

MODELLING OF SINGLE LINK FLEXIBLE MANIPULATOR  
WITH FLEXIBLE JOINT

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To my beloved family Razak , Habibah , Badrol , Haszni , Zila , Fizi , Fiqah , Haziq ,  
Afifah and special person Ayu . Thank for all your support .

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## ABSTRACT

This paper presents a systematic approach to dynamic modelling of a single link flexible manipulator with flexible joint for the case in which the link is oriented vertically. Flexibility is attained by attaching the link to the motor shaft using a pair of springs. The system has two degrees of freedom, corresponding to the rotation of the motor shaft with respect to a coordinate frame fixed to the base, and the rotation of the flexible joint to which the link is attached with respect to the motor. The output of the system is the tip angle, which is given the sum of the motor angle,  $\theta$  and the joint deflection  $\alpha$  with respect to the motor shaft. A dynamic model of the system is developed based on Lagrange's equations of motion. The presence of the gravity is accounted in this model which introduces a non-linearity into the system in the form of a sinusoid, as a result of the potential energy due to gravity. The non-linear model of the dynamics will be linearized by a reasonable assumption. The resulting generalized model is validated through computer simulations and the results will be validated with the existing results. The investigations on the dynamic model in terms of time and frequency responses also are carried out. Besides, a design technique for a vibration suppression of the system is also presented.

## ABSTRAK

Kertas ini mempersembahkan penyampaian sistematik untuk model dinamik pengolah fleksibel satu lengan dengan sendi fleksibel di mana untuk kes ini lengan berada di kedudukan menegak. Fleksibel diperolehi dengan menghubungkan lengan kepada aci motor menggunakan sepasang spring. Sistem ini mempunyai dua darjah kebebasan bersesuaian kepada putaran aci motor merujuk kepada rangka koordinat bertempat kepada tapak dan putaran sendi fleksibel di mana lengan dihubungkan merujuk kepada motor. Keluaran sistem adalah sudut hujung lengan di mana ianya diberikan oleh jumlah sudut motor,  $\theta$  dan penyimpangan sendi  $\alpha$  merujuk kepada aci motor. Model dinamik sistem ini dibangunkan berdasarkan persamaan pergerakan Lagrange. Kewujudan graviti juga diambil kira di dalam model ini di mana ianya menyebabkan ketidak linearan pada sistem dalam bentuk sinus akibat tenaga keupayaan merujuk kepada graviti. Ketidak linearan dinamik model akan dilinearkan dengan andaian yang munasabah. Keputusan menyeluruh model disahkan melalui simulasi komputer dan keputusan tersebut akan dibandingkan dengan keputusan sedia ada. Penyelidikan terhadap dinamik model dalam sambutan masa dan frekuensi juga dilaksanakan. Selain itu, teknik rekaan untuk penyingkiran getaran untuk sistem juga dipersembahkan.

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

Robotic manipulators are widely used to help in dangerous, monotonous, and tedious jobs. Most of the existing robotic manipulators are designed and build in a manner to maximize stiffness in an attempt to minimize the vibration of the end effectors to achieve good position accuracy. This high stiffness is achieved by using heavy material and a bulky design. Hence, the existing heavy rigid manipulators are shown to be inefficient in terms of power consumption or speed with respect to the operating payload.

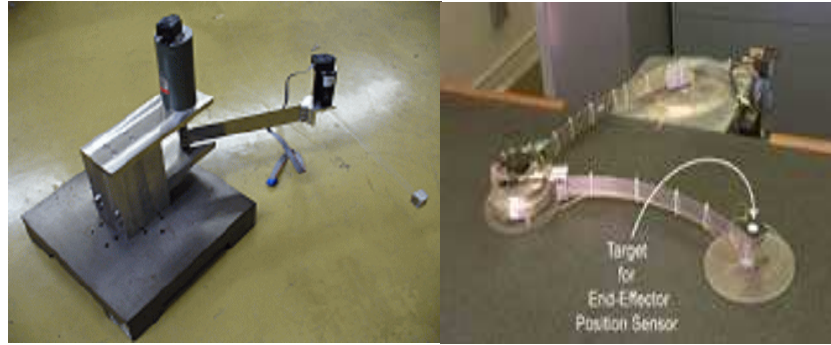
In addition, the operation of high precision robots is severely limited by their dynamic deflection, which persists for a period of time after a move is completed. The settling time required for this residual vibration delays subsequent operations, thus conflicting with the demand of increased productivity. These conflicting requirements between high speed and high accuracy have rendered the robotic assembly task a

challenging research problem. Also, many industrial manipulators face the problem of vibrations during high speed motion.

In order to improve industrial productivity, it is required to reduce the weight of the links and/or to increase their speed of operation. Due to lightweight requirements and high-speed operations, a comprehensive dynamic model that includes the link and/or joint flexibilities is highly needed. Link flexibility is a consequence of the lightweight constructional feature in manipulator arms that are designed to operate at high speed with low inertia. Joint flexibility arises because of the elastic behaviour of the joint transmission elements such as gears, chains and shafts. Compare to the conventional heavy and bulky robots, by introducing link and/or joint flexibility in the mechanical system of robots, its have the potential advantages of lower cost, larger work volume, higher operational speed, greater payload-to-manipulator-weight ratio, smaller actuators, lower energy consumption, better manoeuvrability, better transportability and safer operation due to reduced inertia. Figure 1.1 and Figure 1.2 show some flexible joint and flexible link devices which are used in the robotics application currently.



**Figure 1.1:** Flexible joint



**Figure 1.2:** Flexible link

However, the greatest disadvantage by introducing link and/or joint flexibilities to the robotic mechanical system is the vibration problem due to low stiffness. If the vibration problem cannot be solved, incorporate the flexible link and/or flexible joint in the robotic mechanical system have not been favoured in production industries since this problem will affect the accuracy of end point in response to input commands. In order to overcome this problem, an accurate dynamic model that can characterize with link and/or joint flexibility has to be developed. This is a first step towards designing an efficient control strategy for these manipulators.

The present work is devoted towards establishing dynamic model for a single link flexible manipulator with flexible joint for the case in which the link is oriented vertically. The flexible joint is modelled as a pair of springs and used to attach the link to the motor shaft. The output of the system is the tip angle, which is given the sum of the motor angle,  $\theta$  and the joint deflection  $\alpha$  with respect to the motor shaft. The dynamic model of the system is developed based on Lagrange's equations of motion.

The presence of the gravity is accounted in this model, which introduces a non-linearity into the system in the form of a sinusoid, as a result of the potential energy due



to gravity. The non-linear model of the dynamics will be linearized by assumption that the gravity is equal to zero. The resulting generalized model is validated with the existing results and simulate through computer simulations. The investigations on the dynamic model in terms of time and frequency responses are carried out. Besides, a design technique for a vibration suppression of the system is also presented.

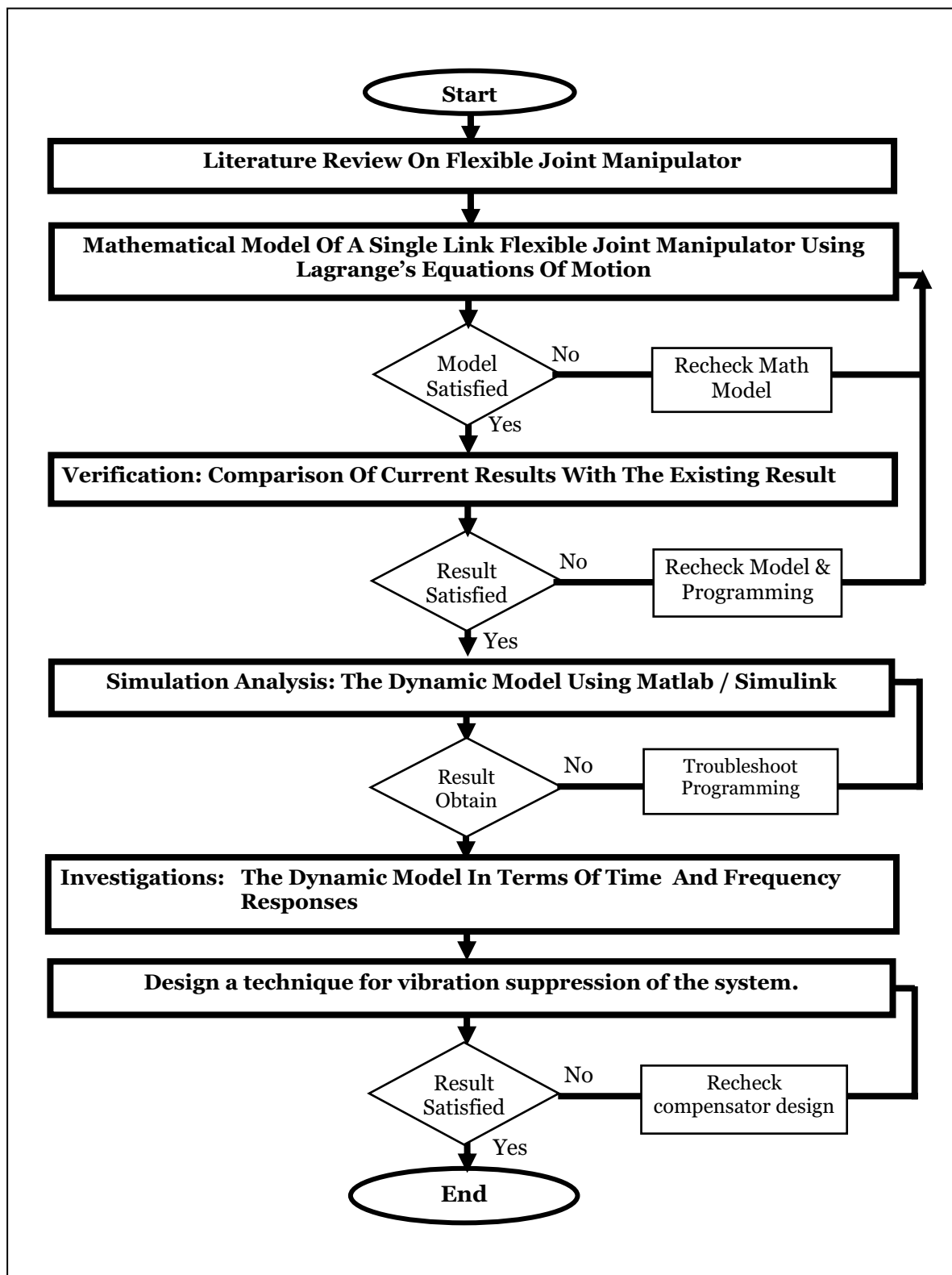
## **1.2 Objective**

The main objective of this project is to obtain a dynamic model of a single link flexible manipulator with flexible joint. Besides, this project has a purpose to study the dynamic characteristics of a single link flexible manipulator with flexible joint.

## **1.3 Scope of Work**

The scope of work is clearly define the specific field of the research and ensure that the entire content of this thesis is confined the scope. It is begun with the literature review on flexible joint manipulator. The next step is to develop the dynamic equations of motion for a single link flexible manipulator with flexible joint using Lagrange's equations of motion. At this stage, the mathematical model will be verified with the existing results.

Then using Matlab and Simulink, the dynamic model will be simulated and followed by the investigations of the dynamic model in terms of time and frequency responses. Finally, a technique for a vibration suppression of the system is designed. The scope of work can be described in terms of flowchart as shown in Figure 1.3.



**Figure 1.3:** Flowchart represents the scope of work

## 1.4 Outline of the Thesis

The thesis presents the implementation or possibly implementation with the improvement of the existing mathematical model of single link flexible manipulator with flexible joint.

Chapter 2 focuses on the literature review, which introduces the overview of flexible joint manipulators. The explanation begins with the previous researches on flexible joint manipulators. This chapter is then described by related researches on modelling of flexible joint manipulators, which is found to be related and facilitate to this project.

Chapter 3 provides the methodology that is used through out the work of this project. It covers the technical explanation of this project and derivation of the dynamic model mathematical equations using related formula. The model also been verified with the existing results.

Chapter 4 deals with the time response results and Matlab's simulation results of the dynamic model. The obtained results are being compared with the existing results for model validation.

Chapter 5 presents the conclusions of the project as well as some constructive suggestions for further development and the contribution of this project. The project outcome is concluded in this chapter. As for future development, some suggestions are highlighted with the basis of the limitation of the effectiveness mathematical equation and simulation analysis executed in this project.

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## LIST OF SYMBOLS

$\theta$	-	angular position of the motor
$\alpha$	-	angular displacement of the flexible joint
$V$	-	potential energy
$K$	-	kinetic energy
$V_g$	-	potential energy due to gravity
$V_s$	-	potential energy due to the springs
$K_h$	-	kinetic energy of the hub
$K_l$	-	kinetic energy of the load
$m$	-	mass of the shaft
$g$	-	gravity constant
$h$	-	height of the center of gravity of the link with respect to the rest position
$J_h$	-	inertia at the motor output
$J_l$	-	inertia of the arm
$K_s$	-	spring stiffness
$R$	-	arm anchor point
$d$	-	body “y” anchor point
$r$	-	the body “x” anchor point
$K$	-	spring stiffness
$F_r$	-	spring restoring force
$L$	-	spring length at rest
$K_m$	-	motor constant

$K_g$	-	gear ratio
$R_m$	-	motor resistance
$\omega$	-	angular velocity of the motor
$i$	-	armature current
$f_t$	-	natural frequency

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