

# TUNNEL LINER SUBJECTED TO EXCAVATION LOADING

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## RINGKASAN

*Satu kajian makmal telah dibuat untuk menganggar momen-momen, daya-daya tujah dan cangga'an dalam suatu dinding terowong yang dikenakan 'beban korekan'. Kesan-kesan kedalaman timbusan ke atas daya tujah, momen dan cangga'an telah dikaji dan keputusan-keputusan dibandingkan dengan penyelesaian-penyelesaian yang dikemukakan oleh Burns dan Richards (1964) dan oleh Morgan (1961) dan penyelesaian secara 'finite element' oleh Mohraz dll. (1975). Penyelidik menyatakan bahawa adanya persetujuan diantara nilai-nilai beban dari ujikaji dengan perhitongan permukaan Morgan (1961) adalah baik dan cadangan kedua boleh digunakan untuk mereka-bentuk dinding-dinding terowong yang tertimbus dengan dalamnya dan dikenakan oleh beban korekan. Dalam masalah timbusan cetek, bagaimanapun, agakan untuk daya tujah, momen dan cangga'an boleh dikira dengan menggunakan persamaan-persamaan untuk dinding tegar.*

## ABSTRACT

*A laboratory study is conducted to estimate moments, thrusts and deformation in a tunnel liner subjected to 'excavation loading'. The effects of thrust, moment and deformation of the depth of burial are examined and the results are compared with the solutions offered by Burns and Richard (1964) and by Morgan (1961) and the finite element solutions of Mohraz et al. (1975). The researcher concludes that the agreement between the experimentally determined load values and those calculated from Morgan (1961) is so good that the latter are applicable to the design of tunnel liners to be deeply buried and subjected to excavation loading. In problems of shallow cover, however, reliable estimates of thrust, moment and deflection can be obtained by using rigid liner equations.*

## Introduction

Because of an increasing demand for urban transit, the design of tunnel support systems has received considerable attention in recent years. At one extreme, 'perfectly flexible liners' are assumed to interact fully with the medium. Such liners do not have to be designed for moments, but they should be designed for full thrust consistent with the initial stress distribution in the medium. This procedure is usually used in designing steel liners. At the other extreme, 'rigid liners' are designed as rigid structures for an assumed set of external loads. The designer assumes that no interaction takes place between the liner and the medium. This procedure is used most often for the design of concrete liners (Peck, 1969).

In most tunnel construction, the tunnel is excavated before the liner is placed. Reasonable estimates of forces and deformations in the liner may be obtained by assuming that the liner is initially unstressed and in contact with the medium directly above it. This loading condition is referred to as 'gravity loading'. If the liner can be inserted into the medium without strain and deformation and the medium inside then excavated, the load experienced by the liner is referred to as 'excavation loading'. Several general solutions for loads on tunnel liners have been proposed (Burns & Richard, 1964; Hoeg, 1968; and Morgan, 1961); virtually no data exist, however, to problems consistent with excavation loading. An experimental program is undertaken to substantiate whether the above equations would be applicable in analyzing tunnel liners subjected to excavation loading.

## Expressions for moment, thrust and deflection

If the tunnel liner is completely rigid, excavation of the soil inside the liner would effect neither the deformation of the liner nor the surrounding soil. As a reasonable approximation the average ring load (thrust) would be (Peck, 1969)

$$T = \frac{1}{2} (1 + K_0) \gamma HR \quad (1)$$

in which  $\gamma$  = unit weight of the soil;  $H$  = depth of soil cover to the tunnel springline;  $R$  = tunnel radius; and  $K_0$  = ratio of the free-field horizontal to vertical soil stresses (i.e., coefficient of earth pressure at rest). The moment,  $M$  at the crown or invert and the springline would be

$$M_c, M_s = \pm \frac{1}{4} (K_0 - 1) \gamma HR^2 \quad (2)$$

If the liner were completely flexible the excavation of the soil inside the tunnel would result in deformation of the liner until the vertical and lateral pressures acting on the liner equalize, or until the tunnel collapsed (Peck, 1969).

All liners are of intermediate stiffness, neither rigid nor flexible.

The liner stiffness with respect to soil stiffness controls the amount of deformation and results in reduction of moment for any given loading condition. Morgan (1961) derived expressions for thrust and moment in the liner in terms of medium and liner properties and liner deflection as follows:

$$M_c = -M_s = P_o R^2 EI (1 + \nu) / 6EI (1 + \nu) + 2R^3 E_c \quad (3)$$

$$T_s = \frac{2}{3} (P_o R) + 2 \left( \frac{P_o R^4 E}{18E I_l (1 + \nu) / (1 - \nu_l^2) + 6R^2 E} \right) \quad (4)$$

$$T_c = \frac{1}{3} (P_o R) + 4 \left( \frac{P_o R^4 E}{18E I_l (1 + \nu) / (1 - \nu_l^2) + 6R^2 E} \right) \quad (5)$$

in which  $\delta$  = deflection;  $E_l$  = modulus of elasticity of the liner, medium;  $I_l$  = moment of inertia per unit length of the liner;  $\nu_l$  = Poisson's ratio of the liner, medium;  $p_o = \gamma H$ ; and  $R$  = radius of the liner.

The properties of the medium have a pronounced effect on the behavior of the tunnel liner. The recent analytical work of Burns & Richard (1964) and Hoeg (1968) can be used to assess quantitatively the stiffness of a liner relative to a soil medium. In these studies, the relative stiffness of the liner to the medium is characterized by two ratios designated as compressibility ratio and flexibility ratio, expressed by

$$C = \frac{\{E / (1 + \nu) (1 - 2\nu)\}}{(E_l t) / (1 - \nu_l^2) R} \quad (6)$$

$$F = \frac{\{E / (1 + \nu)\}}{(6E I_l) / (1 - \nu_l^2) R^3} \quad (7)$$

in which  $t$  = effective thickness of the liner. Burns & Richard (1964) showed that the moments developed in the liner are primarily functions of the flexibility ratio  $F$  and the coefficient of earth pressure at rest,  $K_o$ ; while the thrust depends on the compressibility ratio  $C$  and  $K_o$ . Their equations, which are valid for a deeply buried tunnel only, are given below. For crown or invert, springline

$$T_c, T_s = \frac{1}{2} \left\{ (1 + K_o) b_1 \mp \frac{1}{3} (1 - K_o) b_2 \right\} \gamma H R^2 \quad (8)$$

$$M_c, M_s = \pm \frac{1}{6} (1 - K_o) b_2 \gamma H R^2 \quad (9)$$

$$W_c, W_s = \frac{1}{2} \left\{ (1-\nu) (1+K_o) b_1 C \right. \\ \left. \pm \frac{2}{3} \frac{1-\nu}{1-2\nu} (1-K_o) b_2 F \right\} \frac{\nu HR}{E_c} \quad (10)$$

in which  $b_1 = 1 - a_1$ ;  $b_2 = 1 + 3a_2 - 4a_3$ ;  $a_1 = \frac{\{(1-2\nu)(C-1)\}}{\{(1-2\nu)C + 1\}}$ ;  $a_2 = \frac{(2F+1-2\nu)}{(2F+5-6\nu)}$ ;  $a_3 = \frac{(2F-1)}{(2F+5-6\nu)}$ ; and  $E_c = \frac{E}{(1-\nu)} / (1+\nu)(1-2\nu)$ , the constrained modulus of the medium.

## EXPERIMENTAL STUDY

The experiment was designed to simulate closely the manner in which the tunnel liner deforms as a result of the pressure reduction (due to soil removal) during construction. The test bin consisted of a 1.2 m x 1.2 m x 2.4 m high box with 12.7 mm thick plywood wall (Fig. 1). The hollow cylindrical plexiglass liner 0.8 m long, 114 mm in diameter and 5.5 mm thick had one end sealed; the other end had a porthole for lead wires and air connections. Strain gages were mounted circumferentially at mid-length of the cylinder on both the inner and outer surfaces of the crown, invert and springline. The readings from each set of back-to-back gages enabled the strains in the liner to be resolved into direct and flexural strains. The diametral changes of the cylinder were measured by the two LVDTs.

### Test Procedure

One foot of sand, which served as bedding material, was poured in and compacted prior to placing the liner tube as shown in Fig. 1. Sand that provided the overburden pressure was poured around the liner and compacted to approximately uniform density. In order that the medium around the liner would be stress-free during the loading with sand, appropriate air pressure was applied inside the liner. The two LVDTs were monitored to insure that the liner retained its original circular shape during loading. When the fill reached the specific height, H, air pressure inside the liner was gradually reduced to zero, and the researcher recorded the strains and displacements before and after the release.

## RESULTS AND DISCUSSION

The interior crown and invert as well as the exterior springline

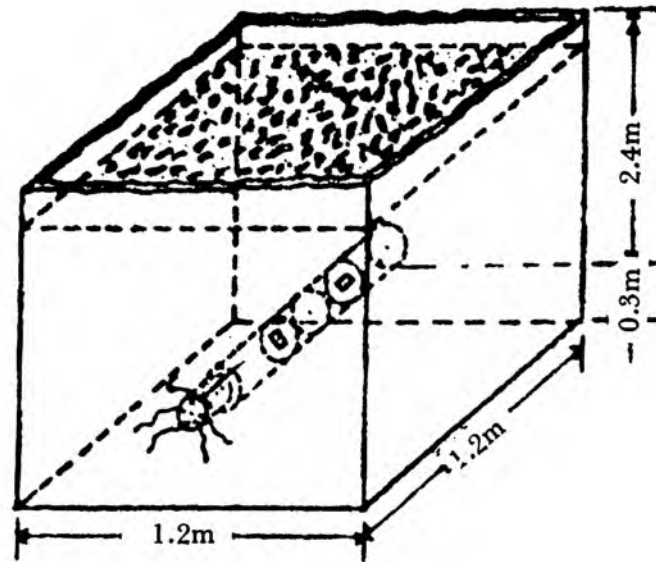


Fig. 1 Test Bin and Model Liner

gages recorded tensile strains, while the other four gages measured compressive strains. The measured strains were resolved into direct and bending strains, by use of which respectively, the thrusts and moments in the liner were calculated as follows:

$$T = \frac{1}{2} E_l A (\epsilon_1 + \epsilon_2) \quad (11)$$

$$M = E_l I (\epsilon_2 - \epsilon_1) / t \quad (12)$$

where  $A$  = cross-sectional area per unit length of the liner;  $\epsilon_1$ ,  $\epsilon_2$  = interior and exterior strains.

The modulus of elasticity of the medium  $E$  which is assumed to vary with the depth is obtained directly from the relation (Wu, 1966)

$$E(\text{kPa}) = 1960 + 420 \sigma'_z \quad (13)$$

where  $\sigma'_z$  (kPa) =  $K_o \gamma H$ . Angle of friction of the sand is assumed to be  $34^\circ$ , the soil unit weight  $\gamma = 15.7 \text{ kN/m}^3$ , the Poisson's ratio  $\nu = 0.362$ , and modulus of elasticity of the liner,  $E = 31 \times 10^6 \text{ kPa}$ .

### Thrust on the Liner

The liner thrusts, calculated using the equations of Morgan (Eqs. 4 and 5), and of Burns & Richard (Eq. 8), as well as Eq. 11, are ana-

lyzed as a function of depth of cover. The analytical as well as experimental liner thrusts are larger than those computed by Eq. 1; however, the former are in good agreement with the approximate solution due to Peck et al. (1972).

$$\frac{T}{\gamma HR} = \frac{1}{2}(1+K_o)(1.2-0.2C) \quad (14)$$

The graphs shown in Fig. 2 are relations between thrust coefficient,  $T/\gamma HR$ , and the dimensionless depth of cover,  $H/D$ . In all solutions, no significant change in the coefficient is observed when  $H/D$  exceeds 10. Comparing the thrust coefficients obtained from various theories one finds that Burns and Richard's equations give the most conservative values.

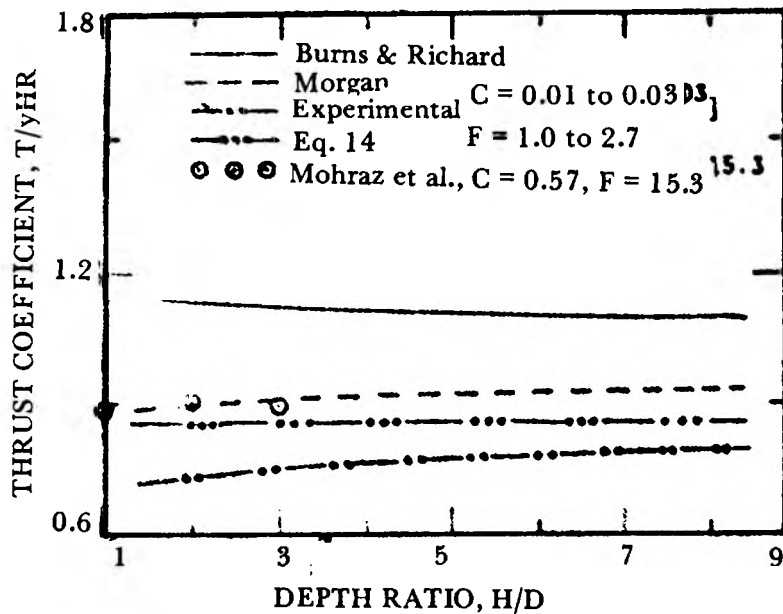


Fig. 2 Variation of Thrust Coefficient (Springline) with Depth of Cover

## Moments

The moments developed in the liner are plotted with respect to the depth of cover (Fig. 3). Equation 2, which assumes no interaction between the liner and the medium, gives a linear variation of moment with respect to  $H$ . In the initial stages when  $H$  is small, all of the curves for the moments, i.e., calculated according to Eq. 9 (Burns and Richard), Eq. 3 (Morgan), and Eq. 12, are close to curve 1 showing little or no interaction. As  $H$  increases, the moments (curves

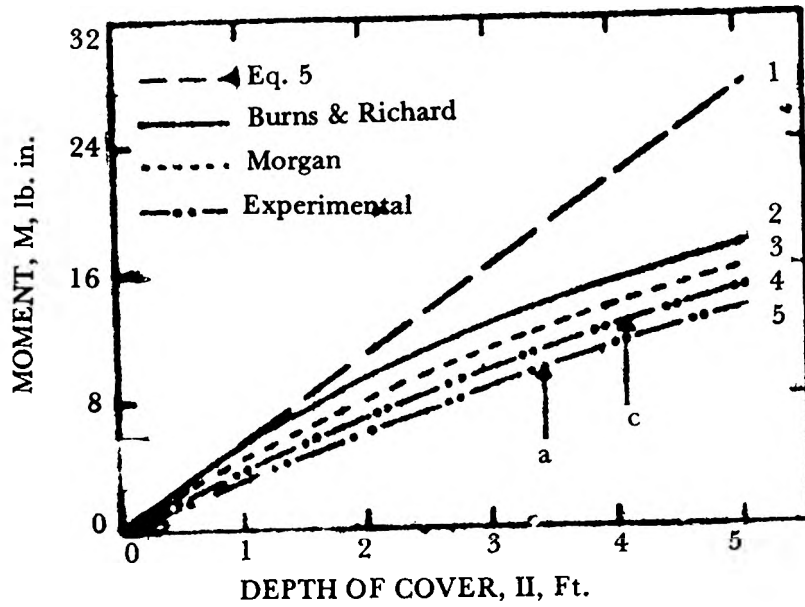


Fig. 3 Variation of Moment with Depth of Cover

2,3,4 and 5) deviate from the linear relation of Eq. 2. The reduction of moment indicates interaction taking place between the liner and the medium.

The moment coefficients calculated using the equations of Morgan and of Burns & Richard are not only larger than the experimental values, but also decrease faster with H/D (Fig. 4). This difference

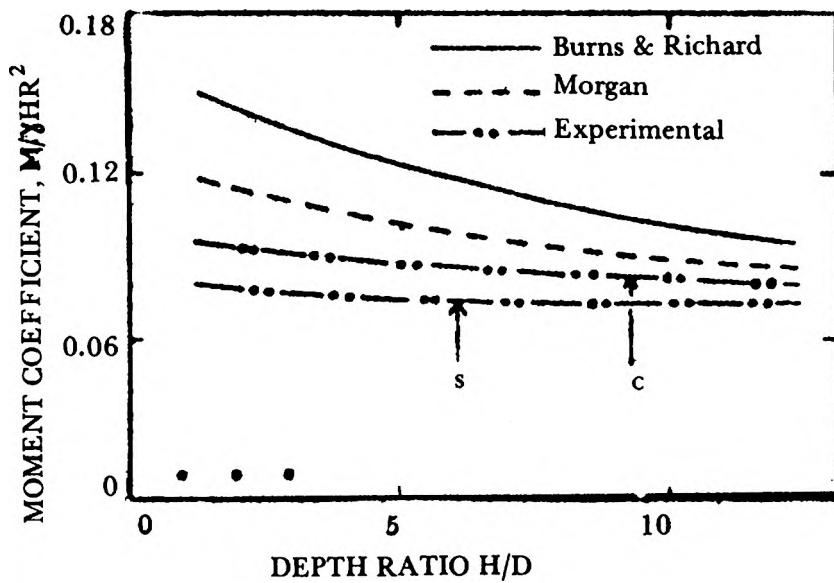


Fig. 4 Variation of Moment Coefficient with Depth of Cover.

may be attributed in part to the inability to maintain the circular shape of the liner during loading. Some ovaling of the liner was observed during the application of surcharge, thereby inducing passive pressure on the sides of the liner and restricting further strains and deformations upon releasing the air pressure.

The observation that the moment coefficients computed from the experimental results are nearly ten times those reported by Mohraz et al. can be explained. The bending moment in a liner is primarily governed by the liner flexibility. The results of Fig. 5 attest

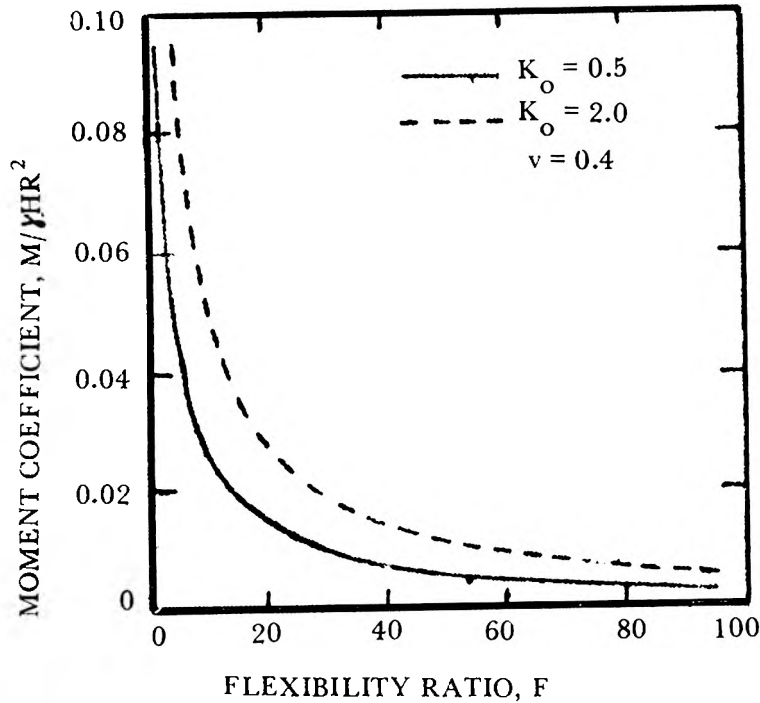


Fig. 5 Variation of Moment Coefficient with Flexibility Ratio (Peck et al, 1972).

to this fact. As shown in Fig. 2, the flexibility ratio is 2.0 for the present experiment and 15.3 for the results obtained by Mohraz et al. (1975). Figure 5 reveals that as the flexibility ratio decreases from 15.3 to 2.0, the moment coefficient increases ten-fold.

### Deflection

The calculated deflections from Burns & Richard's equations (Eq. 10) as well as the experimental values are plotted with respect to the depth of cover (Fig. 6). The variation of deflection computed using the ring equation (Eq. 15) is labelled curve 1.

$$\delta = (1 - K_o) \gamma HR^4 / 12 EI \quad (15)$$



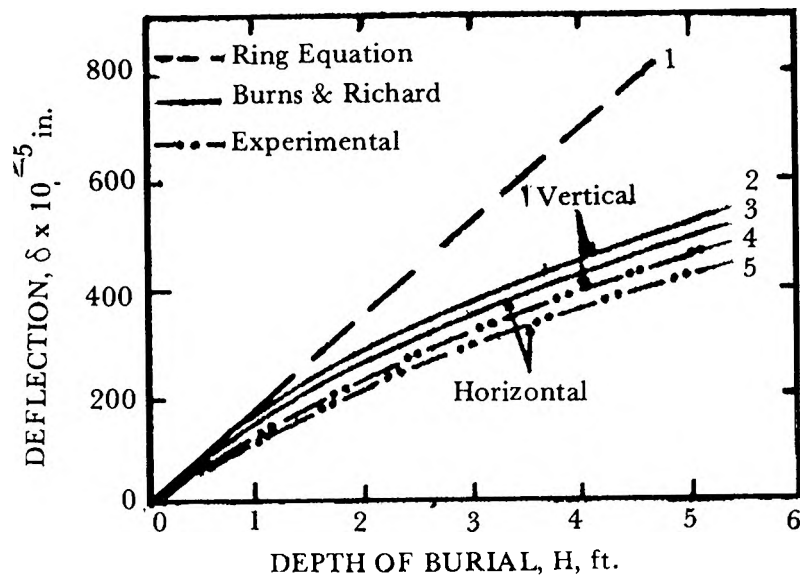


Fig. 6 Variation of Deflection with Depth of Cover.

The deflection curves 2, 3, 4 and 5 are observed to be close to the ring equation curve 1 at low values of H; however, as H increases, the former curves deviate from the latter, an indication of interaction between the liner and the medium. In both sets of data, the vertical deflections are larger than the horizontal deflections. A comparison of the experimental results and those obtained from Burns & Richard reveals good agreement between the two. The approximate expression of Peck et al. (1972) somewhat underpredicts the experimental diameter changes of the liner.

## CONCLUSIONS

1. Compressibility ratio and flexibility ratio are important parameters in defining tunnel liner stiffness.
2. In tunnel liners of shallow cover ( $H/D < 2$ ), reliable estimates of thrust, moment and deflection can be obtained using rigid liner equations.
3. General agreement exists between the experimentally determined moment, thrust and deflection and those calculated using the equations of Morgan. These equations are recommended for the design of deeply buried tunnel liners subjected to excavation loading.
4. Satisfactory estimates of thrust and diameter changes of tunnel liners can be made using Peck et al. modified equations.

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