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INVESTIGATION OF WATER BREAKTHROUGH TIME IN NON-COMMUNICATING LAYERED RESERVOIR

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ABSTRACT

The original Buckley-Leverett fractional flow formula has been extended and more detailed formulation of waterflooding behavior in a multilayered system is presented. In this paper, the layers are assumed to communicate only in the wellbores, and the reservoir may be represented as linear system. Most previous investigations of this nature were limited by assumptions. This study improves on previous work by applying Buckley-Leverett displacement theory to a noncommunicating layered reservoir where permeability, porosity and thickness vary from layer to layer except the oil-water relative permeability and oil viscosity are assumed the same for all layers. Gravity and capillary pressure effects are neglected. These particular considerations have been given to the evaluation of breakthrough time for each layer as a function of cumulative water injection into that layer at the breakthrough. To verify the modified method, calculations were performed a three layered reservoir for three different cases of mobility ratios, then compared with Prats *et al's* method. It is shown that the breakthrough times in the layer with the lowest permeability-thickness product (*kh*) are in very good agreement with Prats *et al's* method. However, breakthrough times for the layer with the highest *kh* are slightly different from Prats *et al*'s method.

Key Words: *Buckley-Leverett Theory, Immiscible Displacement, Two-Phase Flow, Water Breakthrough, and Improved Oil Recovery.*

1.0 INTRODUCTION

Field experience with immiscible displacement usually shows constant producing conditions until breakthrough of the displacing fluid. Then oil production continues at increasing displacing-to-displaced fluid ratios until the economic limit is reached. Three different ideal mechanisms are known that will produce this behavior: (1) relative permeability effect as described by Buckley and Leverett fractional flow formula and rate of frontal advance formula [1], (2) permeability heterogeneity in the vertical stratification and injecivity as considered by Dykstra and Parsons [2], Prats *et al* [3] and others, and (3) different path lengths involved in a real (two-dimensional) flow between wells as described by Dyes *et al* [4]. Without equations, a combination of these factors modified by formation heterogeneity and other known and unknown factors actually does control the behavior of real system. The method presented in this article incorporate the Buckley and Leverett fractional flow formula for a single reservoir with the Dykstra and Parsons concept for multilayered reservoir to determine the breakthrough time for each layer.

Kufus and Lynch [5] presented work which can incorporate Buckley and Leverett theory in the Dykstra and Parsons calculations. Important assumptions Kufus and Lynch have made were that all layers have same relative permeability curves to oil and water and water injection rate in each layer is constant value and dependent only on the absolute permeability and on

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fraction of average water relative permeability to average fractional flow in the current layer, which is made similar to Dykstra and Parsons model. The data presented in this paper were valid only for viscosity ratio of unity.

Rustam [6] modified the Dykstra and Parsons method for 1-D oil displacement by water in such a manner that it would be possible to incorporate the Buckley and Leverett theory. This modification was based on the Buckley and Leverett theory related to two phase homogeneous, horizontal reservoir consisting of two non-communicating layers with different absolute permeabilities. Major assumptions Rustam made were that all layers have different oil-water relative permeability and water injection rate for each layer, a constant pressure gradient across all layers, immiscible incompressible displacement and no capillary or gravity forces.

Dykstra and Parsons [2] presented one of the earliest applications of this model for waterflooding performance. In addition, they assumed that the initial saturations, relative permeabilities were the same for each layer, porosity was the same, displacement was piston-like, fluids were incompressible and injection into each layer was proportional to that layer's permeability capacity. Snyder and Ramey [7] improved on previous work by Buckley and Leverett displacement theory to a noncommunicating layered system where permeability, porosity, initial saturation, residual saturation and relative permeability vary from layer to layer. Snyder and Ramey considered two-phase flow in the displaced region and the injection rate into a layer was proportional to the layer's permeability capacity.

These techniques originate from Buckley and Leverett's work [8.9.10] and consist of prediction methods for waterfloods in stratified formations where each layer has defined homogeneous properties. Each layer's performance is calculated separately, estimating the total or joint performance of the operation with the contribution of each layer's solution.

2.0 DESCRIPTION OF THE METHOD

A model similar to that of Prats *et al* was used in this study. The reservoir was considered to be composed of three layers that communicate only at the wellbores. Each layer is individually homogeneous, but may be different from every other layer. The following properties were allowed to vary between layers: absolute permeability, porosity and thickness. The following assumptions were made: (1) constant width and length for all layers, (2) negligible capillary and gravity forces, (3) constant pressure drop for all layers at a given time, (4) constant oil-water relative permeability for all layers, (5) constant total injection rate for the reservoir (for ease of comparison with other method), (6) water enters each layer in direct proportion to its capacity, *kh*, (7) uniform initial water saturation, and (8) there is no cross-flow between layers.

2.1 Fractional flow formula

The fractional flow formula has been used for many years by reservoir engineers to predict waterflooding performance. Basically, this method assumes that (1) a flood front exist, (2) no water moves ahead of the front, and (3) oil and water move behind the front. If the throughput is constant and the capillary gradient and gravity effects are neglected, the fractional flow equation becomes:

$$f_w = \frac{1}{1 + (k_{ro}/k_{rw})(\mu_w/\mu_o)} \quad (1)$$

Three variables should be considered in determining the fraction of the total fluid flow that consists of water: the viscosity ratio, saturation, and relative permeability ratio. Of these, the viscosity ratio in a given case is essentially constant under the usual waterflooding conditions. The relative permeability ratio is a function of water saturation. The fractional flow of water is, therefore, a function of water saturation, and a curve relating f_w and S_w may easily be determined. The average water saturation $S_{w\ avg}$ behind the flood front at breakthrough can be obtained by constructing a tangent to the f_w versus S_w curve through the initial water saturation, S_{wi}, and reading $S_{w\ avg}$ at $f_w = 1$. (see Figure 1). The water saturation at the outlet face $Max\ S_w$, can be read from the curve at the point of tangency. The mobility ratio M is defined as the ratio of the water mobility at residual oil saturation to the oil mobility at residual water saturation.

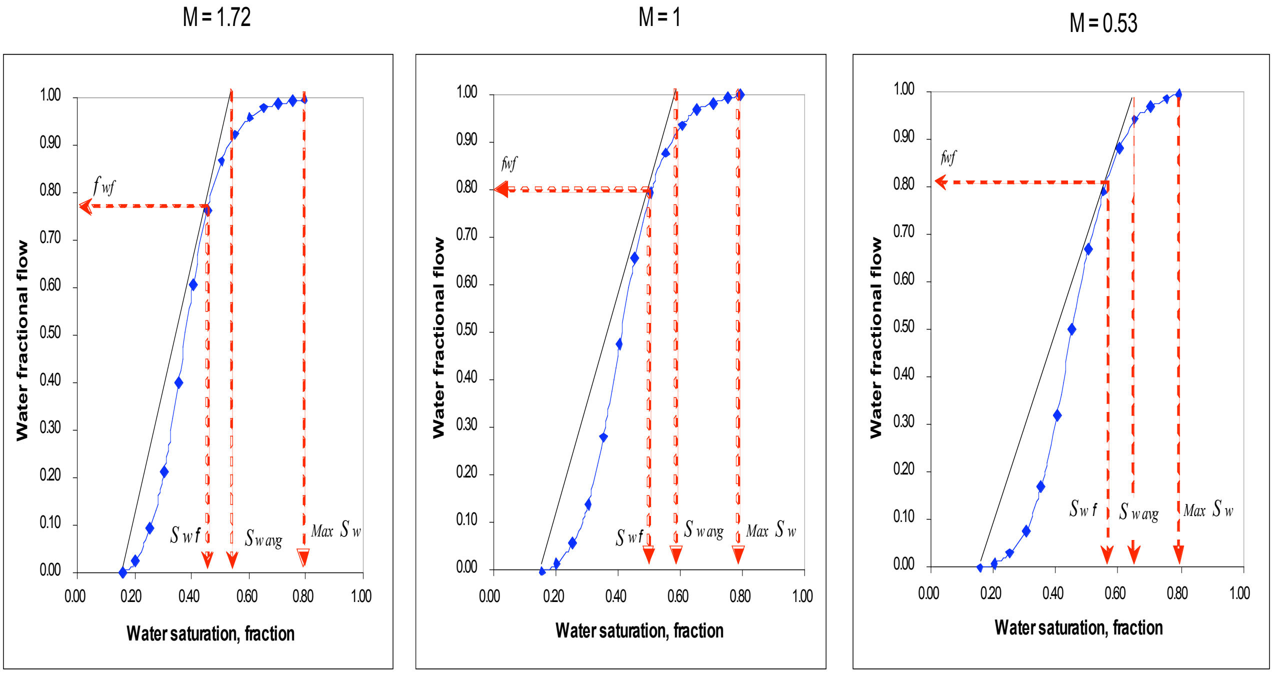

Figure 1 Analysis of the fractional flow curves at three different cases of mobility ratios

2.2 Determination of the Breakthrough Time

Since the saturation at the flood front and the average water saturation behind the flood front were evaluated, the change in the fraction of water flowing with the change in the saturation at flood front $(\Delta f_w / \Delta S_w)_{Swf}$ can be determined. The relative permeability to each phase, oil viscosity and pressure differential across the flow path are assumed to be the same for each layer. Therefore, the water saturation at the flood front and the average saturation behind the flood front should be the same for each layer. Accordingly, the time required for the frontal saturation in a particular layer to reach the producing well can be determined by evaluating the cumulative water injection (W_{ijf}) as a function of the layer pore volume and slope on the fractional flow curve at the breakthrough saturation.

$$W_{ijf} = \frac{7758 A_j h_j \phi_j}{(\Delta f_w / \Delta S_w)_{Swf}} \tag{2}$$

In Equation 2, A_j, h_j, and $Ø_j$ represent the cross sectional area, thickness, and porosity for layer *j* respectively and $(\Delta f_w / \Delta S_w)_{Swf}$ represents the slope of the fractional flow at the flood front saturation which would be the same for each layer. Therefore, the time required for frontal saturation in layer *j* to reach the producing well as a function of cumulative water injection into that layer is calculated as follows:

$$\text{Breakthrough time for layer } j = \frac{\text{Cumulative water injection into layer } j \quad \text{(bbl)}}{\text{Total injection rate (bbl/day)}} \tag{3}$$

The basic reservoir and fluid properties for the waterflood cases are given in **Table 1** and **Table 2** respectively. However, the reservoir properties for each layer were the same for each method in order to facilitate checking our results with Prats *et al's* method.

Table 1: Reservoir and fluid properties, three layers waterflood model

		Case 1	Case 2	Case 3
M	Mobility ratio	1.72	1	0.53
L, ft	Reservoir length	660	660	660
W, ft	Reservoir width	660	660	660
P, psi	Reservoir pressure	2489	2489	2489
μ_o, cp	Oil viscosity	0.723	0.423	0.223
μ_w, cp	Water viscosity	0.270	0.270	0.270
i_w, STB/D	Injection rate	1382	1382	1382
S_{wi}	Initial water saturation	0.153	0.153	0.153
S_{or}	Residual oil saturation	0.210	0.210	0.210
k_{rw} @ S_{or}	Water relative permeability @ S_{or}	0.630	0.630	0.630
k_{ro} @ S_{wi}	Oil relative permeability @ S_{wi}	0.980	0.980	0.980

Table 2 Layers properties

	Layer 1	Layer 2	Layer 3
Average porosity	28%	21%	14.5%
Average permeability	430 md	224 md	110 md
Thickness	21 ft	61 ft	4 ft

3.0 RESULTS AND DISCUSSION

The time required for the frontal saturation to reach the producing phase (breakthrough time) in layered reservoir was studied using the method previously outlined. However, the fractional flow derivation at the breakthrough saturation $(\Delta f_w / \Delta S_w)_{Swf}$ was obtained graphically from fractional flow curve by taking the slope for that saturation. Consequently, the prototype of five-spot is considered to be made of three layers in which all properties vary between layers except relative permeability to each phase and oil viscosity. The results of this method (hereafter called Buckley-Leverett solution) are compared with results obtained by using Prats *et al's* method at three different cases of mobility ratio and presented in Table 3 and Table 4.

The cumulative water injection at breakthrough time into each layer as function of the fractional flow of water and the layer pore volume were calculated by using Equation 2. The results of these calculations were compared with Prats *et al*'s method at three different cases of mobility ratios and reported in Table 3. In general, the results obtained by the modified method are in good agreement with Prats *et al's* results. In case where the mobility ratios were 1 and 1.72, the cumulative water injection for each layer obtained by the modified method is always lesser than Prats *et al's* method, while, the cumulative water injection calculated by the modified method is slightly higher than the Prats *et al's* method when the mobility ratio was 0.53.

Table 3 Comparison of the cumulative water injection in STB at breakthrough time between the modified method and Prats *et al* method

	M = 1.72		**M = 1**		**M = 0.53**	
	Modified method	Prats *et al* method	Modified method	Prats *et al* method	Modified method	Prats *et al* method
Layer 1	123450	126962	139423	142274	158707	157267
Layer 2	273153	280811	308371	314677	351022	347839
Layer 3	12182	12523	13753	14034	15733	15512
Total	398785	420296	461547	470985	525462	520618

The breakthrough time for three layers homogeneous cases based on the modified method were in close agreement with Prats *et al* method for three different cases of mobility ratios. The results presented in Table 4 shows that as the mobility ratio becomes less than one, the results obtained by the modified method become more approximate to the Prats *et al's* method. For instance, when the mobility ratio was 0.53, the results obtained for each layer from the modified method were very similar to the other method. However, for cases where the mobility ratio was unity, the total breakthrough time obtained by the modified method was 7 days lesser than Prats *et al's* method. While in cases where the mobility ratio was 1.72, the total breakthrough time calculated by the modified method was 9 days greater than the other method respectively.

Observing this trend, it can be concluded that the breakthrough times calculated by the modified method for layer one and three which consist of 21 ft and 4 ft respectively of the total thickness were in very good agreement with results obtained by Prats *et al's* method. However, the difference in the total breakthrough times obtained for each method were mainly due to the Layer two which consists of the highest value of the total thickness 61 ft. This difference is probably due to the high thickness of this layer.

Table 4 Comparison of the breakthrough time in days between the modified method and Prats *et al* method

	M = 1.72		**M = 1**		**M = 0.53**	
	Modified method	Prats *et al* method	Modified method	Prats *et al* method	Modified method	Prats *et al* method
Layer 1	89	92	101	103	115	114
Layer 2	197	203	223	228	254	252
Layer 3	9	9	10	10	11	11
Total	295	304	334	341	380	377

Based on the modified method results given in Table 4, the highest mobility ratio gives much earlier breakthrough time compared to other two cases of mobility ratio. The differences of these changes can be attributed to the difference in mobility of each phase, especially when the mobility of the displacing fluid is much higher than the displaced fluid. However, in cases where the mobility ratio was 1.72, the time required for the frontal saturation to reach the producing well or to advance 933 ft (the distance between producing and injection wells) would be around 295 days. This breakthrough time is significant if compared to 39 days increase required time in unity mobility ratio cases or 28 days increase when the mobility ratio was 0.53.

4.0 CONCLUSIONS

For water breakthrough time calculations, the results obtained from the improved Buckley-Leverett's fractional flow equation gave a very good agreement with the Prats *et al*'s results. The results obtained from this study show that previous immiscible design using Buckley-Leverett's theory for a single reservoir together with Dykstra-Parsons concept for a layered reservoir was probably applicable. However, the main attractive capability of this approach is that it can handle the same oil-water relative permeability for each layer. As the value of the mobility ratio becomes less than one, the results obtained from the modified method matched closely those of Prats *et al's* method.

REFERENCES

[1] Buckley S.E., M.C. Leverett. 1942. Mechanism of fluid displacement in sands. *Trans. AIME 1942*, 146: 107-116.

[2] Dykstra H, R.L. Parsons. 1950. The Prediction of Oil Recovery by Waterflooding. *Secondary Recovery of Oil in the United States. Second edition*, API 1950: 160-174.

[3] Prats M., C.S. Matthews, R.L. Jewett, J.D. Baker. 1959. Prediction of injection rate and production history for multifluid five-spot floods. *Trans. AIME* 1959, 216: 98-105.

[4] Dyes A.B., B.H. Caudle, R.A. Erickson. 1954. Oil production after breakthrough-as influenced by mobility ratio. *Trans. AIME* 1954, 201: 81-86.

[5] Kufus H.B, E.J. Lynch. 1959. Linear frontal displacement in multi-layer sands. *Prod. Monthly*. 24: 12, 32.

[6] Rustam R.G. 2005. Modification of the Dykstra-Parsons Method to Incorporate Buckley-Leverett Displacement Theory for Waterfloods. Master's Thesis. Texas A&M University, Petroleum Engineering Department.

[7] Snyder R.W., H.J. Ramey Jr. 1967.Application of Buckley and Leverett displacement theory to noncommunicating layered systems. *Journal of Petroleum Technology.* 1500-1506.

[8] Campbell C.J. April 2001. Peak oil: A turning for mankind. *M. King Hubbert Center for Petroleum Supply Studies. Hubbert Center Newsletter.* 2001/2-1. Petroleum Engineering Department, Colorado School of Mines Golden, CO..

[9] Campbell C.J. July 2002. Forecasting global oil supply 2000-2050. *M. King Hubbert Center for Petroleum Supply Studies. Hubbert Center Newsletter* 2002/3-1. Petroleum Engineering Department, Colorado School of Mines, Golden, CO.

[10] Conway E. June 2004.There's Enough Oil Left to Last for 40 Years, Says BP.. *Telegraph Group Limited,* UK.