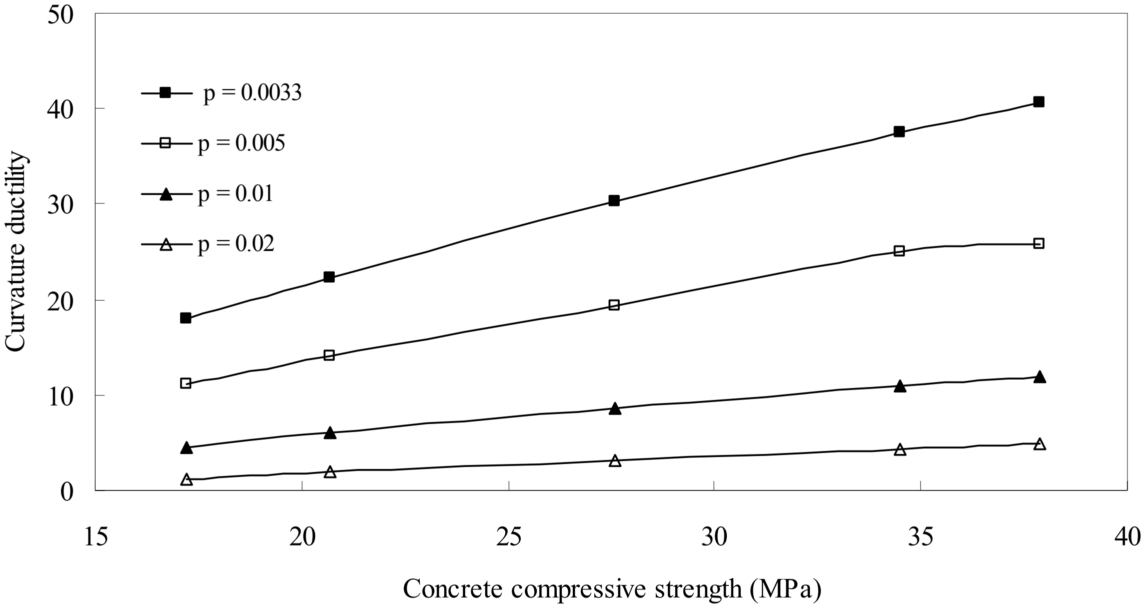

Fig 7. Effect of f'_c on the curvature ductility factors of a singly RC section, f_y = 414 MPa (60 ksi)

CONCLUSIONS

The parabolic stress-strain relationship of concrete and elasto-plastic model of reinforcing steel are utilized for the calculating the curvature ductility factors of singly reinforced concrete sections. The effects of concrete strength and reinforcement content of the sections on the behavior of curvature ductility of the RC sections are analyzed. From the figures it is observed that an increasing reinforcement ratio decreases the curvature ductility of a singly RC section and this pattern is the same for any concrete strength. However, the curvature ductility factors increases as the concrete strength is increased for the same reinforcement content.

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