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Homeomorphisms of Fuzzy Topographic Topological Mapping(FTTM)

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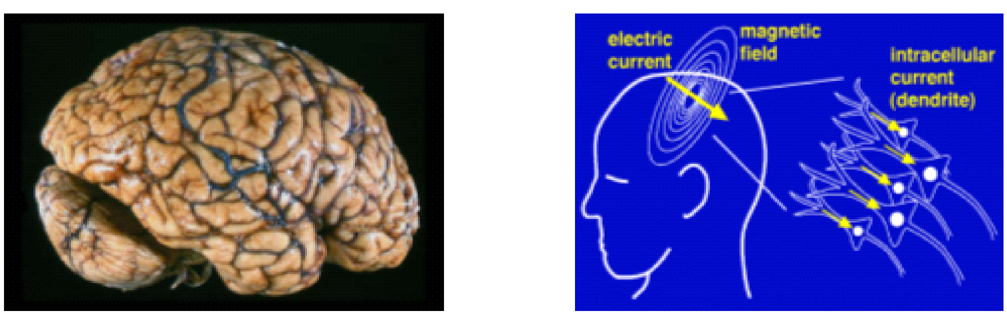
Abstract The mathematical structure of a novel model for detecting the source of a magnetic field is presented. Simulation results show that the orientation of unbounded single current source can be determined with fairly accuracy. The model, FTTM, has the potential to be used for identifying the location of epileptic foci in epilepsy disorder patients.

Keywords Magnetic field, homeomorphic, current source.

1 Introduction

The human brain (Figure 1a) is the most important structure in our body. It is also the most complex organized structure known to exist. There are four lobes in both halves of the cortex: frontal, pariental, temporal and occipital. The outermost layer of the brain is called cerebral cortex. The cerebral cortex has a total surface area of about 2500 cm^2, folded in a complicated way, so that it fits into the cranial cavity formed by the skull of the brain. There are at least 10^{10} neurons in the cerebral cortex [1].



(a) (b)

Figure 1: (a) Human Brain (b) Neuro Magnetic Field

These neurons are the active units in a vast signal-handling network. When information is being processed, small currents flow in the neural system, producing a weak magnetic field (Figure 1b), which can be measured non-invasively by a SQUID (Superconducting Quantum Interference Device). Magnetoencephalography (MEG) is a recording of magnetic fields produced by electrical activity of the neurons in the brain. MEG is completely non-invasive and non-hazardous [1]. When an epileptic patient is having an epileptic fit, the neurons in the brain of the patient produce four times current as compared to a normal brain. The current source is the epileptic foci.

Magnetic field readings obtained from SQUID give information in the process to determine location, direction and magnitude of a current source. This is called neuromagnetic inverse problem. Currently there is only a method for solving this problem, namely Bayesian that needs a priori information (data based model) and it is time consuming [2]. By using Bayesian, forward calculation is used to calculate the magnetic field caused by the current dipole at every possible point. The best location of the current source is determined by minimizing the sum of the squares of the difference between the measured and the calculated value similar to the least squares method. Then, a point with a minimum least squares is the location of a current source [3]. On the other hand, FTTM is a novel model for solving neuromagnetic inverse problem [4]. It does not need priori information and less time consuming.

2 Fuzzy Topographic Topological Mapping (FTTM)

FTTM has been developed to present three-dimensional view of unbounded single current [4]. It consists of three algorithms that link between four components of the model. The first component is Magnetic Contour Plane (MC), the second component is Base Magnetic Plane (BM), the third component is Fuzzy Magnetic Field (FM) and the final component is Topographic Magnetic Field (TM) as shown in Figure 2.

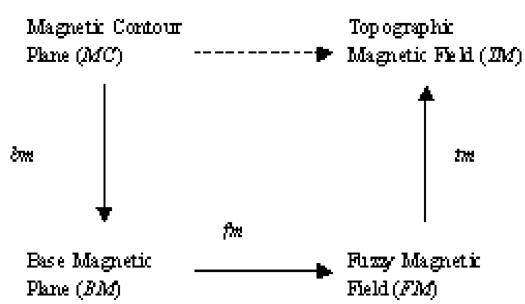


Figure 2: FTTM

MC is a magnetic field on a plane at the level $z = 0$, which is above the current source. The plane is lowered down to BM to a distance of $z = h(h < 0)$. Then the entire BM is fuzzified into a fuzzy environment, FM. Finally, a three-dimensional presentation of FM is plotted, namely TM. All of the components are topological spaces [5]. Furthermore, FTTM is also specially designed to have equivalent topological structures between its components. The outline of the proof inspired by the work on S^2[6] and followed by on E^2[5, 7] of Riemann surfaces.

3 Homeomorphisms Of Components Of FTTM

Each of the components of FTTM is homeomorphic to each other (see Figure 2).

Lemma 3.1 $MC \cong BM$

Proof

Let $bm : MC \to BM \ni bm\left((x,y)_0, \beta_z\right) = \left((x,y)_h, \beta_z\right), \forall \left((x,y)_0, \beta_z\right) \in MC$ such that $h < 0$ and $\beta_z \in \beta \subset R$. bm is a projection from $z = 0$ to $z = -h$. In order to show bm is onto, pick $\omega \in BM$, then $\omega = \left((x,y)_h, \beta_z\right)$ where $\left((x,y)_0, \beta_z\right) \in MC$. Thus $\exists \left((x,y)_0, \beta_z\right) \in MC$ such that $bm\left((x,y)_0, \beta_z\right) = \left((x,y)_h, \beta_z\right)$. bm is one to one since if

$$\begin{aligned}
&bm\left((x_1,y_1)_0, \beta_{z1}\right) = bm\left((x_2,y_2)_0, \beta_{z2}\right)\\
&\Rightarrow \left((x_1,y_1)_h, \beta_{z1}\right) = \left((x_2,y_2)_h, \beta_{z2}\right)\\
&\Rightarrow (x_1,y_1) = (x_2,y_2)\, \text{dan} \beta_{z1} = \beta_{z2}\\
&\Rightarrow x_1 = x_2, y_1 = y_2 \text{dan} \beta_{z1} = \beta_{z2}\,.
\end{aligned}$$

Furthermore bm is open. Pick $\left((a,b)_h, \beta_z\right) \in bm(\theta)$ such that θ is open in MC. Now consider $bm(\theta) = \left\{bm\left((x,y)_0, \beta_z\right) : \left((x,y)_0, \beta_z\right) \in \theta\right\}$. Since bm is a bijection, therefore $\exists$ unique $\left((x,y)_0, \beta_z\right) \in \theta$ such that $bm\left((x,y)_0, \beta_z\right) = \left((a,b)_h, \beta_z\right)$. θ is open, then $\exists N\left((x,y)_0, \beta_z\right) \subset \theta$. Next, consider

$$bm\left(N\left((x,y)_0, \beta_z\right)\right) = \left\{bm\left((x',y')_0, \beta'_z\right) : \left((x',y')_0, \beta'_z\right) \in N\left((x,y)_0, \beta_z\right)\right\}.$$

Clearly, $\left((a,b)_h, \beta_z\right) = bm\left((x,y)_0, \beta_z\right) \in bm\left(N\left((x,y)_0, \beta_z\right)\right)$. Now pick

$$p \in bm\left(N\left((x,y)_0, \beta_z\right)\right).$$

Since bm is a bijection, therefore $\exists$ unique $\left((x',y')_0, \beta'_z\right) \in N\left((x,y)_0, \beta_z\right)$ such that

$$bm\left((x',y')_0, \beta'_z\right) = p.$$

We know $N\left((x,y)_0, \beta_z\right) \subset \theta$ which implies $\left((x',y')_0, \beta'_z\right) \in \theta$. Consequently,

$$p = bm\left((x',y')_0, \beta'_z\right) \in bm(\theta)$$

hence $bm\left(N\left((x,y)_0, \beta_z\right)\right) \subset bm(\theta)$. Notice that

$$\left((a,b)_h, \beta_z\right) \in bm\left(N\left((x,y)_0, \beta_z\right)\right) \subset bm(\theta)$$

and since $\left((a,b)_h, \beta_z\right) \in bm(\theta)$ is arbitrary, therefore

$$\left((a,b)_h, \beta_z\right) \in bm\left(N\left((x,y)_0, \beta_z\right)\right) \subset bm(\theta)$$

for every $((a,b)_h, \beta_z) \in bm(\theta)$ for some corresponding $((x,y)_0, \beta_z) \in \theta$. Thus, $bm(\theta)$ is open. Now the fact θ is also arbitrary, therefore we can conclude $bm(\theta)$ is open in BM for every open θ in MC.

Now, by defining mappings

$$fm : BM \to FM \ni fm((x,y)_h, B_z) = ((x,y)_h, \mu_{B_z})$$
$$tm : FM \to TM \ni tm((x,y)_h, \mu_{B_z}) = (x,y,z)$$

and using similar arguments as above, we can have the following lemmas, respectively. Details of the proof are contained in [5].

Lemma 3.2 $BM \cong FM$.

Lemma 3.3 $FM \cong TM$

Using lemmas 3.1, 3.2 and 3.3, we will have the following theorem.

Theorem 3.4 $MC \cong TM$.
Proof
Since $MC \cong BM$, $BM \cong FM$ and $FM \cong TM$, hence $MC \cong TM$ (Figure 3).

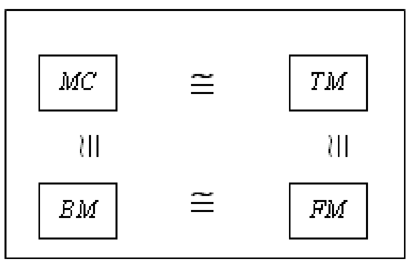


Figure 3: Homeomorphisms of FTTM

These homeomorphisms make up algorithms for determining unbounded single current source [3]. Samples of simulation are given in the following section.

4 Simulation

To apply FTTM, forward calculations were used to generate the magnetic fields data. This is accomplished by executing an algorithm [8] followed by three more algorithms [3] to the second and third components of FTTM. In the final component, the location and the current magnitude can be determined. The following samples of simulations show comparison between the forward and inverse calculations.

In the forward simulations, data 1 is located parallel to the xy- plane and there is no inclination towards the xy-plane. The percentage difference for data 1 is zero. This

Data	Forward Calculation (A)	Inverse Calculation (B)	Differences $\theta_d = \lvert A - B \rvert$	% Difference % θ_d
Data 1	0°	0°	0°	0 %
Data 2	7.59°	7.47°	0.12°	1.6 %
Data 3	14.93°	13.52°	1.41°	9.4 %
Data 4	28.07°	22.1°	5.97°	21.3 %
Data 5	9.96°	9.66°	0.3°	3.0 %
Data 6	21.4°	17.25°	4.15°	19.4 %

Table 1 : Percentage Difference Of The Inclination Of Current Towards The *xy* - plane Between Forward and Inverse Calculation.

implies the successful use of FTTM along with the theoretical distance, h[3] between the measurement plane and the current source. Meanwhile, data 2, 3 and 4, were made to have some inclination towards the xy-plane at increasing angles, respectively. The table shows that as the inclination increases, the percentage error increases.

For data 5 and 6, the current segment inclined to the xy-plane as well as to the x-axis. The error increases as the inclination towards the x-axis increases. Samples of FTTM for Data 4 and 5 are presented in Figure 4 and 5, respectively.

Table 2 shows that the percentage error in the current magnitude is quite proportional to the percentage error obtained for the current location (see Table 1).

5 Conclusion

In this paper, we present the mathematical structure of a novel model, FTTM, for detecting the source of a magnetic field. Simulation results show that the orientation and magnitude of unbounded single current source can be determined with fairly accuracy. The model has the potential to be used for identifying the location of epileptic foci in epilepsy disorder patients.

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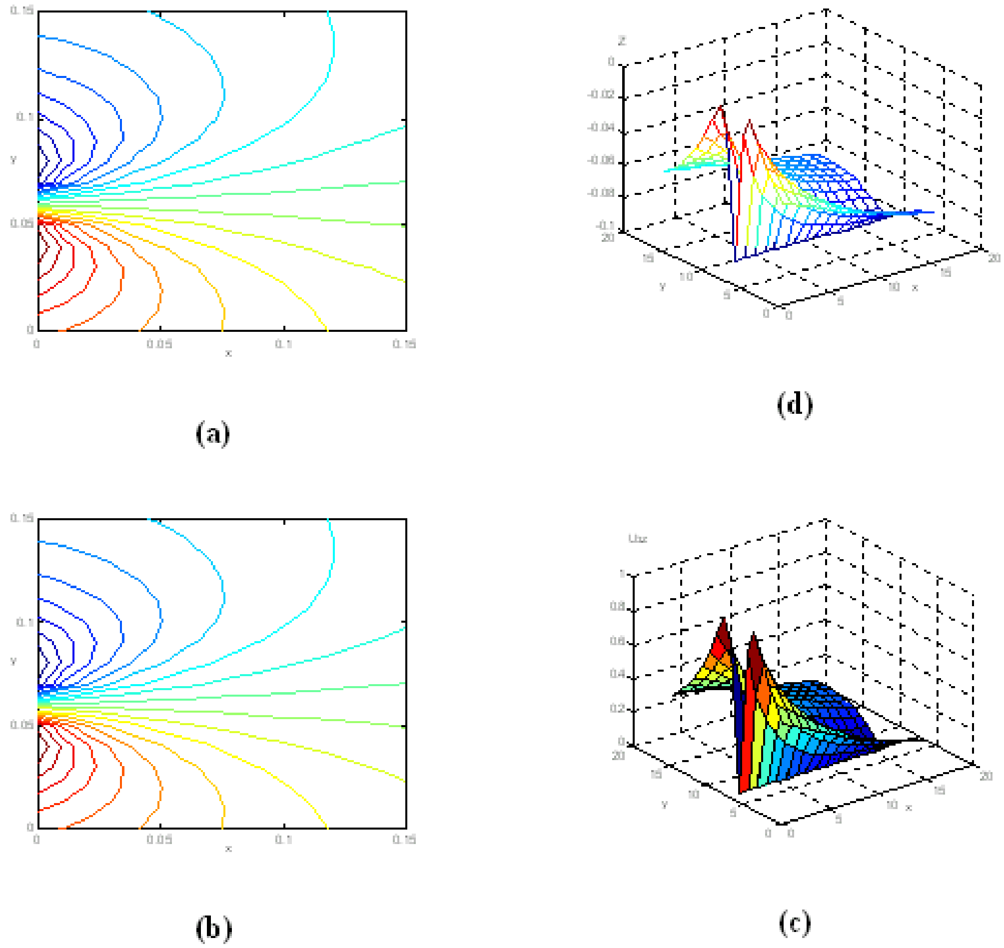

Figure 4: Components of FTTM for Data 4 (a) MC (b) BM (c) FM (d) TM

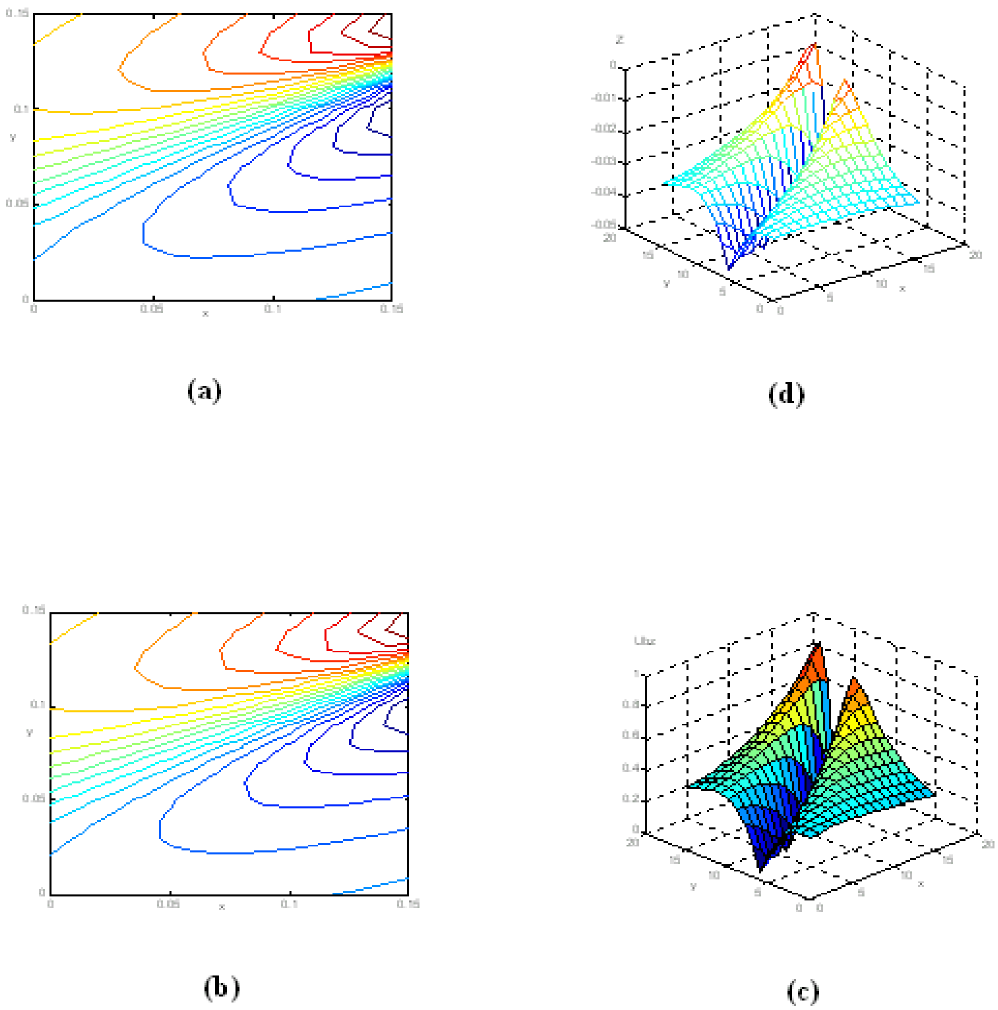

Figure 5: Components of FTTM for Data 5 (a) MC (b) BM (c) FM (d) TM

Data	Forward Calculation (A)	Inverse Calculation (B)	Differences $I_d = \lvert A - B \rvert$	% Difference % I_d
Data 1	2	2	0	0 %
Data 2	3	2.975	0.025	0.8 %
Data 3	2.5	2.43	0.07	2.8 %
Data 4	2	1.672	0.328	16.4 %
Data 5	2.5	2.454	0.046	1.8 %
Data 6	3	2.64	0.36	12.0 %

Table 2 : Percentage Difference of Magnitude of Current Between Forward and Inverse Calculation.

References

[1] M. Hamalainen, R, Hari, J.I. Ilmoniemi, J. Knuutila and O.V. Lounasmaa, *Magnetoencephalography - theory, Instrumentation, and Applications to Non-invasive Studies of The Working of Human Brain,* Reviews of Modern Physics, 65(2)(1993), 430-434.

[2] A. Tarantola and B. Valette, *Inverse Problems: Quest for Information,* J. Geophys, 50(1982), 159-170.

[3] Fauziah Zakaria, *Algoritma Penyelesaiana Masalah Songsang Arus Tunggal Tak Terbatas MEG,* Universiti Teknologi Malaysia: Master Thesis, 2002.

[4] Tahir Ahmad, Rashdi Shah Ahmad, Fauziah Zakaria and Liau Li Yun, *Development of Detection Model for Neuromagnetic Fields,* Proceeding of BIOMED, Kuala Lumpur: Universiti Malaysia, 2000 119-121.

[5] Liau Li Yun, *Homeomorfisma S^2 Antara E^2 Melalui Struktur Permukaan Riemann Serta Deduksi Teknik Pembuktiannya Bagi Homeomorfisma Pemetaan Topologi Topografi Kabur(FTTM),* Universiti Teknologi Malaysia: Master Thesis, 2001.

[6] Tahir Ahmad, *Permukaan Riemann: S^2*, MATEMATIKA, UTM, Jil. 9, Bil. 1(1993), ms. 9-17.

[7] Liau Li Yun & Tahir Ahmad, *Pembinaan Homeomorfisma dari Sfera ke Elipsoid,* MATEMATIKA, UTM, Jilid 19, Bil. 2(2003), ms. 121-138.

[8] Rashdi Shah Ahmad , Tahir Ahmad & C. S. Leong, *Algorithm of Magnetic Flux Density on a Plane Generated by a Finite Length Current Source,* Jurnal TMSK, UiTM Shah Alam , Jilid 3, Bil. 1(Sept. 2001), ms. 63-74.