A high accuracy variant of the iterative alternating decomposition explicit method for solving the heat equation

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Abstract: We consider three level difference replacements of parabolic equations focusing on the heat equation in two space dimensions. Through a judicious splitting of the approximation, the scheme qualifies as an alternating direction implicit (ADI) method. Using the well known fact of the parabolic-elliptic correspondence, we shall derive a two stage iterative procedure employing a fractional splitting strategy applied alternately at each intermediate time step to the one dimensional heat equation. As the basis of derivation is the unconditionally stable (4,2) accurate ADI scheme, this method is convergent, computationally stable and highly accurate.

Keywords: alternating direction implicit (ADI) method; iterative alternating decomposition explicit (IADE) method; fractional splitting.

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1 Introduction

Consider the heat equation,

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial^2 x} \tag{1}$$

subject to given initial and Dirichlet boundary conditions over a rectangular domain with a uniformly spaced network whose mesh points are $x_i = i\Delta x$, $t_j = j\Delta t$. Its corresponding equation in two space is given by,

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial^2 x} + \frac{\partial^2 U}{\partial^2 y}. \tag{2}$$

Following (Mitchell and Griffiths, 1980), a stable and (4,2) accurate three level difference replacement of equation (2) is given by,

$$\begin{aligned}
&\left(1+\left(\frac{1}{12}-\frac{2}{3}\lambda\right)\delta_x^2\right)\left(1+\left(\frac{1}{12}-\frac{2}{3}\lambda\right)\delta_y^2\right)u_{i.j.k+1} \\
&\quad= \frac{2}{3}\lambda\left(\delta_x^2+\delta_y^2+\frac{1}{2}\delta_x^2\delta_y^2\right)u_{i,j,k} \\
&\qquad+\left(1+\left(\frac{1}{12}+\frac{2}{3}\lambda\right)(\delta_x^2+\delta_y^2)\right)u_{i,j,k-1} \\
&\qquad+\left(\frac{1}{12}-\frac{2}{3}\lambda\right)^2\delta_x^2\delta_y^2(2u_{i,j,k}-u_{i,j,k-1})
\end{aligned} \tag{3}$$

where δ is the usual central difference operator and $\lambda = \Delta t/(\Delta x)^2 = \Delta t/(\Delta y)^2$, the mesh ratio for equidistance spacing. The difference scheme equation (3) can be split as follows, to qualify as an alternating direction implicit (ADI) scheme,

$$\begin{aligned}
&\left(1+\left(\frac{1}{12}-\frac{2}{3}\lambda\right)\delta_x^2\right)u_{i,j,k+1/2} \\
&\quad= -\left(\frac{1}{12}-\frac{2}{3}\lambda\right)\delta_y^2(2u_{i,j,k}-u_{i.j.k-1}) \\
&\qquad+\frac{2}{3}\lambda\left(\delta_x^2+\delta_y^2+\frac{1}{2}\delta_x^2\delta_y^2\right)u_{i,j,k} \\
&\qquad+\left(1+\left(\frac{1}{12}+\frac{2}{3}\lambda\right)(\delta_x^2+\delta_y^2)\right)u_{i,j,k-1}
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\left(1+\left(\frac{1}{12}-\frac{2}{3}\lambda\right)\delta_y^2\right)u_{i,j,k+1} &= u_{i,j,k+1/2} \\
&\quad+\left(\frac{1}{12}-\frac{2}{3}\lambda\right)\delta_y^2(2u_{i,j,k}-u_{i,j,k-1}).
\end{aligned} \tag{5}$$

As the temperature reaches steady state over time, $U \to$ constant and $(\partial U/\partial t) \to 0$ and the parabolic equation (5) reduces to the elliptic equation (Laplace's equation),

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \tag{6}$$

whose numerical solution can be solved iteratively using the same ADI technique,

$$\begin{aligned}
&\left(1+\left(\frac{1}{12}-\frac{2}{3}r\right)\delta_x^2\right)u_{i,j}^{*} \\
&\quad=\left(-\left(\frac{1}{12}-\frac{2}{3}r\right)\delta_y^2+\frac{2}{3}r\left(\delta_x^2+\delta_y^2+\frac{1}{2}\delta_x^2\delta_y^2\right)\right. \\
&\qquad\left.+\left(1+\left(\frac{1}{12}+\frac{2}{3}r\right)\left(\delta_x^2+\delta_y^2\right)\right)\right)u_{i,j}^{(p)}
\end{aligned} \tag{7}$$

and

$$\left(1+\left(\frac{1}{12}-\frac{2}{3}\right)\delta_y^2\right)u_{i,j}^{(p+1)} = u_{i,j}^{*} + \left(\frac{1}{12}-\frac{2}{3}r\right)\delta_y^2 u_{i,j}^{(p)} \tag{8}$$

where r is the acceleration parameter.

2 The iterative alternating decomposition explicit (IADE) method

Note from the composite formula equation (3) that the iterative procedure converges if

$$u_{i,j,k-1} = u_{i,j,k} = u_{i,j,k+1} = u_{i,j}$$

for k sufficiently large, leading to

$$\left(\delta_x^2+\delta_y^2+\frac{1}{6}\delta_x^2\delta_y^2\right)u_{i,j} = 0$$

which represents a nine point difference replacement of the Laplace's equation (6). Hence we are motivated by this well known fact of the parabolic-elliptic correspondence (Varga, 1962; Wachspress, 1966; Yanenko, 1971) to develop a new iterative scheme for the solution of equation (1) by considering the following two step iterates corresponding to equations (7) and (8),

$$\begin{aligned}
(\mathbf{I}+\alpha\mathbf{G}_1)\mathbf{u}^{(p+1/2)} &= (-\alpha\mathbf{G}_2+\hat{\omega}(\mathbf{G}_1+\mathbf{G}_2+(1/2)\mathbf{G}_1\mathbf{G}_2) \\
&\quad+\mathbf{I}+\omega(\mathbf{G}_1+\mathbf{G}_2))\mathbf{u}^{(p)}-2r\mathbf{f}
\end{aligned} \tag{9}$$

and

$$(\mathbf{I}+\alpha\mathbf{G}_2)\mathbf{u}^{(p+1)} = \mathbf{u}^{(p+1/2)} + \alpha\mathbf{G}_2\mathbf{u}^{(p)} \tag{10}$$

where

$$\alpha = \frac{1}{12} - \frac{2}{3}r, \quad \omega = \frac{1}{12} + \frac{2}{3}r, \quad \hat{\omega} = \frac{2}{3}r$$

with $r > 0$ being the acceleration parameter of the iterative process.

Noting that $\mathbf{u}^{(p+1)} = \mathbf{u}^{(p)}$ as $p \to \infty$, we have,

at the $(p + 1/2)$ iterate,

$$(\mathbf{I} + \alpha\mathbf{G}_2)\mathbf{u}^{(p+1/2)} = (\mathbf{I} + (\alpha + 2r)\mathbf{G}_1)(\mathbf{I} + (2r\mathbf{G}_2) + \beta\mathbf{G}_1\mathbf{G}_2)\,\mathbf{u}^{(p)} - 2r\mathbf{f} \tag{11}$$

and at the $(p + 1)$ iterate,

$$(\mathbf{I} + \alpha\mathbf{G}_2)\mathbf{u}^{(p+1)} = \mathbf{u}^{(p+1/2)} + \alpha\mathbf{G}_2\mathbf{u}^{(p)} \tag{12}$$

where $\beta = 2r(3\alpha - 2r)/3$, and $\mathbf{G}_1$ and $\mathbf{G}_2$ are lower and upper bidiagonal matrices given by,

$$\mathbf{G}_1 = \begin{bmatrix} 1 & & & & & & \\ k_1 & 1 & & & & & \\ & k_2 & 1 & & & & \\ & & \cdot & \cdot & & & \\ & & & \cdot & \cdot & & \\ & & & & \cdot & \cdot & \\ & & & & k_{m-2} & 1 & \\ & & & & & k_{m-1} & 1 \end{bmatrix}_{(mxm)}, \tag{13}$$

$$\mathbf{G}_2 = \begin{bmatrix} e_1 & h & & & & & \\ & e_2 & h & & & & \\ & & e_3 & h & & & \\ & & & \cdot & \cdot & & \\ & & & & \cdot & \cdot & \\ & & & & & \cdot & \cdot \\ & & & & & e_{m-1} & h \\ & & & & & & e_m \end{bmatrix}_{(mxm)}. \tag{14}$$

Elimination of $\mathbf{u}^{(p+1/2)}$ from equations (11) and (12) leads to the single composite formula,

$$(\mathbf{I} + \alpha\mathbf{G}_1)(\mathbf{I} + \alpha\mathbf{G}_2)\mathbf{u}^{(p+1)} = ((\mathbf{I} + (\alpha + 2r)\mathbf{G}_1)(\mathbf{I} + 2r\mathbf{G}_2) + \beta\mathbf{G}_1\mathbf{G}_2)\mathbf{u}^{(p)} - 2r\mathbf{f} + \alpha(\mathbf{I} + \alpha\mathbf{G}_1) + \mathbf{G}_2\mathbf{u})^{(p)}.$$

As $p \to \infty$, $\mathbf{u}^{(p)}$, $\mathbf{u}^{(p+1)} \to \mathbf{u}$ resulting in

$$\mathbf{A} = \mathbf{G}_1 + \mathbf{G}_2 + \frac{1}{6}\mathbf{G}_1\mathbf{G}_2 \tag{15}$$

and

$$\mathbf{A}\mathbf{u} = \mathbf{f} \tag{16}$$

$\mathbf{A}$ is a tridiagonal matrix which arises from the difference method used to approximate the parabolic equation (1). For example, if the familiar weighted approximation,

$$-\lambda\theta u_{i-1,j+1} + (1 + 2\lambda\theta)u_{i,j+1} - \lambda\theta u_{i+1,j+1} = \lambda(1-\theta)u_{i-1,j} + (1 - 2\lambda(1-\theta))u_{ij} + \lambda(1-\theta)u_{i+1,j}$$

is used with order of accuracy of (1,2), (2,2), (2,4) and (1,2) when θ = 1, 1/2, (1/2 – 1/12λ) and 0 respectively, then its totality can be displayed in matrix form equation (16) as,

$$\begin{bmatrix} a & b & & & & & \\ c & a & b & & & & \\ & c & a & b & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \cdot & \cdot & \cdot \\ & & & & c & a & b \\ & & & & & c & a \end{bmatrix}_{(mxm)} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdot \\ \cdot \\ \cdot \\ u_{m-1} \\ u_m \end{bmatrix}_{j+1} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \cdot \\ \cdot \\ \cdot \\ f_{m-1} \\ f_m \end{bmatrix}. \tag{17}$$

Using equations (11) and (12), we have,

$$\mathbf{u}^{(p+1/2)} = (\mathbf{I} + \alpha\mathbf{G}_1)^{-1}((\mathbf{I} + (\alpha + 2r)\mathbf{G}_1)(\mathbf{I} + (2r\mathbf{G}_2) + \beta\mathbf{G}_1\mathbf{G}_2)\mathbf{u}^{(p)} - 2r\mathbf{f}) \tag{18}$$

and

$$\mathbf{u}^{(p+1)} = (\mathbf{I} + \alpha\mathbf{G}_2)^{-1}(\mathbf{u}^{(p+1/2)} + \alpha + \mathbf{G}_2\mathbf{u}^{(p)}) \tag{19}$$

giving us the following computational formulae at each of the half-iterates,

at the $(p + 1/2)$ iterate,

$$u_i^{(p+1/2)} = (s_{i-1}u_{i-1}^{(p)} + v_i u_i^{(p)} + \hat{s}u_{i+1}^{(p)} - w_{i-1}u_{i-1}^{(p+1/2)} - r^* f_i)/A, \quad i = 1,2,\ldots,m \tag{20}$$

with

$s_0 = v_0 = w_0 = 0$ and $A = 1 + \dfrac{(1+8r)}{12} \neq 0$

at the $(p + 1)$ iterate,

$$u_{m+1-i}^{(p+1)} = (u_{m+1-i}^{(p+1/2)} + d_{m+1-i}u_{m+1-i}^{(p)} + \bar{d}u_{m+2-i}^{(p)} - \bar{d}u_{m+2-i}^{(p+1)})/(1 + d_{m+1-i}) \tag{21}$$

with $d_i \neq 0$ for $i = 1, 2, \ldots, m$ and

$$r^* = -2r, \quad h = \frac{6}{7}b, \quad \alpha = \frac{1+8r}{12},$$

$$q = \frac{r^*(3\alpha - r^*)}{3}, \quad P = \alpha - 2r, \quad \hat{f} = r^* P,$$

$$g = \hat{f} + q, \quad \hat{s} = h(r^* + f + q), \quad \bar{d} = \alpha h,$$

$$\hat{A} = 1 + \alpha, \quad e_1 = \frac{6(a-1)}{7}, \quad k_i = \frac{6c}{6 + e_i},$$

$$e_{i+1} = \frac{6}{7}\left(a - \frac{k_i h}{6} - 1\right), \quad i = 2,3,\ldots,m$$

$$s_{i-1} = Pk_{i-1} + gk_{i-1}e_{i-1}, \quad i = 1,2,\ldots,m$$

$$v_i = 1 + r^* e_i + P + g(hk_{i-1} + e_i),$$

$$w_{i-1} = \alpha k_{i=1}, \quad d_i = \alpha e_i, \quad i = 1,2,\ldots,m.$$

The two-step iterative procedure of equations (20) and (21) corresponds to sweeping through the mesh, involving at each iterate, the solution of an explicit equation. This is continued until convergence is reached, that is, when the convergence requirement $\|\mathbf{u}^{(p+1)} - \mathbf{u}^{(p)}\|_\infty \le \varepsilon$ is met where ε is the convergence criterion.

3 Numerical results

In this experiment we attempted to solve the following heat conduction problem (Saulev, 1964),

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial^2 x} \qquad 0 \le x \le 1 \tag{22}$$

subject to the initial condition

$$U(x, 0) = 4x(1 - x) \qquad 0 \le x \le 1$$

and the boundary conditions

$$U(0, t) = U(1, t) = 0 \quad t \ge 0.$$

The exact solution is given by

$$U(x,t) = \frac{32}{\pi^3} \sum_{k=1,(2)}^{\infty} \frac{1}{k^3} e^{-\pi^2 k^2 t} \sin(k\pi x). \tag{23}$$

Tables 1–3 provide a comparison of the accuracy of the methods under consideration in terms of the absolute errors as well as the root mean square error at the appropriate grid points for mesh ratios $\lambda = 0.1$, $\lambda = 0.5$ and $\lambda = 1.0$ at time levels of $t = 0.05$, $t = 0.25$ and $t = 0.5$. The results in these tables amply demonstrate that this new variant of the IADE (NVIADE) method has comparable accuracy with the highly accurate AGE method of the Peaceman-Rachford (Tien and Chawla, 1989) variant as well as the IADE scheme using the Mitchell-Fairweather variant (Sahimi et al., 1993). This is even more apparent when the (2,4) accurate Douglas finite difference approximation is used to approximate the parabolic equation (1). As the basis of derivation is the unconditionally stable (4,2) accurate ADI scheme, the NVIADE method is convergent and computationally stable and its accuracy compares well with the (4,2) accurate IADE-MF and (2,2) accurate AGE.

Table 1 Absolute errors of numerical solutions $\lambda = 0.1$, $t = 0.05$, $\Delta t = 0.001$, $\Delta x = 0.1$, $eps = 10^{-4}$

X method	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*Average of all absolute errors*	*Root mean square error*	*Number of iterations*
Age												
IMP	1.47×10^{-3}	2.63×10^{-3}	3.34×10^{-3}	3.67×10^{-3}	3.76×10^{-3}	3.67×10^{-3}	3.35×10^{-3}	2.63×10^{-3}	1.47×10^{-3}	2.89×10^{-3}	9.07×10^{-6}	2
CN	9.15×10^{-4}	1.64×10^{-3}	2.09×10^{-3}	2.30×10^{-3}	2.36×10^{-3}	2.30×10^{-3}	2.09×10^{-3}	1.64×10^{-3}	9.15×10^{-4}	1.81×10^{-3}	3.56×10^{-6}	2
DG	1.67×10^{-6}	5.49×10^{-6}	1.15×10^{-5}	1.72×10^{-5}	1.97×10^{-5}	1.73×10^{-5}	1.15×10^{-5}	5.52×10^{-6}	1.60×10^{-6}	1.01×10^{-5}	1.46×10^{-10}	2
NVIADE												
IMP	1.49×10^{-3}	2.64×10^{-3}	3.35×10^{-3}	3.67×10^{-3}	3.75×10^{-3}	3.67×10^{-3}	3.35×10^{-3}	2.64×10^{-3}	1.47×10^{-3}	2.89×10^{-3}	9.08×10^{-6}	2
CN	9.17×10^{-4}	1.64×10^{-3}	2.10×10^{-3}	2.31×10^{-3}	2.36×10^{-3}	2.31×10^{-3}	2.10×10^{-3}	1.64×10^{-3}	9.14×10^{-4}	1.81×10^{-3}	3.57×10^{-6}	2
DG	2.47×10^{-6}	1.50×10^{-7}	1.33×10^{-6}	4.09×10^{-6}	4.32×10^{-7}	1.23×10^{-7}	3.30×10^{-7}	5.89×10^{-7}	4.12×10^{-6}	4.13×10^{-6}	1.22×10^{-11}	2
IADE-MF												
IMP	1.07×10^{-3}	1.88×10^{-3}	2.74×10^{-3}	3.19×10^{-3}	3.36×10^{-3}	3.34×10^{-3}	3.11×10^{-3}	2.59×10^{-3}	1.65×10^{-3}	2.55×10^{-3}	9.07×10^{-6}	4
CN	7.94×10^{-4}	1.43×10^{-3}	1.86×10^{-3}	2.07×10^{-3}	2.14×10^{-3}	2.11×10^{-3}	1.95×10^{-3}	1.55×10^{-4}	8.71×10^{-4}	1.64×10^{-3}	3.53×10^{-7}	3
DG	5.84×10^{-6}	8.46×10^{-6}	3.74×10^{-7}	8.94×10^{-6}	1.38×10^{-5}	1.35×10^{-5}	9.28×10^{-6}	4.48×10^{-6}	1.39×10^{-6}	7.34×10^{-6}	1.35×10^{-10}	3
Exact solution	0.1950648	0.3707705	0.5098716	0.5989617	0.6296137	0.5989617	0.5098716	0.3707705	0.1950648	–	–	–

Table 2 Absolute errors of numerical solutions $\lambda = 0.5$, $t = 0.25$, $\Delta t = 0.005$, $\Delta x = 0.1$, $eps = 10^{-4}$

X method	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*Average of all absolute errors*	*Root mean square error*	*Number of iterations*
Age												
IMP	2.18×10^{-3}	4.15×10^{-3}	5.69×10^{-3}	6.70×10^{-3}	7.04×10^{-3}	6.69×10^{-3}	5.71×10^{-3}	4.14×10^{-3}	2.19×10^{-3}	4.94×10^{-3}	2.76×10^{-5}	2
CN	5.32×10^{-4}	1.02×10^{-3}	1.39×10^{-3}	1.64×10^{-3}	1.72×10^{-3}	1.64×10^{-3}	1.40×10^{-3}	1.01×10^{-3}	5.43×10^{-4}	1.21×10^{-3}	1.65×10^{-6}	2
DG	7.38×10^{-6}	2.05×10^{-5}	3.06×10^{-5}	3.77×10^{-5}	4.11×10^{-5}	4.53×10^{-5}	3.62×10^{-5}	2.76×10^{-5}	1.17×10^{-5}	3.23×10^{-5}	1.20×10^{-9}	2
NVIADE												
IMP	2.23×10^{-3}	4.19×10^{-3}	5.76×10^{-3}	6.77×10^{-3}	7.11×10^{-3}	6.77×10^{-3}	5.75×10^{-3}	4.18×10^{-3}	2.19×10^{-3}	5.00×10^{-3}	2.82×10^{-5}	2
CN	5.47×10^{-4}	1.03×10^{-3}	1.41×10^{-3}	1.65×10^{-3}	1.74×10^{-3}	1.65×10^{-3}	1.40×10^{-3}	1.02×10^{-3}	5.32×10^{-4}	1.22×10^{-3}	1.68×10^{-6}	2
DG	7.38×10^{-6}	2.05×10^{-6}	3.06×10^{-5}	3.77×10^{-5}	4.11×10^{-6}	4.04×10^{-5}	3.57×10^{-5}	2.74×10^{-5}	1.63×10^{-5}	2.85×10^{-5}	9.37×10^{-10}	2
IADE-MF												
IMP	2.20×10^{-3}	4.19×10^{-3}	5.76×10^{-3}	6.76×10^{-3}	7.10×10^{-3}	6.75×10^{-3}	5.74×10^{-3}	4.17×10^{-3}	2.19×10^{-3}	4.98×10^{-3}	2.81×10^{-5}	3
CN	5.37×10^{-4}	1.02×10^{-3}	1.41×10^{-3}	1.65×10^{-3}	1.74×10^{-3}	1.65×10^{-3}	1.41×10^{-3}	1.02×10^{-4}	5.38×10^{-4}	1.22×10^{-3}	1.68×10^{-6}	3
DG	8.14×10^{-6}	2.53×10^{-5}	3.40×10^{-5}	3.97×10^{-5}	4.15×10^{-5}	3.92×10^{-5}	3.29×10^{-5}	2.31×10^{-5}	1.03×10^{-5}	2.82×10^{-5}	9.35×10^{-10}	2
Exact solution	2.704606×10^{-2}	5.144467×10^{-2}	7.080751×10^{-2}	8.323922×10^{-2}	8.7522990×10^{-2}	8.323922×10^{-2}	7.080751×10^{-2}	5.144467×10^{-2}	2.704606×10^{-2}	–	–	–

Table 3 Absolute errors of numerical solutions $\lambda = 1.5$, $t = 0.5$, $\Delta t = 0.01$, $\Delta x = 0.1$, $eps = 10^{-4}$

X method	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*Average of all absolute errors*	*Root mean square error*	*Number of iterations*
AGE												
IMP	6.28×10^{-4}	1.25×10^{-3}	1.64×10^{-3}	1.96×10^{-3}	2.04×10^{-3}	1.93×10^{-3}	1.69×10^{-3}	1.20×10^{-3}	6.98×10^{-4}	1.45×10^{-3}	2.76×10^{-6}	2
CN	7.28×10^{-5}	1.46×10^{-4}	1.90×10^{-4}	2.27×10^{-4}	2.36×10^{-4}	2.24×10^{-4}	1.97×10^{-4}	1.39×10^{-4}	8.12×10^{-5}	1.21×10^{-3}	1.65×10^{-6}	2
DG	1.64×10^{-5}	2.73×10^{-5}	4.33×10^{-5}	4.90×10^{-5}	5.29×10^{-5}	5.10×10^{-5}	3.97×10^{-5}	3.11×10^{-5}	1.17×10^{-5}	3.58×10^{-5}	1.49×10^{-9}	2
NVIADE												
IMP	7.01×10^{-4}	1.32×10^{-3}	1.82×10^{-3}	2.14×10^{-3}	2.25×10^{-3}	2.14×10^{-3}	1.82×10^{-3}	1.33×10^{-3}	7.03×10^{-4}	1.58×10^{-3}	2.82×10^{-6}	2
CN	9.17×10^{-4}	1.64×10^{-3}	2.09×10^{-3}	2.30×10^{-3}	2.36×10^{-3}	2.31×10^{-3}	2.10×10^{-3}	1.64×10^{-3}	9.14×10^{-4}	1.81×10^{-3}	3.56×10^{-6}	2
DG	6.74×10^{-6}	1.52×10^{-5}	2.13×10^{-5}	2.52×10^{-5}	2.65×10^{-6}	2.53×10^{-6}	2.17×10^{-5}	1.59×10^{-5}	8.68×10^{-6}	1.85×10^{-5}	3.90×10^{-10}	2
IADE–MF												
IMP	7.03×10^{-4}	1.33×10^{-3}	1.79×10^{-3}	2.08×10^{-3}	2.16×10^{-3}	2.03×10^{-3}	1.72×10^{-3}	1.25×10^{-3}	6.63×10^{-4}	1.53×10^{-3}	2.62×10^{-6}	4
CN	8.58×10^{-5}	1.73×10^{-4}	2.31×10^{-4}	2.63×10^{-4}	2.67×10^{-4}	2.47×10^{-4}	2.04×10^{-4}	1.43×10^{-4}	7.25×10^{-5}	1.87×10^{-4}	3.99×10^{-8}	3
DG	6.14×10^{-6}	8.45×10^{-6}	1.24×10^{-5}	1.57×10^{-5}	1.75×10^{-5}	1.75×10^{-5}	1.56×10^{-5}	1.18×10^{-5}	6.52×10^{-6}	1.24×10^{-5}	1.72×10^{-10}	3
Exact solution	2.293641×10^{-3}	4.362764×10^{-3}	6.004829×10^{-3}	7.059100×10^{-3}	7.422377×10^{-3}	7.059100×10^{-3}	6.004829×10^{-3}	4.362764×10^{-3}	2.293641×10^{-3}	–	–	–

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