THEORETICAL ANALYSIS FOR EFFECT OF FACET REFLECTIONS ON AMPLIFICATION AND NOISE CHARACTERISTICS IN SEMICONDUCTOR OPTICAL AMPLIFIERS

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ABSTRACT

In most cases, the semiconductor optical amplifiers (SOA) are used with anti-reflection coating for its facets to avoid laser oscillation. Providing effects by residual reflection at the facets are theoretically analyzed. When the reflection by the back facet exists, the SOA shows larger amount of the amplified spontaneous emission as well as the larger intensity noise. When the reflection by the front facet exists, amplification ratio is reduced. However, the spectral linewidth of the inputted optical signal is never changed by amplification in the SOA, even the facets have residual reflectivities.

Keywords— Semiconductor optical amplifier, facet mirror, amplification rate, noise, linewidth

1. INTRODUCTION

Semiconductor optical amplifier (SOA) has excellent features such as having large amplification rate and possible integration with laser or detector. The SOA consist of almost same structure with the semiconductor laser but is provided with anti-reflection coating for the facets to avoid the laser oscillation.

Meanwhile, theoretical analysis of the SOA became difficult by providing the anti-reflection coating, because the Langevin noise source can not be defined without concept of the longitudinal mode, which has been well defined in the laser cavity. Author of this paper proposed a theoretical model to define the longitudinal mode in the SOA without facet mirrors based on the quantum mechanical property of the spontaneous emission, and theoretically analyzed several features of the SOA such as the relative intensity noise (RIN) can be reduced and the spectral linewidth is hardly changed by the amplification [1],[2].

In this paper, operating characteristics such as the amplification rate, the intensity noise, the frequency noise and the spectrum linewidth in the SOA having finite reflectivity at facets are theoretical analyzed. Analyzing manner is basically same as that in Ref.[1] but is added several improvements and suitable approximations.

2. ANALYSING MODEL AND BASIC EQUATIONS

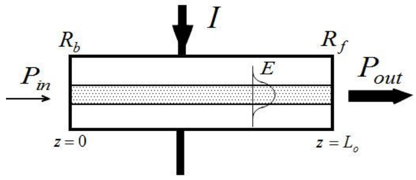

Fig.1 Structure of Semiconductor Optical Amplifier with Facet Mirrors

Structure of the SOA is illustrated in Fig.1. Length of the SOA is L_o . Thickness and width of the active region are d and w , respectively. Power reflectivity of the back and the front facets are R_b and R_a , respectively. Input optical power, output optical power and driving current of the SOA are P_{in} , P_{out} and I, respectively.

We express the electric field component $E(t,\boldsymbol{r})$ of the optical wave with the forward and the backward propagating component to be

$$E=\left\{\widetilde{A}^{(+)}(t,z)e^{-j\beta z}+\widetilde{A}^{(-)}(t,z)e^{j\beta z}\right\}\Phi(x,y)e^{j\omega t}+c.c. \quad (1)$$

where $\widetilde{A}^{(+)}(t,z)$ and $\widetilde{A}^{(-)}(t,z)$ are amplitudes of the forward and the backward waves, respectively. $\Phi(x,y)$ is a normalized field distribution function in the transverse cross section. ω is optical angular frequency and β is propagation constant. Variation of the amplitude is driven from the Maxwell's wave equation as

$$\pm\frac{\partial\widetilde{A}^{(\pm)}}{\partial z}+\frac{1}{v}\frac{\partial\widetilde{A}^{(\pm)}}{\partial z}=\frac{g-\kappa+j\varphi}{2}\widetilde{A}^{(\pm)}+\widetilde{U}^{(\pm)}(t,z) \ , \quad (2)$$

where v is velocity of the propagation given by

$$v=\frac{\beta}{\mu_o\varepsilon_{eff}\,\omega}\approx\frac{c}{n_{eff}} \quad (3)$$

with an effective refractive index n_{eff} . g , κ and φ are the gain coefficient, the loss coefficient and a coefficient for phase variation, respectively. $\widetilde{U}^{(\pm)}(t,z)$ indicates inclusion of the spontaneous emission.

We factorize the amplitudes to absolute value and phase term as

$$\widetilde{A}^{(\pm)}(t,z)=A^{(\pm)}(t,z)e^{j\theta^{(\pm)}(t,z)} \ , \quad (4)$$

Variation of these terms are given to be

$$\pm\frac{\partial A^{(\pm)}}{\partial z}+\frac{1}{v}\frac{\partial A^{(\pm)}}{\partial t}=\frac{g-\kappa}{2}A^{(\pm)}+\mathrm{Re}\left\{\widetilde{U}^{(\pm)}e^{-j\theta^{(\pm)}}\right\} \quad (5)$$

$$\pm\frac{\partial \theta^{(\pm)}}{\partial z}+\frac{1}{v}\frac{\partial \theta^{(\pm)}}{\partial t}=\frac{\varphi}{2}A^{(\pm)}+\frac{1}{A^{(\pm)}}\mathrm{Im}\left\{\widetilde{U}^{(\pm)}e^{-j\theta^{(\pm)}}\right\} \quad (6)$$

Carrying optical powers are given as

$$P^{(\pm)}(z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E^{(\pm)}H^{(\pm)}dxdy=2\sqrt{\frac{\varepsilon_{eq}}{\mu_o}}A^{(\pm)^2}(z) \quad . \quad (7)$$

Here, we introduce a length L_f with which orthogonality for modes induced by the spontaneous emission and the photon number $S^{(\pm)}$ are defined.

$$S^{(\pm)}(t,z)=\frac{2\varepsilon_{eff}L_f A^{(+)^2}(t,z)}{\hbar\omega}=\frac{L_f}{\hbar\omega v}P^{(\pm)}(t,z) \quad (8)$$

Variation of the photon number is given as

$$\frac{dS^{(\pm)}}{dt}=\frac{\partial S^{(\pm)}}{\partial t}+v\frac{\partial S^{(\pm)}}{\partial z}=v(g-\kappa)S^{(\pm)}+vg_e+F^{(\pm)}(t,z) \quad (9)$$

where the gain coefficient is approximately given as follows with electron density *n* and coefficients *a* and *b*, and is separated to components g_e and g_a corresponding to direction of electron transition between the conduction and the valence bands.

$$g=\xi\frac{a(n-n_g)}{1+b\overline{n}}\equiv g_e-g_a \quad (10)$$

$$g_e=\xi\frac{an}{1+b\overline{n}} \quad (11)$$

$$g_a=\xi\frac{an_g}{1+b\overline{n}} \quad (12)$$

Here ξ is the field confinement factor into the active region.

$$\xi=\int_{-d/2}^{d/2}\int_{-w/2}^{w/2}\left|\Phi(x,y)\right|^2 dxdy \quad . \quad (13)$$

The supposed length L_f is given by the coefficient g_e [1]

$$L_f=1/g_e \quad . \quad (14)$$

$F^{(\pm)}(t,z)$ in Eq.(9) is Langevin noise source for the optical emission.

Change of the electron density is given as

$$\frac{dn}{dt}=-\frac{vg\left\{S^{(+)}+S^{(-)}\right\}}{V_f}-\frac{n}{\tau}+\frac{I}{eV_o}+\frac{W(t,z)}{V_f} \quad . \quad (15)$$

where V_o is total volume of the active region

$$V_o=wdL_o \quad , \quad (16)$$

and V_f is a volume corresponding the supposed length L_f

$$V_f=wdL_f \quad . \quad (17)$$

$W(t,z)$ is another Langevin noise source for change of the electron density.

Variation of the optical phase is rewritten from Eq.(6) as

$$\frac{d\theta^{(\pm)}}{dt}=\frac{\partial\theta^{(\pm)}}{\partial t}\pm v\frac{\partial\theta^{(\pm)}}{\partial z}=\frac{v\alpha a'}{2}\{n(z)-\overline{n}(0)\}+T^{(\pm)}(t,z) \quad ,(18)$$

where α is so called linewidth enhancement factor and a' is a tangential coefficient of the gain given by

$$a'=\frac{\partial g}{\partial n}=\frac{\partial g_e}{\partial n}=\xi\frac{a}{1+b\overline{n}} \quad . \quad (19)$$

$T^{(\pm)}(t,z)$ is a noise source for variation of the optical phase.

3. FREQUENCY EXPANSION OF FLUCTUATING TERMS

The noise sources are expanded with noise frequency term

$$\Omega=2\pi f_n \quad (20)$$

in forms of traveling waves [3]

$$F^{(\pm)}(t,z)=\int_{-\infty}^{\infty}F_{\Omega}^{(\pm)}(z)e^{j\Omega(t\mp z/v)}d\Omega \quad (21)$$

$$T^{(\pm)}(t,z)=\int_{-\infty}^{\infty}T_{\Omega}^{(\pm)}(z)e^{j\Omega(t\mp z/v)}d\Omega \quad (22)$$

$$W(t,z)=\int_{-\infty}^{\infty}W_{\Omega}^{(\pm)}(z)e^{j\Omega(t\mp z/v)}d\Omega \quad (23)$$

Corresponding to these expansions, the photon number is expressed with DC (or CW) term and fluctuated term as

$$S^{(\pm)}(t,z)=\overline{S^{(\pm)}}(z)+\int_{-\infty}^{\infty}S_{\Omega}^{(\pm)}(z)e^{j\Omega(t\mp z/v)}d\Omega \quad . \quad (24)$$

Other values of the optical phase $\theta^{(\pm)}(t,z)$, the electron density $n(t,z)$, the gain coefficients $g(t,z)$ and $g_e(t,z)$ are similarly expressed with DC terms and fluctuated terms.

Auto correlation and mutual correlation of the noise sources are given as followings.

$$\left\langle F_{\Omega}^{(\pm)^2}\right\rangle=v\left\{\left(\overline{g}_e+g_a+\kappa\right)\overline{S^{(\pm)}}+\overline{g}_e\right\} , \quad (25)$$

$$\left\langle F_{\Omega}^{(\pm)}F_{\Omega}^{(\mp)}\right\rangle=v\left\{\left(\overline{g}_e+g_a+\kappa\right)\sqrt{\overline{S^{(+)}}\,\overline{S^{(-)}}}+\overline{g}_e\right\} , \quad (26)$$

$$\left\langle T_{\Omega}^{(\pm)^2}\right\rangle=\frac{\left\langle F_{\Omega}^{(\pm)^2}\right\rangle}{4\left\{\overline{S^{(\pm)}}+1/4\right\}^2} \quad , \quad (27)$$

$$\left\langle T_{\Omega}^{(\pm)}T_{\Omega}^{(\mp)}\right\rangle=\frac{\left\langle F_{\Omega}^{(\pm)}F_{\Omega}^{(\mp)}\right\rangle}{4\left\{\overline{S^{(+)}}+1/4\right\}\left\{\overline{S^{(-)}}+1/4\right\}} \quad , \quad (28)$$

$$\left\langle W_{\Omega}^{(\pm)^2}\right\rangle=\left\langle W_{\Omega}^{(\pm)}W_{\Omega}^{(\mp)}\right\rangle=\left\langle W_{\Omega}^{2}\right\rangle =v\left[\left(\overline{g}_e+g_a\right)\left\{\overline{S^{(+)}}+\overline{S^{(-)}}\right\}+\frac{\overline{n}V_f}{\tau}+\frac{V_f}{V_o}\frac{I}{e}\right] \quad (29)$$

$$\left\langle F_{\Omega}^{(\pm)}W_{\Omega}\right\rangle=\left\langle W_{\Omega}F_{\Omega}^{(\pm)}\right\rangle=-v\left\{\left(\overline{g}_e+g_a\right)\overline{S^{(\pm)}}+\overline{g}_e\right\} , \quad (30)$$

$$\left\langle n_{\Omega}^{(\pm)}T_{\Omega}^{(\pm)}\right\rangle=\left\langle n_{\Omega}^{(\pm)}T_{\Omega}^{(\mp)}\right\rangle=0 \quad . \quad (31)$$

By substituting Eqs.(21),(24) and corresponding equations of $n(t,z)$, $g(t,z)$ and $g_e(t,z)$ to Eq.(9), we get variations of the DC and the fluctuated components of the photon number to be

$$\pm\frac{\partial\overline{S^{(\pm)}}}{\partial z}=\left(\overline{g}-\kappa\right)\overline{S^{(\pm)}}+\overline{g}_e \quad , \tag{32}$$

$$\pm\frac{\partial S_\Omega^{(\pm)}}{\partial z}=\left(\overline{g}-\kappa\right)S_\Omega^{(\pm)}+a'\left\{\overline{S}^{(\pm)}+1\right\}n_\Omega^{(\pm)}+\frac{F_\Omega^{(\pm)}}{v} . \tag{33}$$

Similar substitutions to Eq.(15) gives

$$\frac{v\overline{g}\left\{\overline{S}^{(+)}+\overline{S}^{(-)}\right\}}{V_f}=\frac{I}{eV_o}-\frac{\overline{n}}{\tau} \quad , \tag{34}$$

$$n_\Omega^{(\pm)}=\frac{W_\Omega^{(\pm)}-v\overline{g}\left\{S_\Omega^{(\pm)}+S_\Omega^{(\mp)}e^{\pm 2j\Omega z/v}\right\}}{j\Omega V_f+V_f/\tau+va'\left\{\overline{S}^{(+)}+\overline{S}^{(-)}\right\}} . \tag{35}$$

Here, we suppose a relation to make simpler equation as

$$S_\Omega^{(\pm)}+S_\Omega^{(\mp)}e^{\pm 2j\Omega z/v}\approx S_\Omega^{(\pm)}\left\{1+\frac{\overline{S}^{(\mp)}+1}{\overline{S}^{(\pm)}+1}\right\}=S_\Omega^{(\pm)}\frac{\left\{\overline{S}^{(+)}+\overline{S}^{(-)}+2\right\}}{\overline{S}^{(\pm)}+1} \tag{36}$$

Substitution of Eqs.(35) and (36) to Eq.(33) gives

$$\pm\frac{\partial S_\Omega^{(\pm)}}{\partial z}=\left[\overline{g}-\kappa-\frac{va'\overline{g}\left\{\overline{S}^{(+)}+\overline{S}^{(-)}+2\right\}}{j\Omega V_f+V_f/\tau+va'\left\{\overline{S}^{(+)}+\overline{S}^{(-)}\right\}}\right]S_\Omega^{(\pm)}+\frac{a'\left\{\overline{S}^{(\pm)}+1\right\}W_\Omega^{(\pm)}}{j\Omega V_f+V_f/\tau+va'\left\{\overline{S}^{(+)}+\overline{S}^{(-)}\right\}}+\frac{F_\Omega^{(\pm)}}{v} \tag{37}$$

From Eq.(18), we get relations of

$$\pm\frac{\partial\overline{\theta}^{(\pm)}}{\partial z}=\frac{\alpha a'}{2}\left\{\overline{n}(z)-\overline{n}(0)\right\} \quad , \tag{38}$$

$$\pm\frac{\partial\theta_\Omega^{(\pm)}}{\partial z}=\frac{\alpha a'}{2}n_\Omega^{(\pm)}+\frac{T_\Omega^{(\pm)}}{v} . \tag{39}$$

4. BOUNDARY CONDITIONS AT FACETS

The forward and backward waves are connected at the facets. At the back facet R_b at $z=0$

$$\widetilde{A}^{(+)}(t,0)=\sqrt{R_b}\,\widetilde{A}^{(-)}(t,0)+\widetilde{A}_{in}(t) \quad , \tag{40}$$

where $\widetilde{A}_{in}(t)$ is amplitude of the inputted optical signal. At the front facet R_f at $z=L_o$

$$\widetilde{A}^{(-)}(t,L_o)e^{j\beta L_o}=\sqrt{R_f}\,\widetilde{A}^{(+)}(t,L_o)e^{-j\beta L_o} . \tag{41}$$

The inputted optical signal is also factorized as

$$\widetilde{A}_{in}(t)=A_{in}(t)e^{j\theta_{in}(t)} . \tag{42}$$

We substitute Eqs.(4) and (42) to Eqs.(40) and (41) and get relations among the absolute values and phases as

$$A^{(+)}(t,0)=\sqrt{R_b}\,A^{(-)}(t,0)+A_{in}(t) \ , \tag{43}$$

$$\theta^{(+)}(t,0)=\frac{\sqrt{R_b}\,A^{(-)}(t,0)\theta^{(-)}(t,0)+A_{in}(t)\theta_{in}(t)}{A^{(+)}(t,0)} \ , \tag{44}$$

$$A^{(-)}(t,L_o)=\sqrt{R_f}\,A^{(+)}(t,L_o) , \tag{45}$$

$$2\beta L_o+\theta^{(-)}(t,L_o)-\theta^{(+)}(t,L_o)=2p\pi , \tag{46}$$

where p is an integer indicating the longitudinal mode in a laser cavity.

Now, we rewrite the absolute values of the amplitude to the photon number. The photon number of the inputted optical signal is

$$S_{in}(t)=\frac{2\varepsilon_{eff}L_f}{\hbar\omega}A_{in}^2(t)=\frac{(1-R_b)L_f}{\hbar\omega v}P_{in}(t) . \tag{47}$$

Variables $A^{(\pm)}(t,z)$ and $S_{in}(t)$ are expanded with DC terms and fluctuated terms and are substituted to Eqs.(43) to (46), resulting in,

$$\sqrt{\overline{S}^{(+)}(0)}=\sqrt{R_b\,\overline{S}^{(-)}(0)}+\sqrt{\overline{S}_{in}} \quad , \tag{48}$$

$$\frac{S_\Omega^{(+)}(0)}{\sqrt{\overline{S}^{(+)}(0)}}=\sqrt{\frac{R_b}{\overline{S}^{(-)}(0)}}\,S_\Omega^{(-)}+\frac{S_{in\Omega}}{\sqrt{\overline{S}_{in}}} \ , \tag{49}$$

$$\overline{S}^{(-)}(L_o)=R_f\,\overline{S}^{(+)}(L_o) \quad , \tag{50}$$

$$S_\Omega^{(-)}(L_o)=R_f\,S_\Omega^{(+)}e^{-2j\Omega L_o/v} \ , \tag{51}$$

$$\overline{\theta}^{(+)}(0)=\sqrt{\frac{R_b\overline{S}^{(-)}(0)}{\overline{S}^{(+)}(0)}}\,\overline{\theta}^{(-)}(0)+\sqrt{\frac{\overline{S}_{in}}{\overline{S}^{(+)}(0)}}\,\overline{\theta}_{in} \ , \tag{52}$$

$$\theta_\Omega^{(+)}(0)=\sqrt{\frac{R_b\overline{S}^{(-)}(0)}{\overline{S}^{(+)}(0)}}\,\theta_\Omega^{(-)}(0)+\sqrt{\frac{\overline{S}_{in}}{\overline{S}^{(+)}(0)}}\,\theta_{in\Omega} , \tag{53}$$

$$\theta_\Omega^{(-)}(L_o)=\theta_\Omega^{(+)}(L_o) . \tag{54}$$

5. INTENSITY AMPLIFICATION

5.1. DC Terms

We suppose $\overline{n}$, $\overline{g}$ and $\overline{g}_e$ are spatially uniform in the z direction for simple treatment. Then variations of DC terms of the photon number are from Eq.(32) to be

$$\overline{S}^{(+)}(z)=\frac{\overline{g}_e}{\overline{g}-\kappa}\left\{e^{(\overline{g}-\kappa)z}-1\right\}+\overline{S}^{(+)}(0)e^{(\overline{g}-\kappa)z} \ , \tag{55}$$

$$\overline{S}^{(-)}(z)=\frac{\overline{g}_e}{\overline{g}-\kappa}\left\{e^{(\overline{g}-\kappa)(L_o-z)}-1\right\}+\overline{S}^{(-)}(L_o)e^{(\overline{g}-\kappa)(L_o-z)} . \tag{56}$$

By tracing one round trip of the optical wave in the SOA with Eqs.(48),(50),(55) and (56), the photon number of the forwad wave at the front facet is obtained to be,

$$\overline{S}^{(+)}(L_o)=\frac{B_2+\sqrt{B_2^2-B_1B_3}}{B_1} \quad , \tag{57}$$

where

$$B_1\equiv\left\{1-R_bR_f\overline{G}^2\right\}^2 \ , \tag{58}$$

$$B_2\equiv\left\{1-R_bR_f\overline{G}^2\right\}\left\{1+R_b\overline{G}\right\}\overline{Y}+\left\{1+R_bR_f\overline{G}^2\right\}\overline{G}\,\overline{S}_{in} \ , \tag{59}$$

$$B_3\equiv\left\{1+R_b\overline{G}\right\}^2\overline{Y}^2+2\left\{1-R_b\overline{G}\right\}\overline{Y}\,\overline{G}\,\overline{S}_{in}+\left\{\overline{G}\,\overline{S}_{in}\right\}^2 \ , \tag{60}$$

$$\overline{G}\equiv e^{(\overline{g}-\kappa)L_o} \ , \tag{61}$$

$$\overline{Y}\equiv\frac{\overline{g}_e}{\overline{g}-\kappa}\left\{\overline{G}-1\right\} . \tag{62}$$

Output optical power from the SOA is given as

$$P_{out}=\frac{(1-R_f)\hbar\omega v}{L_f}\overline{S}^{(+)}(L_o) . \tag{63}$$

Here, we determine spatially averaged value of the photon numbers by denoting with $=$ such as,

$$\bar{\bar{S}}^{(\pm)} \equiv \frac{1}{L_o}\int_0^{L_o} \bar{S}^{(\pm +)}(z)dz \ . \tag{67}$$

By using these spatially averaged values, Eq.(34) is rewritten as

$$\frac{I}{eV_o} = \frac{\bar{n}}{\tau} + \frac{v\bar{g}}{V_f}\left\{\bar{\bar{S}}^{(+)} + \bar{\bar{S}}^{(-)}\right\} \ . \tag{68}$$

Numerical calculation is performed to find suitable value of $\bar{n}$ and photon umbers by using Eq.(57) to (68) for given current value I.

5.2. Intensity Fluctuated Terms and Intensity Noise

For calculation of fluctuated terms, we suppose that DC terms of $\bar{n}$, $\bar{g}$, $\bar{g}_e$ and $\bar{\bar{S}}^{(\pm)}$ are spatially uniform for simple calculation.

To analyze Eq.(37), we rewrite as followings:

$$X_\Omega \equiv \bar{g} - \kappa - \frac{v a' \bar{g}\left\{\bar{\bar{S}}^{(+)} + \bar{\bar{S}}^{(-)} + 2\right\}}{Y_f} \ , \tag{69}$$

$$C_\Omega^{(\pm)} = \frac{a'\left\{\bar{\bar{S}}^{(\pm)} + 1\right\} W_\Omega}{Y_f} + \frac{F_\Omega^{(\pm)}}{v} \ , \tag{70}$$

$$Y_f = j\Omega V_f + \frac{V_f}{\tau} + v a' \left\{\bar{\bar{S}}^{(+)} + \bar{\bar{S}}^{(-)}\right\} \ . \tag{71}$$

Furthermore, we suppose that the noise sources W_Ω and $F_\Omega^{(\pm)}$ are spatially uniform. Then, solutions of Eq.(37) become

$$S_\Omega^{(+)}(z) = \frac{C_\Omega^{(+)}}{X_\Omega}\left\{e^{X_\Omega z} - 1\right\} + S_\Omega^{(+)}(0)e^{X_\Omega z} \ , \tag{72}$$

$$S_\Omega^{(-)}(z) = \frac{C_\Omega^{(-)}}{X_\Omega}\left\{e^{X_\Omega (L_o - z)} - 1\right\} + S_\Omega^{(-)}(0)e^{X_\Omega (L_o - z)} \ . \tag{73}$$

By tracing one round trip propagation with Eqs.(49), (51), (72) and (73), we get

$$S_\Omega^{(+)}(L_o) = \frac{Y_\Omega^{(+)} + R_b G_\Omega Y_\Omega^{(-)} + G_\Omega \sqrt{\bar{S}^{(+)}(0)/\bar{S}_{in}}\, S_{in\Omega}}{Y_s} \ , \tag{74}$$

with

$$Y_s = 1 - R_b R_f G_\Omega^2 \exp\left(-2j\Omega L_o / v\right). \tag{75}$$

We rewrite the term $Y_\Omega^{(+)} + R_b G_\Omega Y_\Omega^{(-)}$ with the noise sources, and takes auto-correlated value as,

$$\begin{aligned}
&\left\langle S_\Omega^{(+)2}(L_o)\right\rangle \\
&= \Big\{ |K_1|^2 \left\langle W_\Omega^2\right\rangle + 2\,\mathrm{Re}\left(K_1 K_2^*\right)\left\langle W_\Omega F_\Omega^{(+)}\right\rangle \\
&\quad + 2\,\mathrm{Re}(K_1 R_b G_\Omega^* K_2^*)\left\langle W_\Omega F_\Omega^{(-)}\right\rangle \\
&\quad + |K_2|^2 \left\langle F_\Omega^{(+)2}\right\rangle + 2|K_2|^2 R_b \,\mathrm{Re}(G_\Omega)\left\langle F_\Omega^{(+)} F_\Omega^{(-)}\right\rangle \\
&\quad + R_b^2 |G_\Omega K_2|^2 \left\langle F_\Omega^{(-)2}\right\rangle + |G_\Omega|^2 \frac{\bar{S}^{(+)}(0)}{\bar{S}_{in}}\left\langle S_{in\Omega}^2\right\rangle \Big\} \div |Y_s|^2
\end{aligned} \tag{76}$$

where

$$K_1 = \frac{a'(G_\Omega - 1)\left[\bar{\bar{S}}^{(+)} + 1 + R_b G_\Omega \left\{\bar{\bar{S}}^{(-)} + 1\right\}\right]}{Y_f X_\Omega} \ , \tag{77}$$

$$K_2 = \frac{G_\Omega - 1}{v X_\Omega} \ . \tag{78}$$

The relative intensity noise (RIN) for the inputted optical power is defined as

$$RIN_{in} = \frac{\left\langle P_{in\Omega}^2\right\rangle}{\bar{P}_{in}^2} = \frac{\left\langle S_{in\Omega}^2\right\rangle}{\bar{S}_{in}^2} \ , \tag{79}$$

and RIN for the output optical power is evaluated by

$$RIN_{out} = \frac{\left\langle P_{out\Omega}^2\right\rangle}{\bar{P}_{out}^2} = \frac{\left\langle S_\Omega^2(L_o)\right\rangle}{\bar{S}^2(L_o)} \ . \tag{80}$$

6. PHASE FLUCTUATED TERM, FREQUENCY NOISE AND LINEWIDTH

Variation of the phase fluctuation has been given in Eq.(39). If we can suppose that fluctuations $n_\Omega^{(\pm)}$ and $T_\Omega^{(\pm)}$ are almost spatially uniform along, changed value of the phase fluctuation is simply written as

$$\Psi^{(\pm)} \equiv \pm\int_0^{L_o} \frac{\partial \theta_\Omega^{(\pm)}}{\partial z} dz \approx \left\{\frac{\alpha a'}{2} n_\Omega + \frac{T_\Omega^{(\pm)}}{v}\right\} L_o \ , \tag{81}$$

By substituting this equation to the boundary conditions of Eqs.(53) and (54), we get value of the phase to be

$$\theta_\Omega^{(+)}(L_o) = \frac{\Psi^{(+)} + \sqrt{R_b \frac{\bar{S}^{(-)}(0)}{\bar{S}^{(+)}(0)}}\,\Psi^{(-)} + \sqrt{\frac{\bar{S}_{in}}{\bar{S}^{(+)}(0)}}\,\theta_{in\Omega}}{Y_p} \ , \tag{82}$$

with

$$Y_p = 1 - \sqrt{R_b \frac{\bar{S}^{(-)}(0)}{\bar{S}^{(+)}(0)}}\, e^{-2j\Omega L_o / v} \ . \tag{83}$$

The frequency fluctuation is given by time derivative of the phase to be

$$\begin{aligned}
\frac{\partial \theta^{(\pm)}(t,z)}{\partial t} &= \int_{-\infty}^{\infty} j\Omega\, \theta_\Omega^{(\pm)}(z)\, e^{j\Omega(t - z/v)} d\Omega \equiv 2\pi f^{(\pm)} \\
&= 2\pi \left\{\bar{f}^{(\pm)} + \int_{-\infty}^{\infty} f_\Omega^{(\pm)}(z)\, e^{j\Omega(t - z/v)} d\Omega\right\}
\end{aligned} \ . \tag{84}$$

Therefore, the frequency fluctuation (noise) becomes,

$$\begin{aligned}
\left\langle f_\Omega^{(+)2}(L_o)\right\rangle = \Bigg\{ \left(\frac{\Omega L_o}{2\pi v}\right)^2 \Bigg[&\left(\frac{v\alpha a'}{2}\right)^2 \left\{1 + \sqrt{R_b \frac{\bar{S}^{(-)}(0)}{\bar{S}^{(+)}(0)}}\right\}^2 \left\langle n_\Omega^2\right\rangle \\
&+ \left\langle T_\Omega^{(+)2}\right\rangle + R_b \frac{\bar{S}^{(-)}(0)}{\bar{S}^{(+)}(0)}\left\langle T_\Omega^{(-)2}\right\rangle \\
&+ 2\sqrt{R_b \frac{\bar{S}^{(-)}(0)}{\bar{S}^{(+)}(0)}}\left\langle T_\Omega^{(+)} T_\Omega^{(-)}\right\rangle \Bigg] \\
&+ \frac{\bar{S}_{in}}{\bar{S}^{(+)}(0)}\left\langle f_{in\Omega}^2\right\rangle \Bigg\} \div |Y_p|^2
\end{aligned} \tag{85}$$

The auto-correlated value the electron density fluctuation $\left\langle n_{\Omega}^2 \right\rangle$ is also obtained from Eqs.(35) and (76).

The spectrum linewidth is given with the frequency noise as limit of $\Omega \to 0$ to be

$$\Delta f = 4\pi \left\langle f_0^{(+)^2}(L_o) \right\rangle \quad . \tag{86}$$

7. NUMERICAL CALCULATION

As the model of numerical calculation an InGaAsP quantum well structure is supposed. Used parameters are $\lambda = 1.55\mu\text{m}$, $L_o = 1.0\text{mm}$, $w = 2.0\mu\text{m}$, $d = 40\text{nm}$, $\tau = 0.86\text{ns}$, $n_{eff} = 3.5$, $n_g = 2.0\times10^{24}\text{m}^{-1}$, $\alpha = 3.0$, $\kappa = 3.0\times10^3\text{ m}^{-1}$, $\xi = 0.035$, $a = 1.345\times10^{-19}\text{m}^2$, $b = 3.583\times10^{-25}\text{m}^3$, $RIN_{in} = 1.0\times10^{-15}\text{Hz}^{-1}$ and $\left\langle f_{in\Omega}^2 \right\rangle = 1.0\times10^5\text{Hz}$.

7.1. Amplification of DC optical power

Variation of the DC output power $\overline{P}_{out}$ with driving current I without input optical power $\overline{P}_{in} = 0$ is shown in Fig.2. Origin of this output power is the spontaneous emission because there is no input power. The case having finite reflectivities at both the back and the front facets show the laser oscillation with threshold currents of I_{th}= 62mA for $R_b = R_f = 0.3$, and I_{th}= 128mA for $R_b = R_f = 0.01$. The case of $R_b = 0.3$ with $R_f = 0$ is not the laser oscillation but reveals large output power due to the amplified spontaneous emission (ASE) which is generated in backward direction and reflected by the back facet mirror R_b and amplified more in the forward propagation.

Variation of the amplification rate $\overline{P}_{out}/\overline{P}_{in}$ with input optical power $\overline{P}_{in}$ is shown in Fig.3. Larger value of the amplification rate for smaller input power comes from the inclusion of the ASE as shown in Fig.2 and is not true amplification rate. Larger input power range than $\overline{P}_{in} = 200\mu\text{W}$ could be trusted. Situation of $R_b = R_f = 0.3$ show reduction of the amplification rate, because the amplification gain $\overline{g}$ is clamped at the threshold level due to the laser oscillation. Situation of $R_b = 0$ but $R_f = 0.3$ is not suitable to get larger amplification rate, because output power is reduced by the front facet mirror R_f.

Variation of the amplification rate $\overline{P}_{out}/\overline{P}_{in}$ with driving current I for $\overline{P}_{in} = 1\text{mW}$ is shown in Fig.4. It is clear that situation of $R_b = R_f = 0$ is the best, and $R_b = R_f = 0.3$ and $R_b = 0$ but $R_f = 0.3$ are worse. Situations of $R_b = R_f = 0.01$ and $R_b = 0.3$ with $R_f = 0$ are not bad as the amplification rate.

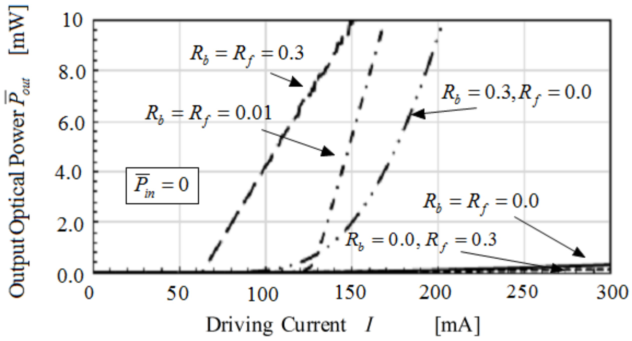

Fig.2 DC output power vs. driving current when the input optical power is zero.

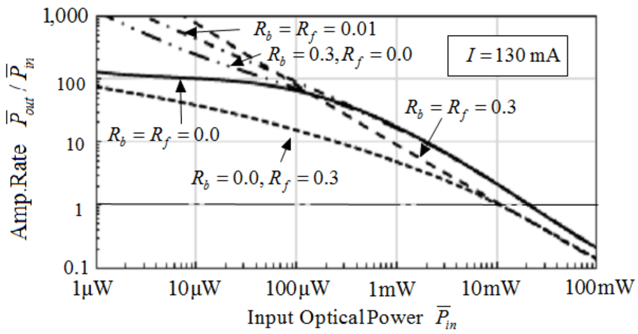

Fig.3 Amplification rate vs. input optical power

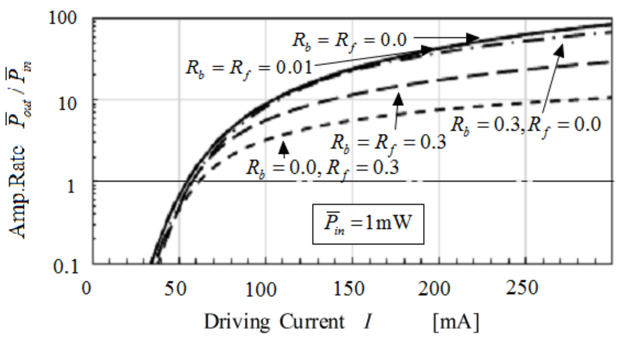

Fig.4 Amplification rate vs. driving current

7.2. Intensity Noise

Noise frequency spectrum when the optical input is $\overline{P}_{in} = 1\text{mW}$ with $RIN_{in} = 10^{-15}\text{Hz}^{-1}$ is shown in Fig.5. RIN_{out} can be reduced from input RIN level. This effect comes from that amplification gain for fluctuated component becomes smaller than that for DC component as found in Eqs.(32) and (37).

Variations of RIN_{out} for the input optical power $\overline{P}_{in}$ and driving current I are shown in Figs.6 and 7. We found that RIN_{out} can be reduced from RIN_{in} for all cases if input optical power and the driving current are large enough.

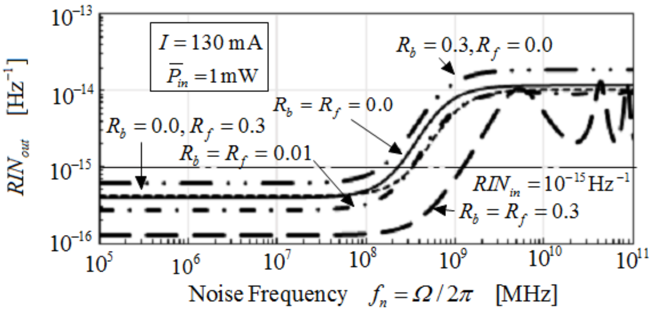

Fig.5 Frequency spectrum of intensity noise when input optical power is 1mW

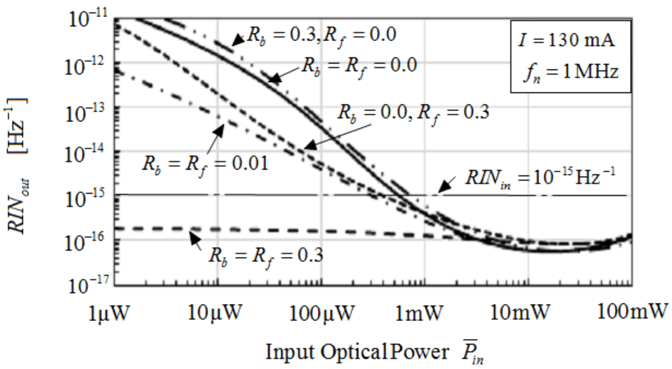

Fig.6 Relative intensity noise vs. input optical power

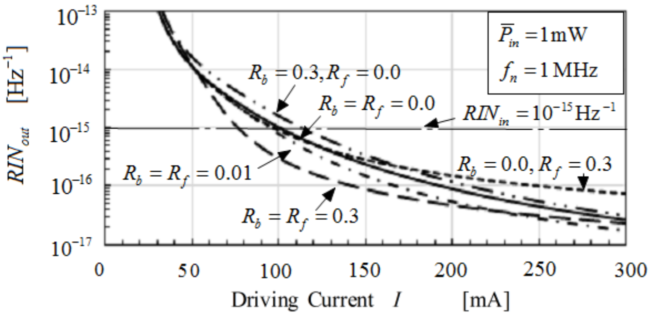

Fig.7 Relative intensity noise vs. driving current

However, we should be careful that situation $R_b = R_f = 0.3$ is in the laser oscillation and can not use as an amplifier as given in Figs.2 to 4. Situation $R_b = 0.3$ with $R_f = 0$ show larger RIN_{out} than that of $R_b = R_f = 0$ because fluctuated power emitted backward direction is reflected by the back facet and amplified in the forward propagation.

7.3. Frequency Noise

Frequency spectrum when $\overline{P}_{in} = 1\text{mW}$ with $\left\langle f_{in\Omega}^2 \right\rangle = 10^5\,\text{Hz}$ is given in Fig.8. The frequency noise does not change from that of the input optical signal at lower frequency region than several 100 MHz for all type SOA. Moreover, the fluctuation of the electron density $\left\langle n_\Omega^2 \right\rangle$ less affect on the frequency noise of the SOA even the SOA is in laser oscillation given by $R_b = R_f = 0.3$ in Fig.8. This is very different character compare with the free running laser oscillation.

The spectral linewidth of the output light from the SOA is given in Fig.9. The linewidth is never change though amplification in the SOA, as found by putting $\Omega = 0$ in Eq.(85).

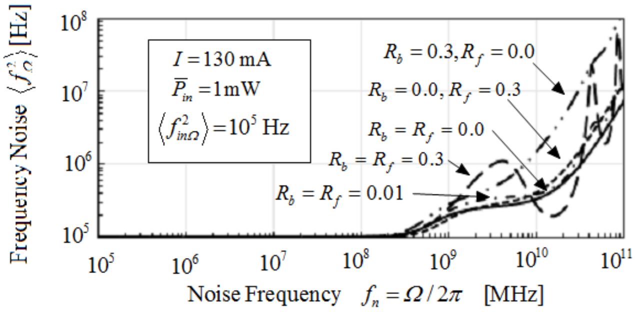

Fig.8 Frequency spectrum of frequency noise when input optical power 1mW.

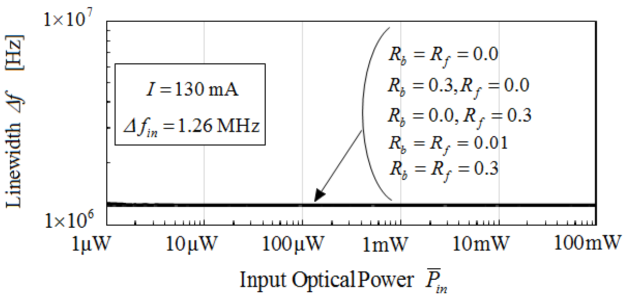

Fig.9 Spectrum linewidth

8. CONCLUSION

Effect of facet reflections on amplification and noise characteristics in the SOA are theoretically analyzed. To get larger amplification with higher power, the residual reflectivity at amplifier facets should be reduced. However, residual reflectivities are not terrible problem to keep specific features of the SOA, which are the RIN level can be reduced and the linewidth is hardly change by the optical amplification in the SOA.

REFERENCES

[1] M. Yamada, “Analysis of Intensity and Frequency Noise in Semiconductor Optical Amplifier”, *IEEE J.Quantum Electron.*, vol.48, no.8, pp.980-990, 2012

[2] M. Yamada, N. Takeuchi, K. Sakumoto and Y.Kuwamura, “Variation of Relative Intensity Noise With Optical Power InGaAsP Semiconductor Optical Amplifier”, *IEEE Photon. Technol. Lett.*, vol.24, no.9, pp.980-990, 2012

[3] A. W. S. Putra, K. Makinoshima, M. Yamada, T. Takamura and S. Ambran, “Investigation of Frequency Noise and Spectrum Linewidth in Semiconductor optical Amplifier”, International Laser Technology and Optics Symposium, 13rd & 14th Oct. 2015, Johor Bahru.