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REINFORCED CONCRETE BEAMS AT ULTIMATE FLEXURAL LIMIT STATE: COMPARISON OF BS 8110 AND EUROCODE 2

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ABSTRACT: BS 8110, the British Code for structural concrete design has been implemented as the code of practice in Malaysia since its first inception. However, the British Standards Institution will cease to revise this structural concrete design code after year 2008. Eurocode 2 will be implemented as the structural concrete design code in the United Kingdom thereafter, with the additional Nationally Determined Parameters (NDP's). It is envisaged that Malaysia will gradually implement Eurocode 2 in lieu of BS 8110 as the standard code of practice. This paper presents a parametric study on flexural strength of singly- and doubly-reinforced rectangular beams based on BS 8110 (1985) and Eurocode 2 (1992). It was found that both codes give almost similar flexural strength for beams reinforced with either mild or high yield steel if the relationship of concrete cube strength and cylinder strength is taken into account. The difference in calculated strength is merely 1% for singly-reinforced beams within the maximum limit for neutral axis depth; and about 2 % of maximum difference. These differences are considered negligible in consideration of steel reinforcement provision. The maximum difference in flexural strength encountered in this study is 7% for doubly-reinforced beams. This may also be considered as insignificant in provision of steel reinforcement. Therefore, it can be concluded that flexural design based on both codes of practice are similar and the structural designer will arrive at the same design following either code. The charts presented in this parametric study can serve as nomographs for the design of singly- and doubly-reinforced rectangular beams and basis for the structural designer to compare the design based on two different codes of practice.

Keywords – beam, flexure, ultimate.

1. INTRODUCTION

Concrete and reinforced concrete are the principal materials used in structural design and engineered construction (MacGregor, 1997; Wang and Salmon 1998; MacGinley and Choo 1990). They can be formed into various shapes and sizes (Mosley et al., 1999) which are only limited by the skills and technology in moulding. In Malaysia, the structural concrete design has been based on British Code BS 8110 (BSI, 1985) since its predecessor CP110 (BSI, 1972). Unfortunately, BS 8110 will be superseded by Eurocode 2 (CEN, 1992) by the year 2008 in the United Kingdom, with the accompanying document containing the Nationally Determined Parameters (NDP's). Therefore, the structural designers in Malaysia may have to implement Eurocode 2 gradually in the structural concrete design after the withdrawal of BS 8110 (Omar et al., 2001). However, Malaysia is not a member country of the European Union, so Malaysia may be adopting the NDP's of United Kingdom if Eurocode 2 is implemented.

In view of the necessity to adopt the new code of practice in replacement of the other, the comparison of the flexural strength based on BS 8110 and Eurocode 2 is reported herein so that the structural designers will get a clear understanding of the similarities and differences in both codes. The 1985 edition of BS 8110 is used instead of the 1997 edition. This is due to the same partial safety factor for steel reinforcement used in the 1985 edition of BS 8110 and Eurocode 2, and therefore the comparison between these codes of practice is more appropriate. The results of the parametric study are plotted in the form of nomographs to serve as design guides for structural designers.

2. ANALYTICAL CONSIDERATIONS

Figures 1 and 2 show the strain and stress distributions of singly- and doubly-reinforced rectangular beam based on the simplified stress block of BS 8110 (BSI, 1985), respectively. The depth of neutral axis *x* is limited to 0.5 times the effective depth *d* so that the beam behaviour is always under-reinforced. The ductility of the beam is therefore ensured by this limit of neutral axis depth in which the beam fails in tension failure at ultimate flexural limit state.

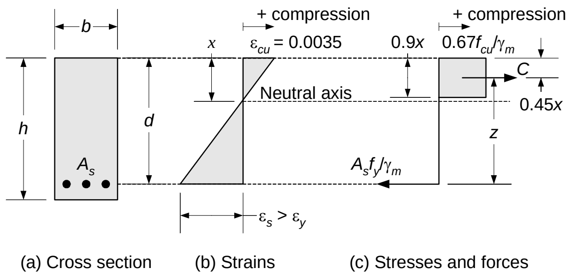

Fig. 1. Strain and stress distributions of a singly-reinforced rectangular beam in accordance to BS 8110 (BSI, 1985)

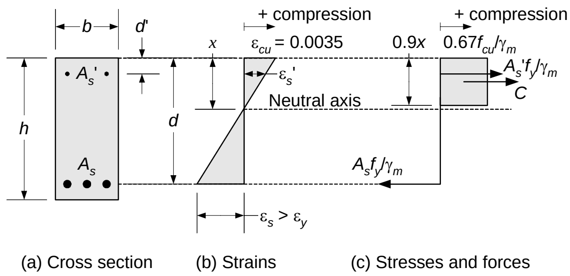

Fig. 2. Strain and stress distributions of a doubly-reinforced rectangular beam in accordance to BS 8110 (BSI, 1985)

Figures 3 and 4 show the strain and stress distributions of a singly- and doubly-reinforced rectangular beam in accordance to the simplified stress block of Eurocode 2 (CEN, 1992), respectively. The depth of neutral axis *x* is limited to 0.45 times the effective depth, *d*, for concrete strength f_{ck} of 35 MPa or less. Otherwise, the neutral axis depth is limited to 0.35 times the effective depth for concrete strength higher than 35 MPa. This is to ensure that the beam will fail in tension failure, that is, exhibiting under-reinforced section behaviour.

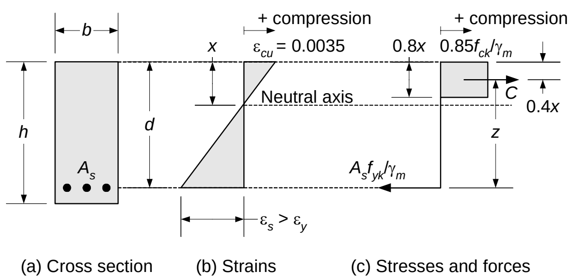

Fig. 3. Strain and stress distributions of a singly-reinforced rectangular beam in accordance to Eurocode 2 (CEN, 1992)

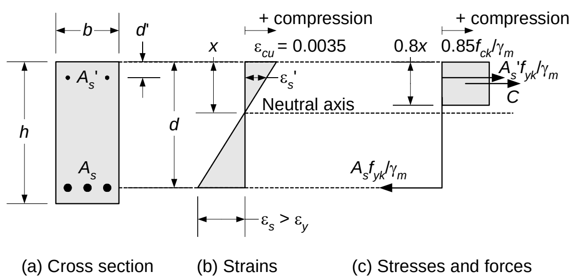

Fig. 4. Strain and stress distributions of a doubly-reinforced rectangular beam in accordance to Eurocode 2 (CEN, 1992)

From Figs. 1 to 4, it is shown that the notations used for cross-sectional properties in both codes of practice are undoubtedly similar, that is, A_s for area of tension reinforcement; A_s' for area of compression reinforcement; b for beam width; d for depth of tension reinforcement; d' for depth of compression reinforcement; and h for overall beam height.

Both design codes are based on limit state design in accordance to strain compatibility and equilibrium of forces in a cross section. Table 1 summaries the similarities and differences in notations and parameters in both design codes. For concrete strength, BS 8110 (BSI, 1985) uses cube strength f_{cu} whereas Eurocode 2 (CEN, 1992) utilizes cylinder strength f_{ck} in the design equations. In order to establish the comparison of beam strength based on the two design codes, the relationship of common cube strength to cylinder strength needs to be established. Beeby and Narayanan (1995) recommended the relationship as shown in Table 2.

The requirement for ductility in beams is more stringent in Eurocode 2 (CEN, 1992) where the maximum allowable neutral axis depth is relatively less than the one in BS 8110 (BSI, 1985) as depicted in Table 1. It is also noted that the depth of concrete compression zone considered in the simplified rectangular stress block has a small difference for both codes, in which Eurocode 2 (CEN, 1992) uses a shallower concrete compression zone.

Table 1. Comparison of notations, parameters and design equations

Parameters	BS 8110 (BSI, 1985)	Eurocode 2 (CEN, 1992)
1. Concrete strength	Cube strength, f_{cu} $f_{cu} \approx f_{ck}/0.8$	Cylinder strength, f_{ck} $f_{ck} \approx 0.8f_{cu}$
2. Partial safety factor, γ_m	For concrete in bending = 1.5 For steel = 1.15	For concrete: Fundamental =1.5 Accidental = 1.3 For steel: Fundamental =1.15 Accidental = 1.0
3. Yield strength of high yield steel	f_y = 460 MPa f_y/γ_m = 400 MPa	f_{yk} = 460 MPa f_{yk}/γ_m = 400 MPa
4. Yield strength of mild steel	f_y = 250 MPa f_y/γ_m = 217 MPa	f_{yk} = 250 MPa f_{yk}/γ_m = 217 MPa
5. Ultimate strain of concrete, ε_{cu}	0.0035 for flexure	0.002 for axial load 0.0035 for flexure
6. Maximum allowable neutral axis depth, x	0.5d (no redistribution)	0.45d for $f_{ck} \leq 35$ MPa 0.35d for $f_{ck} > 35$ MPa 0.25d for plastic analysis
7. Concrete compression zone depth (simplified rectangular stress block)	0.9x	0.8x
8. Ultimate moment of resistance, M_u	$M_u = 0.156f_{cu}bd^2$	For $f_{ck} \leq 35$ MPa: $M_u = 0.167f_{ck}bd^2$ For $f_{ck} > 35$ MPa: $M_u = 0.128f_{ck}bd^2$

Table 2. Relationship of cube strength to cylinder strength (Beeby and Narayanan, 1995)

Cube strength, f_{cu} (MPa)	Cylinder strength, f_{ck} (MPa)
25	20
30	25
37	30
45	35

Other than the three slight dissimilarities in both codes of practice as highlighted above, the other parameters are similar in both codes. Therefore, quantitative comparison on flexural strength of beams based on these codes of practice can be carried out by taking into account of the similarities and dissimilarities for the parametric study. Concrete cube strengths ranging from 25 to 45 MPa were considered in the parametric study, corresponding to cylinder strengths ranging from 20 to 35 MPa. Concrete with higher strength was not considered as strength over 50MPa is considered as high performance concrete in BS 8110 (BSI, 1985), in which the analytical considerations as discussed earlier may not be applicable without modification.

3. RESULTS AND DISCUSSION

The results of a parametric study on flexural strengths of singly-reinforced and doubly-reinforced concrete beams are presented herein to discuss on the flexural design of beams using both codes of practice. Based on the impending discussion, a better understanding of the flexural design in BS 8110 (BSI, 1985) and Eurocode 2 (CEN, 1992) may be revealed.

3.1 Singly-reinforced Beam Sections

Based on the analytical considerations described in the previous section, the flexural strengths of singly-reinforced concrete beams with different concrete strength were calculated. The calculations were carried out for beams with different types of steel, that is, mild steel and high yield steel.

Figure 5 shows the flexural strength of beams reinforced with only tension mild steel bars, or simply, singly-reinforced sections. The limit of reinforcement ratio $100A_s/bd$ in each of the chart of Fig. 5 is the maximum reinforcement ratio which corresponds to the maximum permissible neutral axis depth. Since Eurocode 2 (CEN, 1992) has a more stringent requirement over neutral axis depth, the limit is dominated by this design code as marked in Fig. 5. As the concrete strength increases, this limit also increases.

It can be seen from Fig. 5 that the flexural strength of the beams calculated from both design codes are similar if the relationship of the cube strength and cylinder strength is taken into account. However, Eurocode 2 registered slightly higher flexural strength for concrete strength lower than f_{cu} = 45 MPa/ f_{ck} = 35 MPa. The difference in flexural strength calculated based on both codes within the limit of reinforcement ratio is less than 1%. This small deviation shall not incur any difference in the final design of beams for flexure.

Figure 6 shows the curves for singly-reinforced beams with high yield steel tension reinforcement. Similar to the beams reinforced with mild steel, the flexural strength values calculated in accordance to both codes of practice are identical, of which Eurocode 2 registered slightly higher values for beams with concrete strength lower than f_{cu} = 37 MPa/ f_{ck} = 30 MPa, at reinforcement ratio lower than 1.0. At higher concrete strength or higher reinforcement ratio, BS 8110 gives higher flexural strength values. However, the differences within the limit of reinforcement ratio are less than 1%, which shall be insignificant for the final design of beams in flexure.

As commonly known, the limiting values of reinforcement ratio for high yield steel are smaller due to the higher yield strength as compared to beams reinforced with mild steel.

Similar to beams with mild steel reinforcement, the limit increases with the increase in concrete strength as depicted in Fig. 6.

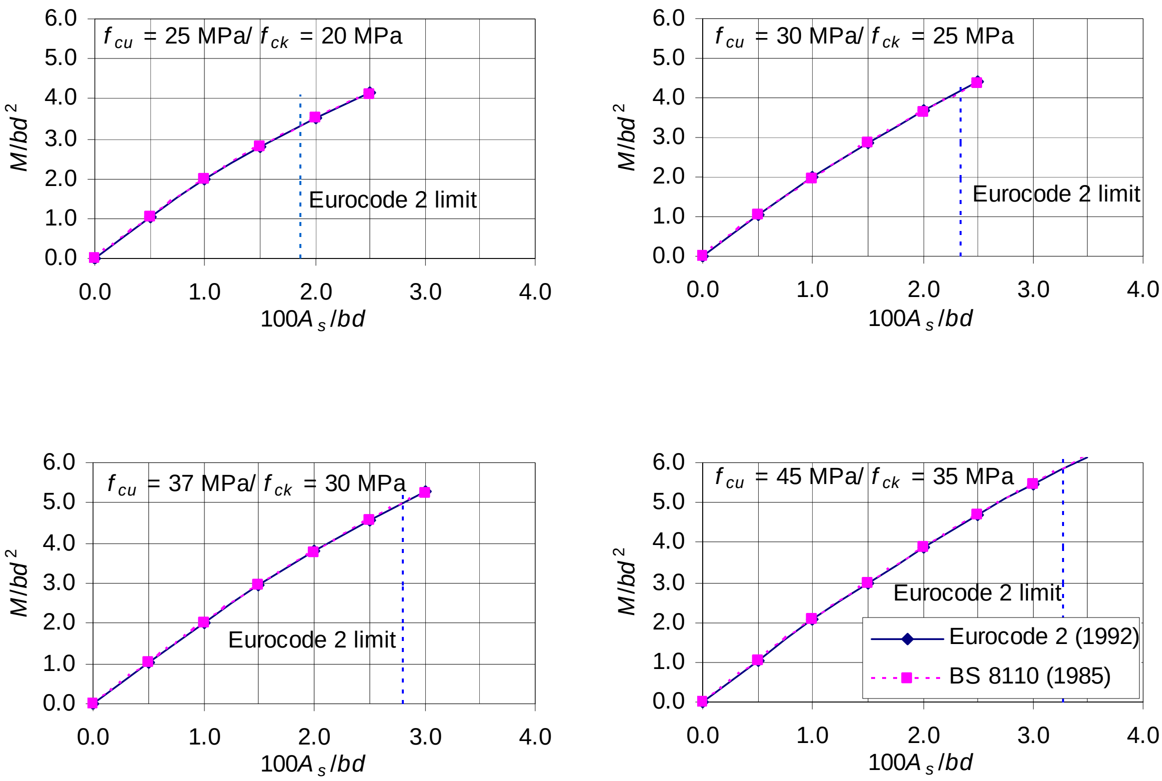

Fig. 5. Flexural strength of singly-reinforced concrete beams with mild steel reinforcement (f_y = f_{yk} = 250 MPa)

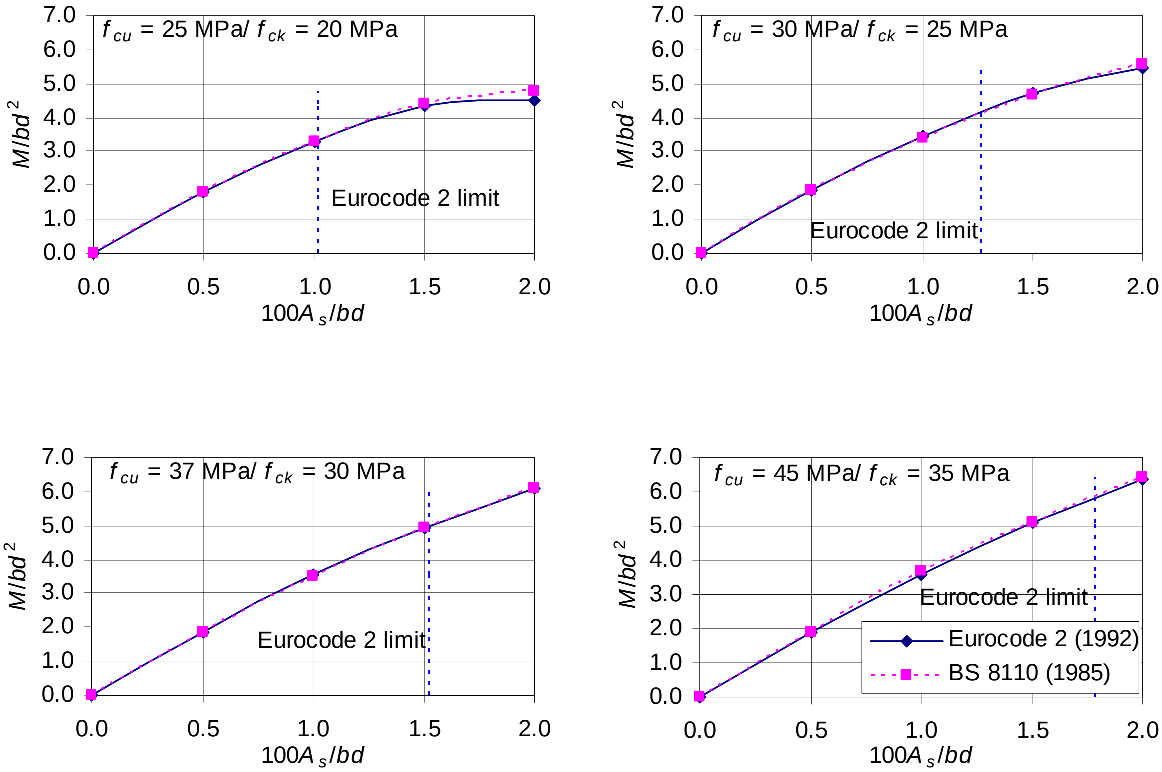

Fig. 6. Flexural strength of singly-reinforced concrete beams with high yield steel reinforcement ($f_y = f_{yk} = 460$ MPa)

Figures 5 and 6 as presented herein can serve as design guides or nomographs for structural designers who intend to compare the design in accordance to both codes of practice. As mentioned earlier, the observed differences are just merely 1%, which can be considered as negligible. Therefore, the same design may be expected from both design codes for singly-reinforced beams.

3.2 Doubly-reinforced Beam Sections

The results of beams with doubly-reinforced sections are reported in this section. Doubly-reinforced beam section is required when the beam needs high flexural strength to resist the high applied moment. Therefore, only beams reinforced with high yield steel are considered herein as doubly-reinforced beams benefited from the high yield strength of this steel type to achieve higher flexural strength. It is uncommon for a doubly-reinforced beam to have mild steel reinforcement as it defeats the purpose of providing high flexural strength.

Figures 7 to 10 show the curves for beams with concrete cube strength of 25, 30, 37 and 45 MPa, corresponding to concrete cylinder strength of 20, 25, 30 and 35 MPa, respectively. These curves were plotted up to the maximum tension reinforcement ratio $100A_s/bd$ of 4.0 as restricted by BS 8110 (BSI, 1985). The amount of compression reinforcement considered is also up to the maximum reinforcement ratio $100A_s'/bd$ of 4.0. The depth of the compression reinforcement was also varied from d'/d ratio of 0.10 to 0.20.

In Fig. 7, it can be seen that the flexural strength of beams increases with the increase in compression reinforcement ratio. For a given tension reinforcement ratio, the tension force in the cross section is constant considering that the reinforcement has reached the yield strength.

The increase in compression reinforcement ratio increases the compression force in the compression reinforcement. As a result, the required amount of compression force from the concrete compression zone to counterbalance the tension force decreases, resulting in decrease in neutral axis depth, hence increases the level arm of the concrete compression force. These effects resulting from the increase in compression reinforcement ratio increases the flexural strength.

It is also shown in Fig. 7 that the flexural strength decreases with the ratio of d'/d, indicating that the increase in depth of compression reinforcement results in the decrease in flexural strength. As the depth of compression reinforcement increases, the level arm of the compression force in the compression reinforcement decreases, resulting in the decrease in flexural strength of the beam section.

A comparison of the BS 8110 curves and Eurocode 2 curves of Fig. 7 reveals that BS 8110 registered higher flexural strength in general. The difference in flexural strength of both codes of practices decreases with the increase in compression reinforcement ratio. The maximum difference in flexural strength is 4% for the beam with $100A_s'/bd$ of 0 at $100A_s/bd$ of 4, that is, a singly-reinforcement beam. This maximum difference is however occurring outside the maximum allowable tension reinforcement ratio of 1.02 (f_{cu} = 25 MPa/ f_{ck} = 20 MPa) according to Eurocode 2 as depicted in Fig. 6. Within the allowable tension reinforcement ratio, the difference in flexural strength between both codes of practice is less than 1%. The allowable tension reinforcement ratio would however increase with the increase in compression reinforcement ratio. It can be observed in Fig. 6 that the allowable tension reinforcement ratio demarcates the curve at the point where it starts to change in gradient more drastically. On the same principle, the allowable limits can be estimated on the other curves in Fig. 7. It can be deduced that the difference in flexural strength of both codes of practice is less than 1% for reinforcement ratio within the allowable limits, indicating that the design of flexural members is similar based on either code.

Similar conclusions can be drawn for beams with higher concrete strength as presented in Figs. 8 to 10, except that the difference in flexural strength between both codes of practice increases with the increase in concrete strength. The maximum difference of 7% occurs in a beam with $100A_s'/bd$ of 0 at $100A_s/bd$ of 4 in Fig. 10, which is a singly-reinforced section. As discussed earlier, this occurs outside the maximum allowable tension reinforcement ratio, of which the limit is 1.79 (f_{cu} = 45 MPa/ f_{ck} = 35 MPa) as depicted in Fig. 6. If we consider only beam sections within the allowable limits, the difference in flexural strength is again less than 1%.

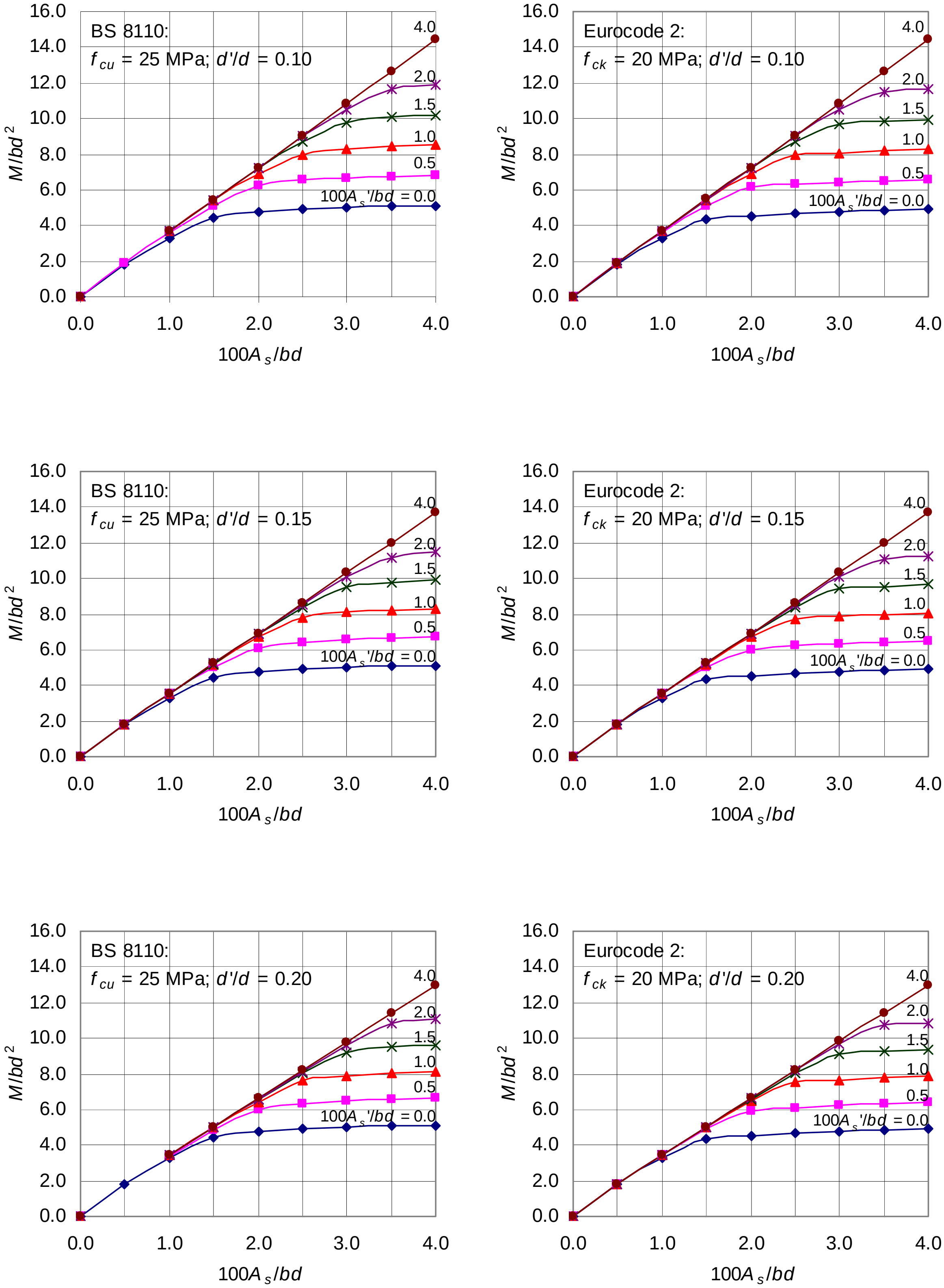

Fig. 7. Flexural strength of doubly-reinforced beams for concrete strength f_{cu} = 25 MPa/ f_{ck} = 20 MPa with high yield steel reinforcement

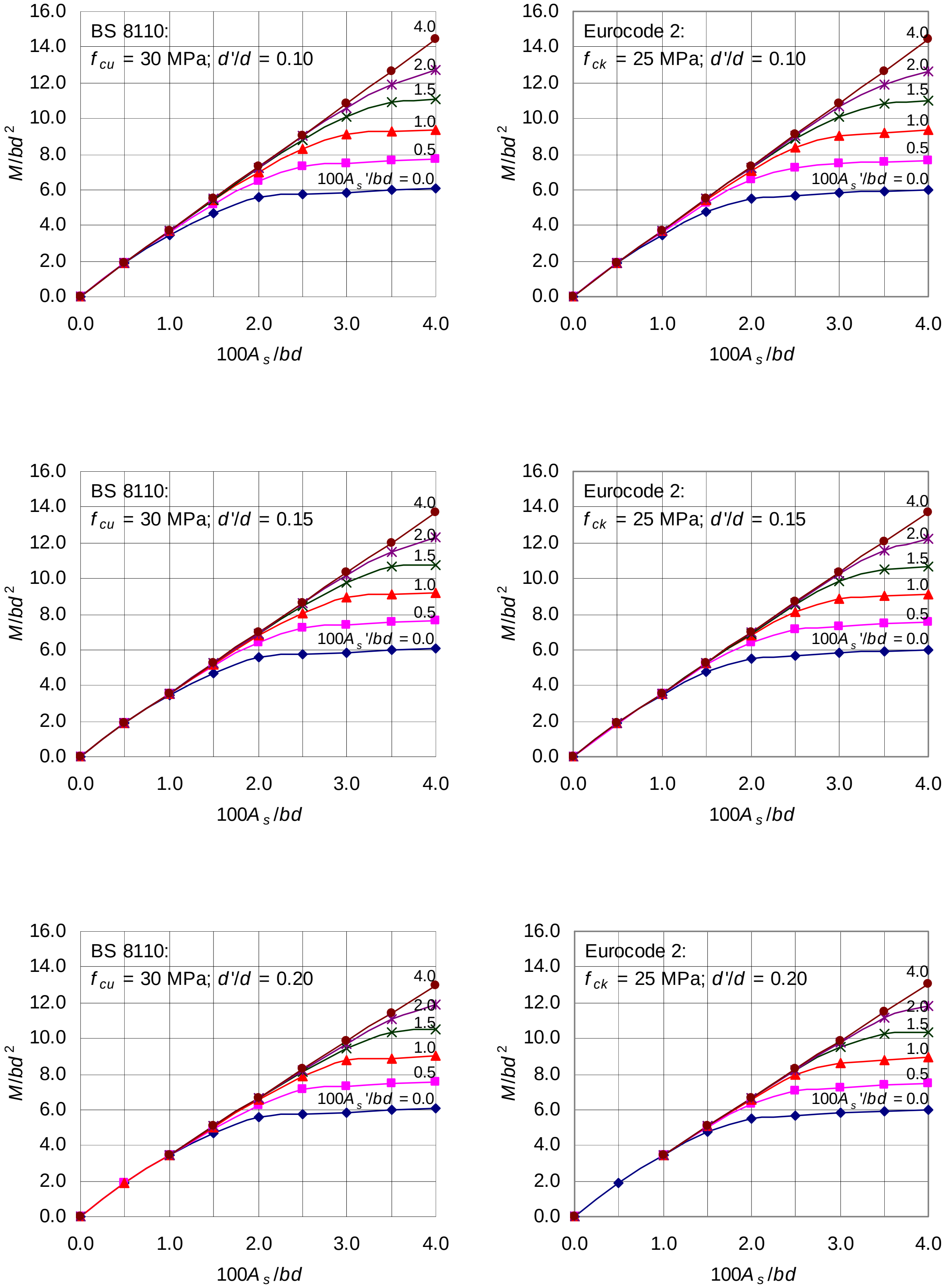

Fig. 8. Flexural strength of doubly-reinforced beams for concrete strength f_{cu} = 30 MPa/ f_{ck} = 25 MPa with high yield steel reinforcement

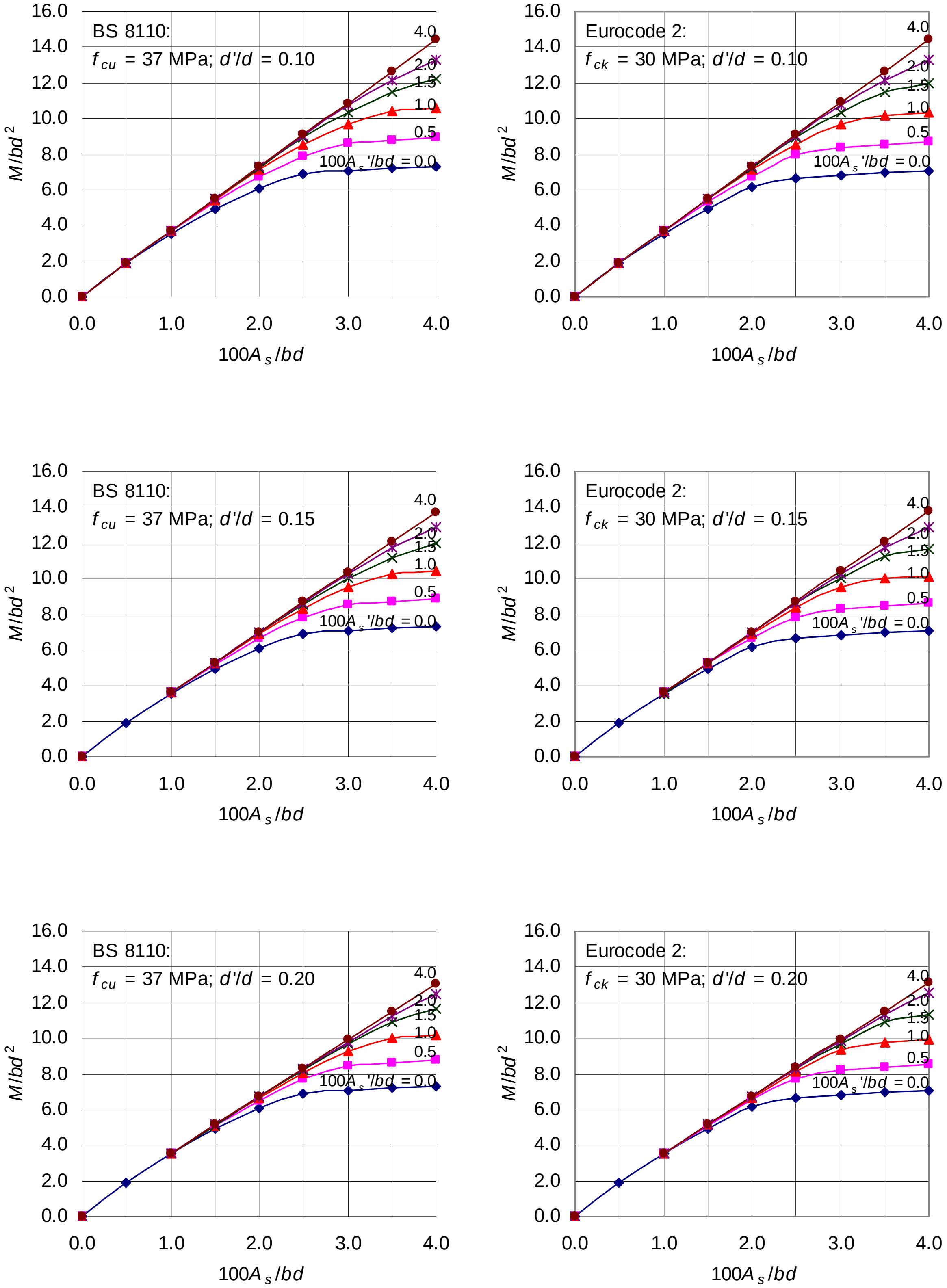

Fig. 9. Flexural strength of doubly-reinforced beams for concrete strength f_{cu} = 37 MPa/ f_{ck} = 30 MPa with high yield steel reinforcement

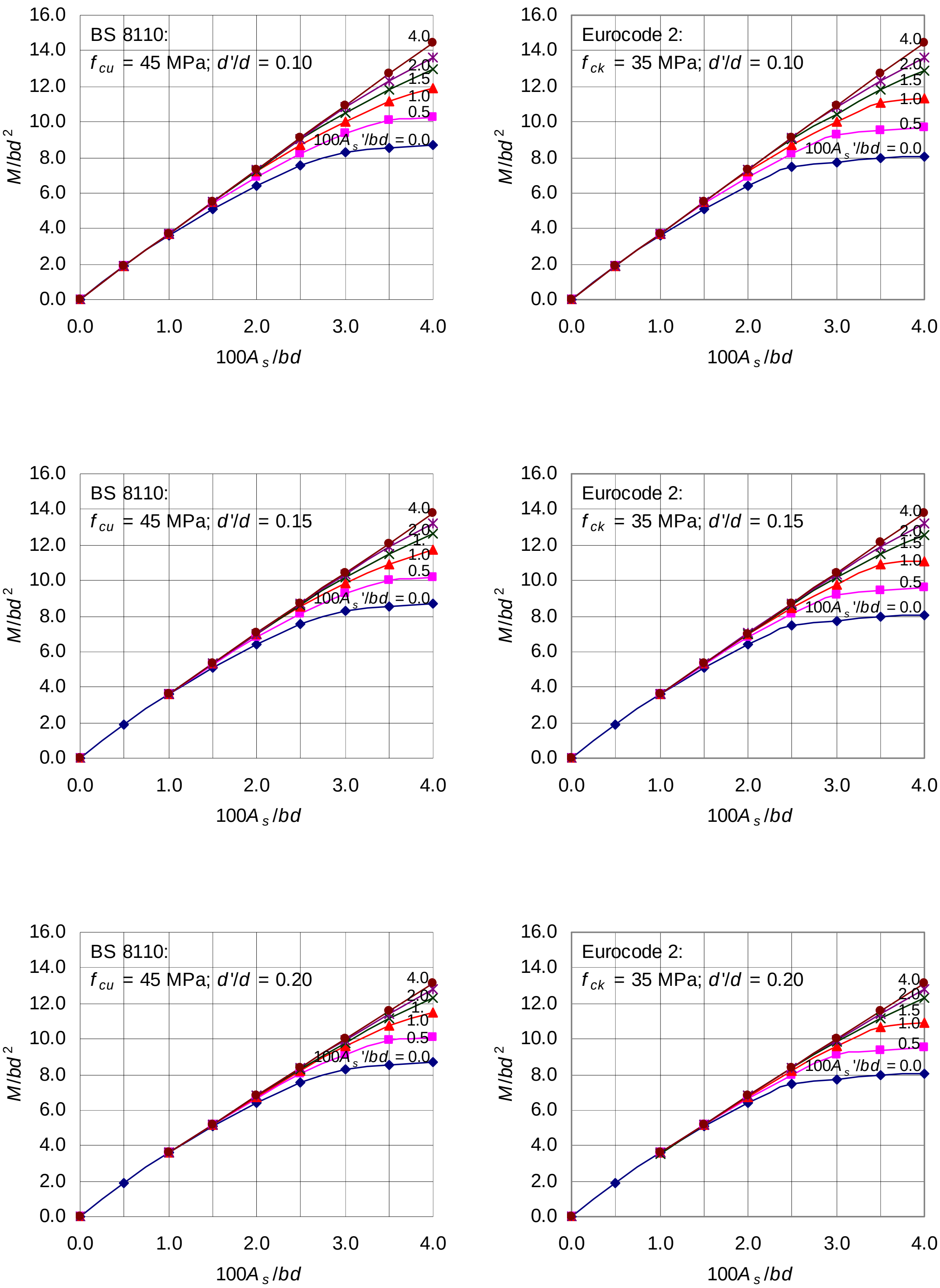

Fig. 10. Flexural strength of doubly-reinforced beams for concrete strength f_{cu} = 45 MPa/ f_{ck} = 35 MPa with high yield steel reinforcement

4. CONCLUSIONS

A comparison of flexural beam strength in accordance to BS 8110 and Eurocode 2 is reported herein. Both codes of practice are based on limit state design in accordance to strain compatibility and force equilibrium. Most of the parameters used in both codes are similar, except for type of concrete strength, limit for neutral axis depth, and depth of concrete compression zone in the simplified rectangular stress block.

When the similarities and dissimilarities have been taken into account, both codes of practice exhibited similar flexural strength for singly-reinforced and doubly-reinforced beams within the limit of reinforcement ratio. The registered differences in flexural strength for singly-reinforced beam sections based on both codes within the limit of reinforcement ratio are well below 1%, indicating that the final design of singly-reinforced beams should be the same based on either code. For the doubly-reinforced beams, the maximum difference in flexural strength outside the allowable limits of reinforcement ratio is 7%. However, the difference is again less than 1% if only beams within the allowable limits of reinforcement ratio are considered. These indicate that both codes of practice arrive at similar design for the main reinforcement of flexural members. The nomographs as presented can be used to compare the flexural strength calculated based on both codes.

Similar study can be conducted to compare the shear and torsional strengths of beams based on both codes. It is envisaged that this kind of study will further enhance the understanding of structural designers who will be adopting Eurocode 2 in lieu of BS 8110 beyond the year 2008 for structural concrete design.

5. REFERENCES

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